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From the Library of
Elmer Adelbert Lyman, A.B.
1886
Instructor in Mathematics
1890-1898

QA
101
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1837

THE
COMMON SCHOOL
ARITHMETIC:

IN WHICH THE

RULES ARE EXPLAINED IN THE PLAINEST AND MOST
CONCISE METHODS EXTANT,

WITH

MANY IMPORTANT IMPROVEMENTS,

CONTAINING

PROOFS IN EACH RULE, WITH DEMONSTRATIONS FROM THE MOST SIMPLE
AND EVIDENT PRINCIPLES; TOGETHER WITH REASONS FOR EVERY
RULE, ACCOMPANIED WITH A SERIES OF QUESTIONS ON
THE NATURE AND APPLICATION OF THE SAME.

TO WHICH IS PREFIXED

AN ALPHABETICAL INDEX TO RULES AND SUBJECTS,

AND ADDED

A DICTIONARY OF ARITHMETICAL TERMS,
NOT FOUND IN ANY OTHER TREATISE.

BY P. E. BATES BOTHAM.

SEVENTH EDITION REVISED AND IMPROVED.

HARTFORD: CONN.
PUBLISHED BY HENRY BENTON.

1837

QA
101
B75
1837

Notice. The Author of this Treatise (Mr. Botham) has a practical knowledge of the subject on which he has written: he has been in the business of teaching, in this, and the adjoining States for a number of years. He has made himself *thoroughly* conversant with *more* than SIXTY different treatises on Arithmetic including recent publications in the United States;—besides, those of England, France and Germany; and is not ignorant of Eular, Lacroix, Day and Colburn's Algebras. Moreover, he has used Daboll, Colburn, Adams, Smith and Ruger's books in teaching Arithmetic, and has attentively observed the progress of his pupils in each book, *by which means*, he found *each* and *all of these books*, in some respects faulty, deficient, or redundant. He now presents to the public the result of his observations and practical experience under the title of "*The Common School Arithmetic*," which book you are requested to carefully and impartially examine, *then* decide whether it is the "best and cheapest Arithmetic" ever offered to the public for the use of our Common Schools.

N. B.—In this book there are no Rules or Questions for solution copied from Daboll, Pike, Dilworth or Bonnycastle, *which* FACT cannot be said of any other like publication, *however recent.*

Entered, according to Act of Congress, in the year 1832, by Henry Benton, in the Clerk's Office of the District Court of Connecticut.

Hist. of sci.
Mrs E. A. Lyman
6-26-36
32490

DEDICATION.

THIS WORK IS RESPECTFULLY SUBMITTED

TO THE

TEACHERS IN THE UNITED STATES.

Gentlemen. In deciding upon the adoption of this work, your Brother trusts, that there is but one point for investigation: *Is this* Arithmetic *a true and faithful report of the principles of the science, and of the Federal Currency? And is it adapted to the business of life? Is it what is needed? Is it the desideratum? Is it calculated to remove those obstacles which have hitherto retarded the progress of youth?* Let a candid, liberal, and impartial public decide.

PREFACE.

Although this work is intended for Common Schools, yet it will be found to contain a clear and full explanation of the fundamental principles of arithmetic. The number of examples for solution, in the higher rules, exceeds that of any other book. The methods of solution are simplified and rendered perfectly intelligible. The *synthetic*, *analytic inductive* and *productive* systems of instruction have been adopted in the prosecution of this work. Scholars, who shall be properly conducted through this volume, will have acquired a knowledge of arithmetic adequate to all the purposes of common business.

The first edition of this treatise having been so well received, induced the publisher to issue a new STEREOTYPED EDITION, with greater claims to public patronage.

The errors of the first edition were unimportant, yet they have been minutely considered and *thoroughly* corrected. The work is *now* entirely free from errors.

This treatise differs from all others—and ought not to be classed with those that have preceded it. A valuable discovery may occasionally fall to the lot of the humblest votary of science, even where those of prouder name have failed.

Hartford, Nov. 1833. THE AUTHOR.

*** The author of this treatise has followed *implicitly* the orthography of Dr. Webster's "*duodecimo*" Dictionary, and "Elementary Spelling Book."

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Hora

THE

COMMON SCHOOL ARITHMETIC.

INTRODUCTORY LESSONS.

DESIGNED TO TEACH THE BEGINNER HOW TO READ FIGURES.

THE lessons that follow the figures that have their amount expressed in words, are to be read by the pupils in classes.

The questions at the foot of each page, are for examination and to be answered by the pupil at recitation.

LESSON I.

three.	two.	four.	seven.	nine.	eight.	five.	six.	one.	naught.	ten.	thirteen.	sixteen.	eleven.	eighteen.	twelve.
3	2	4	7	9	8	5	6	1	0	10*	13	16	11	18	12

LESSON II.

8 7 9 3 2 1 4 2 0 5 4 7 9 6 8 1 2 10 11 12 13
10 16 10 11 14 18 18 12 13 16 18 14 10 11 12

LESSON III.

fourteen.	fifteen.	seventeen.	nineteen.	twenty.	twenty-three.	twenty-six.	twenty-one.	twenty-five.	twenty-four.	twenty-eight.	twenty-seven.	twenty-two.	twenty-nine.	thirty.
14	15	17	19	20	23	26	21	25	24	28	27	22	29	30

LESSON IV.

26 19 14 17 15 20 24 28 25 27 22 21 23 14 29
30 17 19 20 15 25 28 24 22 27 21 14 29 26 23

Name the nine figures that stand for numbers; and the character which of itself is of no value. *A.* One, Two, Three, Four, Five, Six, Seven, Eight, Nine, and a Cipher.—What figures stand for ten? What for thirteen? For Eighteen? For Eleven? What figures stand for thirty? For forty? For thirty-five? For twenty?—Two fours express what number? Four and nine express what number?

---

* *Read* the figures, one and naught are *ten;* one and three are *thirteen;* &c.

LESSON V.

thirty-four.	thirty-three.	thirty-six.	thirty-five.	thirty-one.	thirty-seven.	thirty-two.	thirty-eight.	forty.	forty-one.	forty-five.	forty-eight.	forty-four.	forty-nine.	forty-three.
34	33	36	35	31	37	32	38	40	41	45	48	44	49	43

LESSON VI.

32 33 36 35 34 38 41 36 40 31 40 43 49 45 44
34 35 33 36 31 48 44 45 41 43 36 40 37 43 41

LESSON VII.

forty-two.	forty-six.	fifty.	forty-seven.	fifty-three.	fifty-six.	fifty-two.	fifty-nine.	fifty-one.	fifty-four.	fifty-eight.	fifty-five.	fifty-seven.	sixty-one.	sixty.
42	46	50	47	53	56	52	59	51	54	58	55	57	61	60

LESSON VIII.

46 42 47 50 56 52 53 57 59 57 55 51 61 60 54
50 60 42 46 56 47 61 53 55 52 58 51 59 54 57

LESSON IX.

sixty-two.	sixty-nine.	sixty-three.	sixty-one.	sixty-eight.	sixty-four.	sixty-seven.	sixty-five.	seventy.	sixty-six.	seventy-three.	seventy-eight.	seventy-two.	seventy-nine.	seventy-one.
62	69	63	61	68	64	67	65	70	66	73	78	72	79	71

LESSON X.

61 62 63 69 64 68 65 79 66 73 79 72 71 70 64
72 70 61 66 73 64 67 65 70 73 78 71 79 69 68

LESSON XI.

seventy-four.	seventy-six.	seventy-five.	eighty.	eighty-three.	eighty-five.	eighty-one.	eighty-four.	eighty-two.	seventy-seven.	eighty-six.	eighty-nine.	eighty-seven.	ninety.	eighty-eight.
74	76	75	80	83	85	81	84	82	77	86	89	87	90	88

Three and two express what number?—Name the figures that stand for thirty-one;—for twenty-one— for forty-one—for twelve, &c. Which is the most, five and six, or five and two? What figures stand for eighty-eight? What do five and nine stand for?—five and four? Two fives stand for what number? Six and two express what number? Six and seven what? Seven and one express what number? Seven before a 0 express what number? Seven and nine express what number? Nine and a 0 make what number? A 0 placed after eight are how many? Two nines are for what number?—two eights?—two sevens? two sixes? two threes? two twos?

LESSON XII.

75 74 76 77 90 89 86 82 84 81 85 83 80 88 87
88 80 83 85 84 82 87 89 81 86 77 80 74 76 84

LESSON XIII.

thirty-nine.	ninety-one.	ninety-four.	ninety-two.	ninety-three.	ninety-five.	ninety-seven.	ninety-six.	ninety-eight.	ninety-nine.	one hundred.
39	91	94	92	93	95	97	96	98	99	100

LESSON XIV.

91 93 90 95 99 98 96 97 95 93 92 94 100
100 51 69 71 82 91 92 32 22 32 42 52 92

☞ Let the pupil study the foregoing lesson till entirely familiar to him.

73	2	82	1	32	10	6	21	9	63
74	3	83	2	33	11	6	22	9	64
75	4	84	3	34	12	6	23	9	65
76	5	85	4	35	13	6	24	9	66
77	6	86	6	36	14	6	25	9	67
78	7	87	6	37	15	6	26	9	68
79	8	88	7	38	16	6	27	9	69

☞ 1, placed at the left hand of any two figures, in the previous lessons, makes the value *one hundred greater.*

LESSON I.

112 one hundred twelve.
120 one hundred twenty.
125 one hundred twenty-five.
130 one hundred thirty.
160 one hundred sixty.
180 one hundred eighty.
190 one hundred ninety.
150 one hundred fifty.
170 one hundred seventy.
161 one hundred sixty-one.
141 one hundred forty-one.
181 one hundred eighty-one.
164 one hundred sixty-four.
136 one hundred thirty-six.
140 one hundred forty.
147 one hundred forty-seven.
152 one hundred fifty-two.
110 one hundred ten.
121 one hundred twenty-one.
131 one hundred thirty-one.
164 one hundred sixty-four.
191 one hundred ninety-one.
171 one hundred seventy-one.
151 one hundred fifty-one.
138 one hundred thirty-eight.
173 one hundred seventy-three.

LESSON II.

110 112 130 190 131 130 110 140 147 121 120
125 170 110 138 140 160 130 180 160 152 180

LESSON III.

102 one hundred and two.
105 one hundred and five.
101 one hundred and one.
185 one hundred eighty-five.
115 one hundred fifteen.
145 one hundred forty-five.

107 one hundred and seven.
108 one hundred and eight.
109 one hundred and nine.
103 one hundred and three.
106 one hundred and six.
104 one hundred and four.

199 one hundred ninety-nine.
111 one hundred eleven.
113 one hundred thirteen.
117 one hundred seventeen.
114 one hundred fourteen.
118 one hundred eighteen.

LESSON IV.

104	105	101	103	116	101	111	118	117	102
117	199	107	108	109	102	113	114	105	104

LESSON V.

307 three hundred and seven.
409 four hundred and nine.
508 five hundred and eight.
607 six hundred and seven.
717 seven hundred seventeen.
863 eight hundred sixty-three.
937 nine hundred thirty-seven.
785 seven hundred eighty-five.
270 two hundred seventy.

603 six hundred &c.
708 seven hundred &c.
807 eight hundred &c.
903 nine hundred &c.
327 three hundred &c.
468 four hundred &c.
537 five hundred &c.
749 seven hundred &c.
230 two hundred thirty.

LESSON VI.

307	603	468	603	863	307
409	708	537	708	937	503
508	807	749	807	785	508
607	903	230	903	270	607

LESSON VII.

1000 one thousand. 1002 one thousand and two.
2373 two thousand three hundred seventy-three.
4763 four thousand seven hundred sixty-three.
3072 three thousand and seventy-two.
5384 five thousand three hundred eighty-four.
6730 six thousand seven hundred thirty.
7302 seven thousand three hundred and two.
8327 eight thousand three hundred twenty-seven
9999 nine thousand nine hundred ninety-nine.
4087 four thousand and eighty-seven.
3160 three thousand one hundred sixty.
9100 nine thousand one hundred.
8301 eight thousand three hundred and one.
5327 five thousand three hundred twenty-seven.
9372 nine thousand three hundred seventy-two.
5875 five thousand eight hundred seventy-five.
9863 nine thousand eight hundred sixty-three.
13728 thirteen thousand seven hundred twenty-eight.
11563 eleven thousand five hundred sixty-three.
37101 thirty-seven thousand one hundred and one.

30138 thirty thousand one hundred thirty-eight.
47180 forty-seven thousand one hundred eighty.
41836 forty-one thousand eight hundred thirty-six.
12001 twelve thousand and one.
14768 fourteen thousand seven hundred sixty-eight.
23472 twenty-three thousand four hundred seventy-two.
47321 forty-seven thousand three hundred twenty-one.

☞ The following columns of figures should be read by the pupils in classes.

2609
3796
6984
9165
7956
6923
9162
6207
1399

6396
9850
2036
4325
5985
6302
3108
9054
2290
6654

1695
4304
3957
3169
6901
7539
9054
9164
9962

11577
5906
4117
2139
44403
3444
74310
16039
1746

NUMERATION.

This is the right placing and just valuing of figures in any number. The following illustrations will show its nature and use.

Characters.	*Numerated.*	*Common Names.*	*Proper mode of expressing them in words.*
1	is a Unit.*	*One*	one Unit.
2	two Units.	*Two*	two Units.
3	three Units.	*Three*	three Units.
4	four Units.	*Four*	four Units.
5	five Units.	*Five*	five Units.
6	six Units.	*Six*	six Units.
7	seven Units.	*Seven*	seven Units.
8	eight Units.	*Eight*	eight Units.

What have you learned since you began the study of arithmetic? To read figures, that is, tell what number certain figures stand for, as 47 is for forty-seven. How many figures are used to express the numbers you have learned? Only *nine*—All differing in their form. There is one other character used; what is it called? The cipher; it is of no value of itself, but joined to a figure it makes its value ten times greater. Can any number be expressed by those figures and the cipher? Yes—one figure by itself shows its own real value, as 7 9; but when placed together, (thus 79,) they have a local value, known by the place they occupy, and determined by the number used. When figures stand alone, do they always express the same number? They do, because they are of value in and of themselves. Name all the different numbers that can be made out of 472? Ans. 4 7 2 is the real value, 472, 724, and 247 are local values. How many figures express *twelve?* Two. *Five?* One. *Three hundred eighty?* Three. How many ciphers and figures in *Five hundred?* One figure, two ciphers.

---

* All numbers begin with a unit or units, except 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, &c. The cipher has no value of itself; it shows there are no units of the unit order in 10, 20, &c. If the cipher had not been placed by the side of 1, the 1 would be in the unit order.

9	nine Units.	*Nine*	nine Units.
0	naught, cipher, or Zero.	*Naught*	a Cipher.
10	are no Units one Ten.	*Ten*	one ten.
11	one Unit one Ten.	*Eleven*	ten and one.
12	two Units one Ten.	*Twelve*	ten and two.
13	are three Units one Ten.	*Thirteen*	ten and three.
20	are no Units two Tens.	*Twenty*	two tens.
21	one Unit two Tens.	*Twenty-one*	two tens & one.
22	two Units two Tens.	*Twenty-two*	two tens & two.
29	nine Units two Tens.	*Twenty-nine*	two tens & nine.
90	no Units nine Tens.	*Ninety*	nine tens.
93	three Units nine Tens.	*Ninety-three*	nine tens & three.
99	nine Units nine Tens.	*Ninety-nine*	nine tens & nine.
100	no U. no Tens one hun.	*One hundred*	ten tens, &c.

How to read and numerate numbers.

Rule :—Begin at the right hand, read each figure as if it stood alone, naming the name of its order : then read the whole figures in the number, beginning at the left hand and end with the unit figure.

Read and numerate the following figures ; 232, 375, 1000 2100, 3238, 4376, 5867, 6735, and 7823.

OPERATION.

232 two units, three tens, two hundred. *Two hundred thirty two.*

375 five units, seven tens, three hundred. *Three hundred seventy-five.*

1000 no units, no tens, no hundreds, one thousand. *One thousand.*

2100 no units, no tens, one hundred, two thousand. *Two thousand one hundred.*

3238 eight units, three tens, two hundred, three thousand. *Three thousand two hundred thirty-eight.*

4376 six units, seven tens, three hundred, four thousand. *Four thousand three hundred seventy-six.*

What is a unit? A number expressing *one* whole thing, written thus 1. Three units are how many? Three ones. Five? Five ones. Nine? Nine ones. One ten and 1 unit are how many? Ten and 1. What number is it? *Eleven.* One ten and five units? Ten and five. What is the name? *Fifteen.* What are six tens called? *Sixty.* What are four tens called? *Forty.* What are eight tens and *two units* called? *Eighty-two.* What are one hundred seven tens and five units called? *One hundred seventy-five.* What is three? Three units or ones. What thirteen? One ten and three. What twenty-five? Two tens and five. What does *teen* signify? *And ten.* What is the meaning of *ty?* It means *tens* added to two, three, four, &c. What does *fourteen* mean? *Four and ten.* Twenty? Add *ten* to two. How may these numbers be read · 30, 60, 152 and 25?

5867 seven units, six tens, eight hundred, five thousand. *Five thousand eight hundred sixty-seven.*

6735 five units, three tens, seven hundred, six thousand. *Six thousand seven hundred thirty-five.*

7823 three units, two tens, eight hundred, seven thousand. *Seven thousand eight hundred twenty-three.*

ILLUSTRATION. *Last Example.*—The 3 [first figure] is three units or ones—the 2 is in ten's place, and is two tens, or *twenty*—the 8 is in hundred's place, and is eight hundred times one, or *eight hundred*—the 7 is in thousand's place, and is seven thousand times one, or *seven thousand.*

☞ The pupils should practice numerating and reading the following numbers, with the book open, till perfectly familiar to them.

8734	6783	7342	90812	5702	6324
6347	7832	6347	60183	2347	3726

NUMERATION TABLES.

No. I.

NOTE. This table shows that a cipher denotes the want of a number; it also increases the value of a figure ten fold, when placed on the right hand of whole numbers.

Millions.	Hun. of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	
.	.	.	.	.	.	1	*Read*—One.
.	.	.	.	.	1	0	*Read*—Ten.
.	.	.	.	1	0	0	*Read*—One hundred.
.	.	.	1	0	0	0	*Read*—One thousand.
.	.	1	0	0	0	0	*Read*—Ten thousand.
.	1	0	0	0	0	0	*Read*—One hundred thousand.
1	0	0	0	0	0	0	*Read*—One million.

No. II.

NOTE. Those words at the head of this Table are applicable to any sum or number, and can readily be applied to any number.

Millions.	Hun. of thous.	Tens of thous.	Thousands.	Hundreds.	Tens.	Units.	
.	.	.	.	.	.	7	Seven.
.	.	.	.	.	8	6	Eighty-six.
.	.	.	.	4	3	2	Four hundred, thirty-two.
.	.	.	7	3	5	4	Seven thousand, three hund. fifty-four.
.	.	8	6	2	8	3	Eighty-six thou. two hun. eighty-three.
.	9	4	2	3	7	1	Nine hun. forty-two thou. three hun. 71.
5	3	3	6	4	2	5	Five millions, 336 thousand, 425.

No. III.

Note. This table is designed to assist the young learner in writing numbers where ciphers come between or after the significant figures.

Millions.	Hun. of thous.	Tens of thous.	Thousands.	Hundreds.	Tens.	Units.	
.	.	.	.	.	.	8	8, eight.
.	.	.	.	.	1	0	10, ten.
.	.	.	.	1	0	4	104, one hundred and four.
.	.	.	1	0	0	1	1001, one thousand and one.
.	.	1	0	0	6	0	10060, ten thousand and sixty.
.	1	3	0	0	5	0	130050, one hun. thirty thous. and 50.
1	6	0	1	5	0	0	1601500, one million 601 thous. 5 hund.

No. IV.

Note. For the easier reading of this Table, the figures are divided into three periods each, and placed at the right hand of the Table.

	100 of millions.	10 of millions.	Millions.	Hun. of thous.	Tens of thous.	Thousands.	Hundreds.	Tens.	Units.	Millions.	Thousands.	Units.
Places	1	2	3	4	5	6	7	8	9			
	9	8	7	6	5	4	3	2	1	987,	654,	321
		9	8	7	6	5	4	4	2	98,	765,	442
			9	8	7	6	5	4	3	9,	876,	543
				9	8	7	6	5	4		987,	654
					9	8	7	6	5	3d period.	98,	765
						9	8	7	6		9,	876
							9	8	7			987
								9	8		2d period.	98
									9			9
												1st period.

No. V.

	Millions. 3d period.	Thousands. 2d period.	Units. 1st period.
(1.)	8 9 2,	1 2 0,	6 1 4
(2.)	1 2 8,	3 7 4,	3 8 7
(3.)	3 1 0,	2 3 7,	1 0 7
(4.)	7 0 6,	5 2 7,	0 2 3
(5.)	1 3 1,	6 1 2,	3 7 5
(6.)	5 2 3,	1 0 1,	0 0 8
(7.)	1 6 3,	7 2 4,	3 7 8
(8.)	9 7 1,	3 8 1,	2 4 3

Note. This table teaches how to read large numbers by pointing them off into periods of three figures each.

(1.) Eight hundred ninety-two millions, one hundred twenty thousand, six hundred fourteen.

(2.) One hundred twenty-eight millions, three hundred seventy-four thousand, three hundred eighty-seven.

(3.) Three hundred ten millions, two hundred thirty-seven thousand, one hundred and seven.

(4.) Seven hundred and six millions, five hundred twenty-seven thousand, and twenty-three.

(5.) One hundred thirty-one millions, six hundred twelve thousand, three hundred seventy-five.

(6.) Five hundred twenty-three millions, one hundred and one thousand, and eight.

(7.) One hundred sixty-three millions, seven hundred twenty-four thousand, three hundred seventy-eight.

(8.) Nine hundred seventy-one millions, three hundred eighty-one thousand, two hundred forty-three.

ILLUSTRATION.

The *first* right hand period is called the *unit period*; and contains the *orders* called *units, tens* and *hundreds*.

The *second period* is called the *thousand period*; and contains the *orders* called *thousands, tens of thousands*, and *hundreds of thousands*.

The *third period* is called the *million period*, and contains the *orders* called *millions, tens of millions* and *hundreds of millions*.

☞ The following are the periods which must be learned in *succession*, beginning with the *highest*, as well as with the *lowest;* thus,

First Period, Unit.
Second Period, Thousand.
Third Period, Million.
Fourth Period, Billion.
Fifth Period, Trillion.
Sixth Period, Quadrillion.
Seventh Period, Quintillion.
Eighth Period, Sextillion.

Eighth Period, Sextillion.
Seventh Period, Quintillion.
Sixth Period, Quadrillion.
Fifth Period, Trillion.
Fourth Period, Billion.
Third Period, Million.
Second Period, Thousand.
First Period, Unit.

What is the first period? the third? the fifth? the second? the fourth? the seventh? the sixth? the eighth?

☞ Require the pupils to read the numbers expressed in the following columns, without recourse to the preceding tables.

3865	7012	6410	13786	87053	76384
4378	7605	4732	65302	12010	68732
8723	6738	7862	83720	23801	38630

212783	1000101	423781237
837437	3807302	747832876
738206	6303209	400138723

Retrospective Observations.

Figures have an absolute and local value, determined by the place they occupy. No three figures can express more than *nine hundred and ninety-nine*, (999.)—Two no more than *ninety-nine*

(99.)—And one no more than simply *nine*, (9.) Any sum may be expressed by 1, 2, 3, 4, 5, 6, 7, 8 and 9, which are significant figures, for they are expressive of number, except 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 1000, 10,000, &c., which consist of one significant figure, and one or more ciphers. The cipher has no value of itself; but when placed at the right of figures; *then* it increases the value of integers TEN fold.

In arithmetical notation, *And* means *add* a 0, or put a 0 in its place. Sometimes the 0 is placed after the next figure, instead of before it, thus, 140; 130; 160, &c. The *and* is then omitted, and is read one hundred forty, &c. A 0 is also placed in the middle of significant figures, to fill up a vacancy, thus, 104; 706; 403; &c., then read, one hundred and four; seven hundred and six, &c. In some instances, *and* signifies the want of two or more ciphers. Examples—1001; 20007; 500008; &c.—*read* one thousand and one; twenty thousand and seven; five hundred thousand and eight.

NOTATION.

This is expressing numbers by figures. It is likewise the right placing of figures according to their value.

There are four methods by which numbers can be expressed or written, viz: by perpendicular lines, [| | |]; in words, [three]; by the Arabic figures, [3] and by the Roman letters, [III].

The Letters used in the Roman method are I, V, X, L, C, D, M, and IƆ.

N. B. A less number placed after a greater, increases that number; if placed before a greater, it diminishes it, thus VI are 6; XI are 11: but IV are 4; IX are 9. Again, LX are 60; CX are 110: but XL are 40: XC are 90. A line drawn over a number increases its value *one thousand times, thus*: $\overline{XX}$ *twenty thousand.*

ROMAN TABLE.

Letters.	*Value.*	*Letters.*	*Value.*
I	1	V	5
II	2	VI	6
III	3	VII	7
IV	4	VIII	8

What value does one figure express standing alone? Simple. When two figures are used to express a number, what is the value of the first at the right hand? Simple. At the left? Local.

How do you determine the value of figures? By the place they occupy. Is a unit or one a number? It is, because it answers the question, how many? As how many feet are there in 12 inches? *One.* Do all numbers begin with a unit? Those with significant figures do; but those that begin with a cipher do not. What does a cipher show? The want of a number. Give an example? Two, naught,—which is twenty. When does a cipher supply a vacancy in a number? When it is placed among significant figures; as 101, 102, 103, 104, &c.

What is Numeration? It is valuing figures according to their location. Give an example! Units, tens, hundreds, thousands. At which hand do you begin to numerate? Right. At which to read? Left. Give an example of reading figures! 478, *four hundred seventy-eight.*

IX	9	LXX	70
X	10	LXXI	71
XI	11	LXXV	75
XII	12	LXXVI	76
XIII	13	LXXVIII	78
XIV	14	LXXIX	79
XV	15	LXXX	80
XVI	16	LXXXV	85
XVII	17	LXXXIX	89
XVIII	18	XC	90
XIX	19	XCIV	94
XX	20	XCVI	96
XXI	21	XCIX	99
XXII	22	C	100
XXIII	23	CIX	109
XXIV	24	CX	110
XXV	25	CXII	112
XXVI	26	CXV	115
XXVII	27	CXVII	117
XXVIII	28	CXIX	119
XXIX	29	CXX	120
XXX	30	CXXX	130
XXXI	31	CXL	140
XXXIV	34	CL	150
XXXVI	36	CXC	190
XXXIX	39	CC	200
XL	40	CCX	210
XLIV	44	CCC	300
XLVI	46	CCCC	400
XLIX	49	D or IↃ	500
L	50	DC	600
LI	51	DCC	700
LIV	54	DCCC	800
LVI	56	DCCCC	900
LIX	59	M or CIↃ	1000
LX	60	MD	1500
LXI	61	MDC	1600
LXV	65	MM	2000
LXIX	69	MDCCCXXX	1830

How many methods have we of expressing numbers? Four. By words, by making perpendicular lines, by the Arabic figures, and by the Roman letters. How many letters are used to express numbers, and what are they? The number is seven, the letters are I, V, X, L, C, D, M. What if a less number be placed after a greater? It increases the number. Give an example! X is ten; if I be annexed, it becomes eleven. If a lesser number be placed before a greater, what will be the result? It will diminish the larger. As V is five, but prefix I it is *but* four. What does X stand for? IX? XIX? XXV? A line drawn over a number increases its value how much? 1000 times.

Note to Teachers.—The Addition, Carrying, Multiplication, and Subtraction tables ought to be committed to memory previous to using the slate. The teacher can best explain to the pupil the use of the inverted comma (') ſound in the following tables.

ADDITION TABLE.

0 and	0	is 0	0 and	4	are 4	0 and	7	are 7
0	1	1	4	0	4	7	0	7
1	0	1	4	1	5	7	1	8
1	1	are 2	4	2	6	7	2	9
1	2	3	4	3	7	7	3	1'0
1	3	4	4	4	8	7	4	1'1
1	4	5	4	5	9	7	5	1'2
1	5	6	4	6	1'0	7	6	1'3
1	6	7	4	7	1'1	7	7	1'4
1	7	8	4	8	1'2	7	8	1'5
1	8	9	4	9	1'3	7	9	1'6
1	9	1'0						
0 and	2	are 2	0 and	5	are 5	0 and	8	are 8
2	0	2	5	0	5	8	0	8
2	2	4	5	1	6	8	1	9
2	3	5	5	2	7	8	2	1'0
2	4	6	5	3	8	8	3	1'1
2	5	7	5	4	9	8	4	1'2
2	6	8	5	5	1'0	8	5	1'3
2	7	9	5	6	1'1	8	6	1'4
2	8	1'0	5	7	1'2	8	7	1'5
2	9	1'1	5	8	1'3	8	8	1'6
			5	9	1'4	8	9	1'7
0 and	3	are 3	0 and	6	are 6	0 and	9	are 9
3	0	3	6	0	6	9	0	9
3	1	4	6	1	7	9	1	1'0
3	2	5	6	2	8	9	2	1'1
3	3	6	6	3	9	9	3	1'2
3	4	7	6	4	1'0	9	4	1'3
3	5	8	6	5	1'1	9	5	1'4
3	6	9	6	6	1'2	9	6	1'5
3	7	1'0	6	7	1'3	9	7	1'6
3	8	1'1	6	8	1'4	9	8	1'7
3	9	1'2	6	9	1'5	9	9	1'8

Questions to be answered Orally.

8 and	8?	A. 16	4 and	4?	A. 8	6 and	6?	A. 12	5 and	5?	A. 10	3 and	3?	A. 6
8	1?	9	1	4?	5	1	6?	7	1	5?	6	1	3?	4
1	8?	9	5	4?	9	6	7?	13	5	6?	11	3	2?	5
8	7?	15	4	8?	12	5	6?	11	7	5?	12	4	3?	7
3	8?	11	6	4?	10	6	8?	14	5	2?	7	3	1?	4

8 and	4 ?	A. 12	4 and	1 ?	A. 5	4 and	6 ?	A. 10	3 and	5 ?	A. 8	7 and	3 ?	A. 10
2	8 ?	10	2	4 ?	6	6	9 ?	15	5	9 ?	14	3	6 ?	9
8	9 ?	17	4	5 ?	9	8	6 ?	14	4	5 ?	9	5	3 ?	8
5	8 ?	13	9	4 ?	13	6	5 ?	11	5	1 ?	6	3	4 ?	7
8	3 ?	11	4	7 ?	11	2	6 ?	8	8	5 ?	13	6	3 ?	9
7	8 ?	15	8	4 ?	12	6	1 ?	7	5	3 ?	8	3	5 ?	8
4	8 ?	12	4	9 ?	13	7	6 ?	13	2	5 ?	7	2	3 ?	5
8	6 ?	14	3	4 ?	7	6	3 ?	9	5	7 ?	12	3	9 ?	12
6	8 ?	14	4	2 ?	6	9	6 ?	15	6	5 ?	11	8	3 ?	11
8	2 ?	10	7	4 ?	11	6	4 ?	10	5	8 ?	13	3	7 ?	10
9	8 ?	17	4	6 ?	10	6	3 ?	9	9	5 ?	14	9	3 ?	12
8	5 ?	13	4	3 ?	7	6	2 ?	8	5	4 ?	9	3	8 ?	11

9 and	9 ?	A. 18	7 and	7 ?	A. 14	2 and	2 ?	A. 4	1 and	1 ?	A. 2
1	9 ?	10	1	7 ?	8	1	2 ?	3	[illegible]	1 ?	10
9	3 ?	12	7	8 ?	15	2	3 ?	5	1	8 ?	9
8	9 ?	17	9	7 ?	16	4	2 ?	6	5	1 ?	6
4	9 ?	13	7	1 ?	8	2	8 ?	10	1	4 ?	5
9	4 ?	13	8	7 ?	15	9	2 ?	11	3	1 ?	4
9	2 ?	11	7	3 ?	10	2	7 ?	9	1	7 ?	8
7	9 ?	16	6	7 ?	13	7	2 ?	9	6	1 ?	7
9	1 ?	10	7	2 ?	9	2	4 ?	6	1	9 ?	10
2	9 ?	11	3	7 ?	10	6	2 ?	8	7	1 ?	8
9	5 ?	14	9	9 ?	18	2	1 ?	3	1	6 ?	7
6	9 ?	15	5	7 ?	12	3	2 ?	5	4	1 ?	5
9	8 ?	17	7	4 ?	11	2	5 ?	7	1	3 ?	4
3	9 ?	12	2	7 ?	9	8	2 ?	10	8	1 ?	9
9	6 ?	15	7	5 ?	12	5	2 ?	7	1	5 ?	6
5	9 ?	14	7	6 ?	13	2	6 ?	8	2	1 ?	3
9	7 ?	16	4	7 ?	11	2	9 ?	11	1	2 ?	3

CARRYING TABLE.

1	1		21	1	2	41	1	4	61	1	6	81	1	8
2	2		22	2	2	42	2	4	62	2	6	82	2	8
3	3		23	3	2	43	3	4	63	3	6	83	3	8
4	4		24	4	2	44	4	4	64	4	6	84	4	8
5	5		25	5	2	45	5	4	65	5	6	85	5	8
6	6		26	6	2	46	6	4	66	6	6	86	6	8
7	7		27	7	2	47	7	4	67	7	6	87	7	8
8	8		28	8	2	48	8	4	68	8	6	88	8	8
9	9		29	9	2	49	9	4	69	9	6	89	9	8
10	0	1	30	0	3	50	0	5	70	0	7	90	0	9
11	1	1	31	1	3	51	1	5	71	1	7	91	1	9
12	2	1	32	2	3	52	2	5	72	2	7	92	2	9
13	3	1	33	3	3	53	3	5	73	3	7	93	3	9
14	4	1	34	4	3	54	4	5	74	4	7	94	4	9
15	5	1	35	5	3	55	5	5	75	5	7	95	5	9
16	6	1	36	6	3	56	6	5	76	6	7	96	6	9
17	7	1	37	7	3	57	7	5	77	7	7	97	7	9
18	8	1	38	8	3	58	8	5	78	8	7	98	8	9
19	9	1	39	9	3	59	9	5	79	9	7	99	9	9
20	0	2	40	0	4	60	0	6	80	0	8	100	0	10

Explanation. When a column of figures amount to 9 or a less sum, set down that figure. None to carry to the next column. If a column, when added amount to 12, set down 2.

Questions to be answered Orally.

For 11? (*See Table.*) For 20? (*See Table.*) For 97? (*See Table.*) Which figure set down? For 23? (*See Table.*) For 40? (*See Table.*) For 38? (*See Table.*) Which figure set down? For 8? (*See Table.*) For 50? (*See Table.*) For 29? (*See Table.*) Which figure set down? For 30? (*See Table.*) For 60? (*See Table.*) For 19? (*See Table.*) Which figure set down? For 80? (*See Table.*) For 70? (*See Table.*) For 9? (*See Table.*) Which figure set down? For 6? (*See Table.*) For 90? (*See Table.*) For 88? (*See Table.*) Which figure set down? For 39? (*See Table.*) For 100? (*See Table.*) For 96? (*See Table.*) Which figure set down? For 63? (*See Table.*) For 10? (*See Table.*) For 81? (*See Table.*) Which figure set down?

MULTIPLICATION TABLE.

1	time	0	are	0*	4	times	0	are	0	7	times	0	are	0
1		1		1	4		1		4	7		1		7
1		2		2	4		2		8	7		2		1'4
1		3		3	4		3		1'2	7		3		2'1
1		4		4	4		4		1'6	7		4		2'8
1		5		5	4		5		2'0	7		5		3'5
1		6		6	4		6		2'4	7		6		4'2
1		7		7	4		7		2'8	7		7		4'9
1		8		8	4		8		3'2	7		8		5'6
1		9		9	4		9		3'6	7		9		6'3
2	times	0	are	0	5	times	0	are	0	8	times	0	are	0
2		1		2	5		1		5	8		1		8
2		2		4	5		2		1'0	8		2		1'6
2		3		6	5		3		1'5	8		3		2'4
2		4		8	5		4		2'0	8		4		3'2
2		5		1'0	5		5		2'5	8		5		4'0
2		6		1'2	5		6		3'0	8		6		4'8
2		7		1'4	5		7		3'5	8		7		5'6
2		8		1'6	5		8		4'0	8		8		6'4
2		9		1'8	5		9		4'5	8		9		7'2
3	times	0	are	0	6	times	0	are	0	9	times	0	are	0
3		1		3	6		1		6	9		1		9
3		2		6	6		2		1'2	9		2		1'8
3		3		9	6		3		1'8	9		3		2'7
3		4		1'2	6		4		2'4	9		4		3'6
3		5		1'5	6		5		3'0	9		5		4'5
3		6		1'8	6		6		3'6	9		6		5'4
3		7		2'1	6		7		4'2	9		7		6'3
3		8		2'4	6		8		4'8	9		8		7'2
3		9		2'7	6		9		5'4	9		9		8'1

---

* A 0 is never multiplied by a figure. When a 0 terminates a multiplier it is omitted in multiplying, and is joined to the product.

10 times	1 are	1‘0	11 times	1 are	1‘1	12 times	1 are	1‘2
10	2	2‘0	11	2	2‘2	12	2	2‘4
10	3	3‘0	11	3	3‘3	12	3	3‘6
10	4	4‘0	11	4	4‘4	12	4	4‘8
10	5	5‘0	11	5	5‘5	12	5	6‘0
10	6	6‘0	11	6	6‘6	12	6	7‘2
10	7	7‘0	11	7	7‘7	12	7	8‘4
10	8	8‘0	11	8	8‘8	12	8	9‘6
10	9	9‘0	11	9	9‘9	12	9	10‘8
10	10	10‘0	11	10	11‘0	12	10	12‘0
10	11	11‘0	11	11	12‘1	12	11	13‘2
10	12	12‘0	11	12	13‘2	12	12	14‘4

SUBTRACTION TABLE.

☞ This table and the questions which follow include every principle in subtracting.

0 from	0 leaves	0
0	1	1
1	1	0
1	2	1
2	2	0
2	3	1
2	4	2
2	5	3
2	6	4
2	7	5
2	8	6
2	9	7
0 from	3 leaves	3
3	3	0
3	4	1
3	5	2
3	6	3

3 from	7 leaves	4
3	8	5
3	9	6
0 from	4 leaves	4
4	4	0
4	5	1
4	6	2
4	7	3
4	8	4
4	9	5
0 from	5 leaves	5
5	5	0
5	6	1
5	7	2
5	8	3
5	9	4

0 from	6 leaves	6
6	6	0
6	7	1
6	8	2
6	9	3
0 from	7 leaves	7
7	7	0
7	8	1
7	9	2
0 from	8 leaves	8
8	8	0
8	9	1
0 from	9 leaves	9
9	9	0

Obs. When the figure, in the *Minuend*, is less than the Subtrahend—unity or *one* is prefixed to that figure when subtracting, according to the following illustration.

Suppose 7 were to be taken from 3, what must be done? Say, 7 from 13 leaves 6.

Suppose 1 from 0? say, 1 from 10 leaves 9.
2 from 0? say, 2 from 10 leaves 8.
2 from 1? say, 2 from 11 leaves 9.

Suppose 3 from 0? say, 3 from 10 leaves 7.
3 from 1? say, 3 from 11 leaves 8.
3 from 2? say, 3 from 12 leaves 9.
4 from 0? say, 4 from 10 leaves 6.
4 from 1? say, 4 from 11 leaves 7.
4 from 2? say, 4 from 12 leaves 8.
4 from 3? say, 4 from 13 leaves 9.
5 from 0? say, 5 from 10 leaves 5.
5 from 1? say, 5 from 11 leaves 6.
5 from 2? say, 5 from 12 leaves 7.
5 from 3? say, 5 from 13 leaves 8.
5 from 4? say, 5 from 14 leaves 9.
6 from 0? say, 6 from 10 leaves 4.
6 from 1? say, 6 from 11 leaves 5.
6 from 2? say, 6 from 12 leaves 6.
6 from 3? say, 6 from 13 leaves 7.
6 from 4? say, 6 from 14 leaves 8.
6 from 5? say, 6 from 15 leaves 9.
7 from 0? say, 7 from 10 leaves 3
7 from 1? say, 7 from 11 leaves 4
7 from 2? say, 7 from 12 leaves 5
7 from 3? say, 7 from 13 leaves 6
7 from 4? say, 7 from 14 leaves 7
7 from 5? say, 7 from 15 leaves 8.
7 from 6? say, 7 from 16 leaves 9.
8 from 0? say, 8 from 10 leaves 2.
8 from 1? say, 8 from 11 leaves 3.
8 from 2? say, 8 from 12 leaves 4.
8 from 3? say, 8 from 13 leaves 5.
8 from 4? say, 8 from 14 leaves 6.
8 from 5? say, 8 from 15 leaves 7.
8 from 6? say, 8 from 16 leaves 8.
8 from 7? say, 8 from 17 leaves 9.
9 from 0? say, 9 from 10 leaves 1.
9 from 1? say, 9 from 11 leaves 2.
9 from 2? say, 9 from 12 leaves 3.
9 from 3? say, 9 from 13 leaves 4.
9 from 4? say, 9 from 14 leaves 5.
9 from 5? say, 9 from 15 leaves 6.
9 from 6? say, 9 from 16 leaves 7.
9 from 7? say, 9 from 17 leaves 8.
9 from 8? say, 9 from 18 leaves 9.

Questions to be answered Orally.

9 from 10?	A. 1	3 from 10?	A. 7	8 from 8?	A. 0	1 from 8?	A 7
9 9?	0	3 4?	1	8 10?	2	1 10?	9
9 11?	2	3 11?	8	8 9?	1	1 9?	8
9 13?	4	3 6?	3	8 17?	9	1 7?	6
9 18?	9	3 12?	9	8 11?	3	1 3?	2
9 14?	5	3 7?	4	8 16?	8	1 4?	3
9 12?	3	3 9?	6	8 12?	4	1 6?	5
9 15?	6	3 5?	2	8 14?	6	1 1?	0
9 16?	7	3 8?	5	8 13?	5	1 2?	1
9 17?	8	3 3?	0	8 15?	7	1 5?	4
4 from 9?	A. 5	7 from 16?	A. 9	2 from 10?	A. 8	5 5?	A. 0
4 7?	3	7 7?	0	2 2?	0	5 14?	9
4 4?	0	7 10?	3	2 3?	1	5 10?	5
4 10?	6	7 8?	1	2 11?	9	5 12?	7
4 13?	9	7 13?	6	2 9?	7	5 8?	3
4 12?	8	7 9?	2	2 7?	5	5 11?	6
4 8?	4	7 11?	4	2 5?	3	5 6?	1
4 5?	1	7 12?	5	2 8?	6	5 9?	4
4 6?	2	7 15?	8	2 4?	2	5 7?	2
4 11?	7	7 14?	7	2 6?	4	5 13?	8
6 from 7?	A. 1	6 from 15?	A. 9	6 from 8?	A. 2	6 from 12?	A. 6
6 9?	3	6 14?	8	6 6?	0	6 13?	7
6 10?	4	6 11?	5	10 10?	0		

DIVISION TABLE.

Obs. Beginners are troubled and puzzled to find how many times the Quotient figure is contained in the Dividend. This table together with the explanations in the Rule of Division at § 27, will entirely obviate the difficulty.

in		*time.*	*rem.*	*in*		*time.*	*rem.*	*in*		*time.*	*rem.*
2	2	1		2	19	9	1	3	19	6	1
2	3	1	1	3	3	1		3	20	6	2
2	4	2		3	4	1	1	3	21	7	
2	5	2	1	3	5	1	2	3	22	7	1
2	6	3		3	6	2		3	23	7	2
2	7	3	1	3	7	2	1	3	24	8	
2	8	4		3	8	2	2	3	25	8	1
2	9	4	1	3	9	3		3	26	8	2
2	10	5		3	10	3	1	3	27	9	
2	11	5	1	3	11	3	2	3	28	9	1
2	12	6		3	12	4		3	29	9	2
2	13	6	1	3	13	4	1	4	4	1	
2	14	7		3	14	4	2	4	5	1	1
2	15	7	1	3	15	5		4	6	1	2
2	16	8		3	16	5	1	4	7	1	3
2	17	8	1	3	17	5	2	4	8	2	
2	18	9		3	18	6		4	9	2	1

DIVISION TABLE, CONTINUED.

	in	*time.*	*rem.*		*in*	*time.*	*rem.*		*in*	*time.*	*rem*
4	10	2	2	5	19	3	4	6	19	3	1
4	11	2	3	5	20	4		6	20	3	2
4	12	3		5	21	4	1	6	21	3	3
4	13	3	1	5	22	4	2	6	22	3	4
4	14	3	2	5	23	4	3	6	23	3	5
4	15	3	3	5	24	4	4	6	24	4	
4	16	4		5	25	5		6	25	4	1
4	17	4	1	5	26	5	1	6	26	4	2
4	18	4	2	5	27	5	2	6	27	4	3
4	19	4	3	5	28	5	3	6	28	4	4
4	20	5		5	29	5	4	6	29	4	5
4	21	5	1	5	30	6		6	30	5	
4	22	5	2	5	31	6	1	6	31	5	1
4	23	5	3	5	32	6	2	6	32	5	2
4	24	6		5	33	6	3	6	33	5	3
4	25	6	1	5	34	6	4	6	34	5	4
4	26	6	2	5	35	7		6	35	5	5
4	27	6	3	5	36	7	1	6	36	6	
4	28	7		5	37	7	2	6	37	6	1
4	29	7	1	5	38	7	3	6	38	6	2
4	30	7	2	5	39	7	4	6	39	6	3
4	31	7	3	5	40	8		6	40	6	4
4	32	8		5	41	8	1	6	41	6	5
4	33	8	1	5	42	8	2	6	42	7	
4	34	8	2	5	43	8	3	6	43	7	1
4	35	8	3	5	44	8	4	6	44	7	2
4	36	9		5	45	9		6	45	7	3
4	37	9	1	5	46	9	1	6	46	7	4
4	38	9	2	5	47	9	2	6	47	7	5
4	39	9	3	5	48	9	3	6	48	8	
5	5	1		5	49	9	4	6	49	8	1
5	6	1	1	6	6	1		6	50	8	2
5	7	1	2	6	7	1	1	6	51	8	3
5	8	1	3	6	8	1	2	6	52	8	4
5	9	1	4	6	9	1	3	6	53	8	5
5	10	2		6	10	1	4	6	54	9	
5	11	2	1	6	11	1	5	6	55	9	1
5	12	2	2	6	12	2		6	56	9	2
5	13	2	3	6	13	2	1	6	57	9	3
5	14	2	4	6	14	2	2	6	58	9	4
5	15	3		6	15	2	3	6	59	9	5
5	16	3	1	6	16	2	4	7	7	1	
5	17	3	2	6	17	2	5	7	8	1	1
5	18	3	3	6	18	3		7	9	1	2

DIVISION TABLE, CONTINUED.

	in	time.	rem.		in	time.	rem.		in	time.	rem.
7	10	1	3	7	54	7	5	8	36	4	4
7	11	1	4	7	55	7	6	8	37	4	5
7	12	1	5	7	56	8		8	38	4	6
7	13	1	6	7	57	8	1	8	39	4	7
7	14	2		7	58	8	2	8	40	5	
7	15	2	1	7	59	8	3	8	41	5	1
7	16	2	2	7	60	8	4	8	42	5	2
7	17	2	3	7	61	8	5	8	43	5	3
7	18	2	4	7	62	8	6	8	44	5	4
7	19	2	5	7	63	9		8	45	5	5
7	20	2	6	7	64	9	1	8	46	5	6
7	21	3		7	65	9	2	8	47	5	7
7	22	3	1	7	66	9	3	8	48	6	
7	23	3	2	7	67	9	4	8	49	6	1
7	24	3	3	7	68	9	5	8	50	6	2
7	25	3	4	7	69	9	6	8	51	6	3
7	26	3	5	8	8	1		8	52	6	4
7	27	3	6	8	9	1	1	8	53	6	5
7	28	4		8	10	1	2	8	54	6	6
7	29	4	1	8	11	1	3	8	55	6	7
7	30	4	2	8	12	1	4	8	56	7	
7	31	4	3	8	13	1	5	8	57	7	1
7	32	4	4	8	14	1	6	8	58	7	2
7	33	4	5	8	15	1	7	8	59	7	3
7	34	4	6	8	16	2		8	60	7	4
7	35	5		8	17	2	1	8	61	7	5
7	36	5	1	8	18	2	2	8	62	7	6
7	37	5	2	8	19	2	3	8	63	7	7
7	38	5	3	8	20	2	4	8	64	8	
7	39	5	4	8	21	2	5	8	65	8	1
7	40	5	5	8	22	2	6	8	66	8	2
7	41	5	6	8	23	2	7	8	67	8	3
7	42	6		8	24	3		8	68	8	4
7	43	6	1	8	25	3	1	8	69	8	5
7	44	6	2	8	26	3	2	8	70	8	6
7	45	6	3	8	27	3	3	8	71	8	7
7	46	6	4	8	28	3	4	8	72	9	
7	47	6	5	8	29	3	5	8	73	9	1
7	48	6	6	8	30	3	6	8	74	9	2
7	49	7		8	31	3	7	8	75	9	3
7	50	7	1	8	32	4		8	76	9	4
7	51	7	2	8	33	4	1	8	77	9	5
7	52	7	3	8	34	4	2	8	78	9	6
7	53	7	4	8	35	4	3	8	79	9	7

DIVISION TABLE, CONTINUED.

	in	time.	rem.		in	time.	rem.		in	time.	rem.
9	9	1		9	36	4		9	63	7	
9	10	1	1	9	37	4	1	9	64	7	1
9	11	1	2	9	38	4	2	9	65	7	2
9	12	1	3	9	39	4	3	9	66	7	3
9	13	1	4	9	40	4	4	9	67	7	4
9	14	1	5	9	41	4	5	9	68	7	5
9	15	1	6	9	42	4	6	9	69	7	6
9	16	1	7	9	43	4	7	9	70	7	7
9	17	1	8	9	44	4	8	9	71	7	8
9	18	2		9	45	5		9	72	8	
9	19	2	1	9	46	5	1	9	73	8	1
9	20	2	2	9	47	5	2	9	74	8	2
9	21	2	3	9	48	5	3	9	75	8	3
9	22	2	4	9	49	5	4	9	76	8	4
9	23	2	5	9	50	5	5	9	77	8	5
9	24	2	6	9	51	5	6	9	78	8	6
9	25	2	7	9	52	5	7	9	79	8	7
9	26	2	8	9	53	5	8	9	80	8	8
9	27	3		9	54	6		9	81	9	
9	28	3	1	9	55	6	1	9	82	9	1
9	29	3	2	9	56	6	2	9	83	9	2
9	30	3	3	9	57	6	3	9	84	9	3
9	31	3	4	9	58	6	4	9	85	9	4
9	32	3	5	9	59	6	5	9	86	9	5
9	33	3	6	9	60	6	6	9	87	9	6
9	34	3	7	9	61	6	7	9	88	9	7
9	35	3	8	9	62	6	8	9	89	9	8

OBS. Pupils should be required to *add* by *calculation*, and not by *counting*, as young scholars are very apt to do. If a teacher will but *be thorough*, at the *commencement*, in these respects, much time and labor will be saved.

Add 3 2 7 3 1 4 5 and 6 together without counting or making marks for the figures or their amount.

Method—3 to 2 are 5; 7 are 12; 3 are 15; 1 are 16; 4 are 20; 5 are 25; and 6 are 31—*The amount.*

☞ The following figures are to be added by the pupil, in the manner just described; having the book open at recitation.

Add 7 3 4 6 5 2 and 8 together. *Amt.* 35
Add 8 9 3 4 7 3 and 5 together. *Amt.* 39
Add 9 3 7 8 6 5 and 7 together. *Amt.* 45
Add 7 5 8 6 2 4 and 3 together. *Amt.* 35
Add 6 8 7 4 5 9 and 5 together. *Amt.* 44
Add 3 4 5 6 9 2 and 7 together. *Amt.* 36
Add 6 1 8 3 4 5 and 3 together. *Amt.* 30

Add 3 4 7 9 1 3 and 7 together. *Amt.* 34
Add 7 3 2 1 8 7 and 1 together. *Amt.* 29
Add 9 8 7 6 5 and 7 together. *Amt.* 42
Add 1 9 7 8 6 and 4 together. *Amt.* 35

Obs. Pupils, before they commence Addition, should practice writing figures on the slate, till they be able to place them in straight lines, both perpendicular and horizontal, as in the following examples.

123456	123456	123456
789012	789012	789012
345678	345678	345678
901234	901234	901234
567890	567890	567890
123456	123456	123456
789012	789012	789012
345678	345678	345678

☞The following lessons are intended to exercise the learner in expressing the value of numbers by Arabic figures.

Rule:—Write the unit figure *first*, being on the right hand; then tens, then hundreds, according to the Numeration Table. When there is an AND in the sum put a *Cipher*, as explained in the 3d Numeration Table.

LESSON I.

Ninety-five. One hundred. Two hundred and five. Eight hundred twenty-seven. Six hundred and eight. Nine hundred forty-six. Seven hundred and seven. One hundred nineteen. Two hundred and one. Four hundred and nine. Two hundred and seven. Nine hundred and eight. Nine hundred eighty. Three hundred and six.

LESSON II.

One thousand. One thousand and seven. Three thousand and twenty-one. Four thousand and nine. Three thousand one hundred. Six thousand five hundred nineteen. Three thousand one hundred and one. Six thousand eight hundred and ten. Seven thousand nine hundred and fourteen. Thirteen thousand. Eleven thousand and one. Ten thousand and eight. Nine thousand and seven. Thirty thousand.

LESSON III.

Three units four tens; two units six tens; three units nine tens; ten and four; ten and five; one ten and eight units; two tens and nine units; four tens and six units; five tens and three units; two tens and one; five tens and two; eight tens and eight units; seven tens and four units; three tens and five; four tens and five; ten tens; ten tens and one unit; eight tens; five tens; six tens; four tens; three tens; two tens, &c.

QUESTIONS ON NUMERATION.

1. What is the first thing to be learned in arithmetic? To read the figures. What is a figure? It represents a number. What is number? A collection of units. What is a unit? It represents one thing of any kind, and is the origin of all numbers. What are the figures 1, 2, 3, 4, to 9 called? Digits. What does that mean? It is from the Latin *digitus*, which means fingers. [See Art. 9. Part II.] Who made the nine digits? The Indians; and the Arabs introduced them into Europe; the Moors into Spain, and John of Basingstoke into England, about 700 years ago. Can all numbers be expressed by the nine digits? Not without using the cipher, which is a character. What is the use of the cipher? It denotes the absence of a number: it supplies a vacancy: it fills a place where no value is to be expressed: it serves to bring figures into their proper place: to supply the place of a single character for ten; and placed at the right of figures it increases the value ten times. Why is the cipher of no value? Because it is of no value in numeration: in addition: in subtraction: in multiplication or division—if added to or subtracted from any number it neither increases or lessens the sum.

2. What are cardinal numbers? Those that show the number of things. Name them! 1, 2, 3, 4, 5, 6, &c. What are ordinal numbers? Such as show the order of things. Which are they? First, secondly, thirdly, fourthly, lastly, &c. What are even numbers? Such as can be divided by 2, without a remainder. Name them! 4, 8, 12, 16. What are odd numbers? Such as have a remainder, if divided by 2. What are they? *Ans.* 5, 7, 9, 13. What are prime numbers? Such as are divisible only by *one*; they are 3, 11, 17. What are perfect numbers? Those that are equal to their aliquot parts. Three of them are 6, 28, 496. There are only 7 others that are known at present. What are whole numbers? Such as represent an entire thing, 4, 6. What are abstract numbers? Those that relate only to numerical value, as 7 is for the number *seven*. What are concrete numbers? They are figures which stand for the sum or number of persons, beasts, or things spoken of. Give an example! 5 men; 3 oxen; 8 trees. What are broken numbers? Those that represent less than the whole. Name them! Half, a third, one-fifth, &c.

3. What is the first place in numeration called? Units. The second? Third? Fourth? How are figures to be valued? According to their distance from the place of units. Can such value arise by ciphers? Yes, if they be annexed to figures, as *six naught* (60)—the 6 is removed from unit's to ten's place by having the 0 annexed to it. What is the use of numeration? It gives names to figures, that we may read them. Illustrate it by an example! Do ciphers prefixed to a *whole* number affect the value? They do not. Ought any whole number to begin with a cipher? No. Why? How many words are

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Remarks.—When you can read, numerate, and write numbers, you then understand Numeration and Notation. The next step is Addition, which is putting numbers or quantities together to find their total sum.

What is notation? Writing figures for words. At which hand do you begin to write figures? Right. Give an example! When I put down on the slate three, four, seven, I then write, or express numbers in figures (347.).

there used in numeration? Thirty-five. Name them! One, two, three, four, five, &c. to twenty; then twenty, thirty, forty, fifty, sixty, &c. to one hundred; then thousand, million, billion, trillion, quadrillion, quintillion, sextillion.

4. In what proportion do numbers increase from unit's place towards the left hand? In ten fold proportion. Why is the Numeration Table made to consist of *twelve* places of figures rather than any other? Because they make 4 *even* periods. What is meant by a period? A number expressed by three figures, of which the right hand *one* signifies so many units, the second so many tens, the third so many hundreds, &c. Why are three figures called a period? Because, if the number be increased above three places, there is still the same periodical return of value; every third figure to the left will always be *hundreds*, if ever so far extended.

5. How many methods of numeration are there? Two, the English and French. In what do they differ? The English divide the periods into *six* figures; the French into three. Which method is practiced? The French. Why? Because it is both convenient and common. Give an example of both methods! What is meant by the *Radix* of a system? The number by which we reckon. What is the radix of the decimal notation? Ans. Ten.

ARITHMETIC.

CHAPTER I.

§ 1. The first primary principle in Arithmetic is

ADDITION.

It is the method of putting together two or more numbers in order to find the whole amount, or total sum of several numbers. This rule may have a few or many given numbers, which are to be brought into one sum of a like denomination. Addition is of one denomination, when the numbers are all Dollars, Cents, Mills, Acres or Yards.

Ex. Add 11123423138351 and 83543516451442 together.

OPERATION.	EXPLANATION.
11123423138351	To find the answer or amount of the
83543516451442	numbers here given to be added.
94666939589793	

Rule.—Begin at figure 2, the unit column, and say, 2 and 1 are 3, which set down directly under 2; proceed to the next column; say, 4 and 5 are 9, (9[*]); then go to the next, and say, 4 and 3 are 7, (7); then go to the next, say, 1 and 8 are 9, (9); then go on to the next column, say, 5 and 3 are 8, which set down directly under the column added; for this must always be done. Finish the operation as commenced.

The operation as now performed is finding the sum of several numbers.

Obs. Always begin to add at the foot of the column on the right hand, and proceed to the left, till all the columns are added.

Exercises for the Slate.

1.
87235284232218
11643713654371
98878997886589

2.
43265431287654
31721558711244
74986989998898

3.
12322341321153
34254125437612
46576466758765

4.
712123712368
127836274310[†]
839959986678

---

[*] The figures included in the ( ) must be set down in the operation.
[†] A 0 is not mentioned in adding.

5.	6.
231578235	0321251212622
623301210	7132101121101
124110124	
978989569	7453352333723

☞ Questions to be answered by the pupils in classes.

What is the first rule in Arithmetic? Addition. How are the figures placed as to value? Units first; tens next; hundreds next; and so on to millions. How many columns of figures may there be in an addition sum? From two to any number possible.

How are figures in Addition generally placed? One above the other in columns. Does it make any material difference whether the greater or lesser figures be placed first or last? It does not; because 7 and 9 are 16—so are 9 and 7. At which hand do you begin to add? On the right, at the foot of the column. Why? Because units must first be added.

§ 2. In the foregoing examples (§ 1) no single column amounted to more than *nine*: but in this §, the principle of carrying for *ten*, will be introduced and explained.

EXAMPLE.	EXPLANATION.
38976482	To find the sum total of the numbers here
91214509	given to be added.
*130190991	

RULE.—Begin at the unit figure 9, and say, 9 and 2 are 11; set down 1: one to carry, which added to 8 make 9, set down 9 under 0: none to carry; then say, 5 and 4 are 9, (9): none to carry; then say, 4 and 6 are 10, (0): 1 to carry, which added to 1 make 2 and 7 are 9, (9): none to carry; now say, 2 and 9 make 11, (1): 1 to carry, which added to 1 make 2, and 8 are 10, (0): 1 to carry, which added to 9 make 10, and 3 are 13, which set down. The operation is now performed.

OBS. The adding of *one* to the next column, when the figures amount to over 9, (as in the foregoing example) is carrying *one* for every *ten*.

Note. The above operation and its explanation virtually illustrates the whole principle of Addition.

Exercises for the Slate.

1.	2.	3.
456789134523	486734251821	87683452183
614121715497	837161737298	45321245621
1070910850020	1323895989119	133004697804

4.	5.	6.
93453281561	5873582733425	97384384978
12315728238	6724329714780	78476975612
27234213242	1372182348278	57194367598
133003223041	13970094796483	233055728188

☞ Questions to be answered by the pupils in classes.

How many are 5 and 3? Eight. Why? Because the figure *five* (5) is for five units or ones, [11111] and three (3) *for three* units or ones, [111]; which brought into *one* sum is equal to eight ones, [11111111] written thus. 8.

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* Set down the whole amount of the last column on the left hand.

When a column of figures amount to 9 or a less sum, what figure must be set down? That figure. How many to carry to the next column? None. Why? For the column does not amount to any tens. It takes *ten* ONES or units to make one 10. Suppose a column, when added amounts to 12; what must be set down? 2. Why? Because there are *two* units and ONE ten in 12. How many must be carried or added to the *first* figure of the next column? 1. Why? Because the one stands in tens' place, and must be located in that denomination; *Ten* in units' place make ONE in tens' place; and ten tens make *one* in the place of hundreds.

How many units and tens in 16 integers? Six units and one ten. Why? Because 6 occupies the first place in numeration. And because the 1 stands in tens' place, or the second place of numerical notation.

Why do you carry *one* for every TEN? Because numbers increase in a tenfold proportion; and because 10, in an inferior column, is just equal to *one* in a superior column.

§ 3. Let the pupil proceed to the performance of the following operations. The answers are given in the Key,* which ought not to be consulted by the pupil till an answer shall have been obtained.

1.	2.	3.
46797654	43786524	89756898
97383637	98372857	93598637
43654219	63724365	14637481
37416374	74285231	72343332

☞ The pupil may write down in figures the following sums, and find their amount.

1. Add Four thousand three hundred fifty-four—Two hundred sixty-three—Nine hundred twenty-one, and sixty-eight, together. *Ans.* Five thousand six hundred and six.

2. Find the amount of Nine hundred twenty-six—Six thousand four hundred sixty-three; and One hundred sixty-nine. *Ans.* Seven thousand five hundred fifty-eight.

3. How do you set down and add eleven thousand, eleven hundred, eleven? *Ans.* 12111

4. How do you write fourteen thousand, fourteen hundred, fourteen, and add it? *Ans.* 15414

DEFINITION OF TERMS IN ADDITION.

A definition is an explanation of what is meant by any word or phrase.

It is *essential* to a complete definition, that it perfectly distinguishes the thing defined from every thing else.

Addition, the bringing of numbers or quantities together to find their amount.

Amount, the sum total of two or more numbers, brought into one.

Answer, the sum total of numbers, the result of an operation.

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* Page 230.

Carry, the adding of one denomination to another according to its ratio.

Column, a perpendicular line of figures. A *line* is the distance between two points, and has length only.

Row, a number of figures ranged in a *right* line.

Figure, the representation of one or more units.

Unit, the least whole number.

Number, one or more units

Adding, the act of putting numbers together.

The Rule for performing operations in Addition may be given thus:

1. Begin at the right hand column, and add together, upward, all the figures in it, and place the amount, if less than ten, under that column.

2. If the amount be just ten, place a cipher there, and carry one to the next left hand column.

3. If more than ten, or two or more even tens, set down all there is over, and carry one for each even ten to the next left hand column.

4. Proceed in this way through all the columns, and set down the full amount under the left hand column.

§ 4. The design of the following examples is to discover whether the pupil has been thorough in the solutions of previous questions, and to prevent him from transcribing the answers.

Find the sum of these two numbers, *viz*:

873687		475837
243787	and	632868
879634		678371
1997108		1787076.

Now add their sum into one total amount.

OPERATION.
1997108
1787076
3784184 *Amount.*

☞ In like manner perform the following

EXAMPLES.

1.	2.	3.	4.
22321	23432	110331	222311
41332	42212	224212	131232
12123	13124	103123	101221
13220	21101	220320	234031

Amt. of Nos. 1, *and* 2. 188865. *Amt. of Nos.* 3, *and* 4. 1346781.

5.	6.	7.	8.
321586	543214	943587	898989
984564	679846	846597	999874
876912	487532	756498	487563

Amt. of Nos. 5, *and* 6. 3893654. *Amt. of Nos.* 7, *and* 8. 4933108.

9.	10.	11.	12.
48642	42345	745213	376529
49421	37512	656329	487632
38764	48623	167432	474621

Amt. of Nos. 9, *and* 10. 265307. *Amt. of Nos.* 11, *and* 12. 2907756.

13.	14.	15.
84317427	78481387	13287342
73827241	87321874	38127438
23747287	33872337	85323471
21874881	81347824	23218137
34782838	23185933	13487312

Amt. of Nos. 13, 14, *and* 15 716202729.

§ 5. A Rule showing how to add by tens.

1. First, add the figures till they amount to ten or over, but not to 20—then place a dot.

2. Add all that is over ten to the next figure, till the whole column is added, then set down at the foot of the column all that is over *even* Tens.

Lastly, count the dots, calling each a unit—carry the amount to the next column; so proceed till all the columns are added.

EXAMPLE.

76.4.
.44 7
38.7.
.73 6.
49.7
39.2
.47.3.
38 2
.79.6.
36.7
.86.3.
.73 9
6843

EXPLANATION.—Say, 9 and 3 are 12—put a . opposite three [3.]; then say, 2 and 7 are 9, and 6 are 15—place a . opposite six, [6.] as before; then add the 5 to 2, which make 7, and 3 are 10—here place a . [3.]; now say, 2 and 7 are 9, and 6 are 15—here place a . [6.]; and add the 5 to 7, which make 12—place a . opposite seven, [7.]; then add the 2 to 7, which make 9, and 4 make 13—place a . opposite four [4.]

In like manner perform the remaining columns.

☞ Let the learner perform the following examples, and apply the dots as above.

84317427	78481387	13287342
73827241	87321874	38127438
23747287	33872337	85323471
21874881	81347824	23218137
34782838	23185933	13487312
28834713	93821738	87411841
48327232	13484813	22783487
75323831	38748381	81234871
83422134	21314819	31837482
474457584	471579106	396711381

Examples including every principle in adding.

1.	2.	3.	4.
7474221	978467	.8 7. 8 9. 7. 1	8. 7. 9. 6 4
1112126	989678	.9 8 9. 5. 6. 7	8. 6. 7. 8. 9.
1211651	18.....	3 5. 6. 7 9. 4.	5 7. 6 5 3
9797998	15....	4 3 2 9. 8 6	1 0 2 9 6
	17...	26 5 8 3 1 8	2 2 2 1 1
	10...		2 3 2 4 0 6
	13..		
	15		
	1968145		

5.	6.	7.	8.
51221			
95331	81541		
78241	98963	765432	476981
86832	86532	765432	000000
98611	99654	765432	763821
89764	98743	765432	000000
500000	465433	3061728	1240802

9.	10.	11.	12.
101010101	1020403	21437	67
102030202	2080503	1321	123
402010201	7010602	112	1274
201020301	3030101	21	13218
101010103	1070201	1	654321
907080908	14211810	22892	669003

13.	14.	15.	16
176142	1456172	6	2138741
274	2131	22	1326
4	231	313	212
471472	12	4471	17
469	1176621	58567	1
648361	2635167	213577	2140297

THEORETICAL PRINCIPLES.

1. Any given number may be increased by putting another number to it, which operation is called *Addition.* To find the sum of two or more numbers is also *Addition.* The numbers proposed to be added, are supposed to be the several parts which make up a sum; and their total *the whole;* Proving that the whole is equal to all its parts.

2. Units of one order must stand under each other;—units must first be added, then tens, then hundreds, &c. that we may save figures, and carry on the combination as we go from one order to

another. We set down units of one column, and carry the tens to the next. The whole amount is written down, at the last column, for, there is no order to which it can be added. Ten units of one order, make *one ten* of the next higher; therefore we reserve the tens of a higher order, and add them to a like denomination, which is *carrying one for every ten.*

3. Ciphers are not mentioned in adding, they neither increase nor lessen the value of figures: they make no value when they form an entire column, and their amount is a cipher. When orders of units, &c. are wanting in any column, the figures that stand above or below of the same order, are added in with those of a like denomination. When the amount of a column exceeds 9, the whole amount may be written down, if the *next* be comprised *entirely* of ciphers.

QUESTIONS ON ADDITION.

1. How may a number be increased? How is the amount of several numbers found? Does this increase the numbers added? *A.* No, the answer shows the whole amount in one sum. The whole is equal to all its parts, how can this be proved? What is Addition? What are the terms used in addition? Define each of them! What is *a definition?* What is essential to one? [*For answer, see Theoretical Principles and* § 3.]

2. What is the first step in Addition? Why should units stand under units, &c.? Why first added? What is set down under each column? *A.* Its own value. What is carried to the next? *A.* The units of the next order. What is this operation called? Why is this done? How do you perform an operation in Addition? [*See Rule*, § 3.]

3. What is the reason that ciphers are not mentioned in adding? What is the amount of a column of ciphers? When can the whole amount of a column be set down? When orders of units, &c. are omitted in some columns, how are the figures added? [*See Theoretical Principles, and* § 5, p. 35.]

CHAPTER II.

§ 6. When a number is to be added to itself several times, it may be quickly done by a shorter Rule, called

MULTIPLICATION.

It repeats a number of like terms as many times as the other given number contains a unit. In this rule there are two numbers given or implied, to find a third. The number to be repeated or added to itself, is generally greater than the number of times it is repeated. When performing an operation, the larger number is placed first, the smaller directly under it. The number to be repeated or added to itself is the number that is Multiplied. The number that repeats or adds the other to itself, is the number multiplied by. The sum made by multiplying is the answer.

Multiply 3432432 by 2.

OPERATION.	EXPLANATION.
3432432	To find the answer or product
2	here given to be multiplied,
6864864	

Rule.—Begin by saying, 2 times 2 are 4, which set down directly under 2, [the figure multiplied]: then go to the next, say, 2 times 3 are 6, which set down, again, say, 2 times 4 are 8, which set down: then say, again, 2 times 2 are 4, which set down: again, 2 times 3 are 6, which set down: then again, 2 times 4 are 8, which set down: lastly, say, 2 times 3 are 6, which set down as before. The operation, now performed, is multiplying one number by another, and is multiplication.

Exercises for the Slate.

1.	2.	3.	4.	5.
324323	2323223	121212121	3213213	212121212
2	3	4	3	4
648646	6969669	484848484	9639639	848484848

6.	7.	8.	9.
24342	231231	123412	121212
2	3	2	4
48684	693693	246824	484848

§ 7. In the previous § the learner formed an idea of what multiplication is; but not of *carrying*. The principle will now be introduced and explained.

OPERATION.	EXPLANATION.
345648342 3 1036945026	To perform the given operation.

Rule.—Begin by saying, 3 times 2 are 6: none to carry; then say, 3 times 4 are 12, set down 2: 1 to carry to the next product. Again, say, 3 times 3 are 9, and 1 to carry from 12 make 10, [*see the inverted comma in the Multiplication Table*;] set down 0; then say, 3 times 8 are 24, and 1 to carry from 10 make 25, (5): 2 to carry to the next. Again, say, 3 times 4 are 12, and 2 to carry from 25, make 14, (4); 1 to carry: then say, 3 times 6 are 18, 1 to carry from 14 make 19, (9); 1 to carry. Again, 3 times 5 are 15, 1 to carry from 19 make 16, (6); 1 to carry: then say, 3 times 4 are 12, and 1 to carry from 16 make 13, (3); 1 to carry—lastly, say, 3 times 3 are 9, 1 to carry from 13 make 10, which set down, being the whole amount. The above operation is carrying for 10 in multiplication, as in Addition.

Exercises for the Slate.

1.	2.	3.	4.
7689754	6896785	7879786	8978987
4	5	6	7
30759016	34483925	47278716	62852909

5.	6.	7.	8.
8989898	9999999	6987434	8698675
8	9	3	4
71919184	89999991	20962302	34794700

9.	10.	11.	12.
234374382	87654734	8765437978	453728321
7	8	9	5
1640620674	701237872	78888941802	2268641605

☞ Questions to be answered by the pupils in classes.

What is meant by multiplying a number? It is saying, 2 times 4 are 8, 3 times 3 are 9, 4 times 4 are 16, &c. 5 times 8 are how many? 40. What must be set down? 0. How many to carry to the next product? 4. Why? Because there are four tens in forty. 9 times 7 are how many? Set down what? Carry how many to the next? 8 times 3 are how many? Set down what? Carry how many to the next? 7 times 2 are how many? Set down what? Carry how many to the next?

Obs. The answers to the following examples are found in the Key, which let the pupil examine after he shall have performed each operation.

1.	2.	3.	4.	5.	6.
67895	67985	36789	47867	46789	86789
7	9	8	6	5	7

DEFINITION OF TERMS IN MULTIPLICATION.

Multiplication, is adding a given number to itself, as many times as there are units in the repeating number. They are called the *Multiplicand*, which is the number to be repeated. The *Multiplier*, which shows how many times the multiplicand is to be repeated or added to itself; and are called Factors.

Product, the amount of the Multiplicand, repeated a given number of times.

Multiply, the act of repeating a number; or the adding of a like sum to itself to find the result.

ILLUSTRATION.

Multiply	7261	*Multiplicand.*
By	7	*Multiplier.*
	50827	*Product.*

Obs. 7261 is the Multiplicand; 7 the Multiplier; 50827 the Product; and 7261 and 7 are the Factors, or producers of 50827. The Multiplicand is first written down: then the Multiplier, and the result of multiplying is the Product.

Multiply 327 by 3, by 4; and bring their products into one sum total.

OPERATION.

327	327	981 *First Product,*
3	4	1308 *Second Pro.*
981	1308	2289 Sum Total.

☞ In like manner perform the following Examples.

Multiply	4723	by 5; 6.	*Sum Total.*	51953
	3634	by 7; 8.	*Sum Total.*	54510
	7349	by 3; 4.	*Sum Total.*	51443
	8734	by 7; 3.	*Sum Total.*	87340
	87341	by 7; 3.	*Sum Total.*	873410
	3625	by 4; 6.	*Sum Total.*	36250
	6381	by 9; 4.	*Sum Total.*	82953

§ 8. When the multiplier consists of more figures than one.

Rule:—Multiply the *first* figure of the multiplicand by each figure in the multiplier, separately. Then take the second, and proceed as with the first; so continue to do with the others, (if any,)—*but* always place the first figure of the product directly under the figure you multiply by. There will be in such operations two or more lines of figures, which add together, and their sum will be the answer or product required.

Ex. Mult. 3723472 by 12 Mult. 32472321 by 132

OPERATIONS.

3723472	32472321
12	132
7446944	64944642
3723472	97416963
44681664	32472321
	4286346372

Exercises for the Slate.

1.	2.	3.
4732337623	3872138	83721383
13	35	24
61521039099	135524830	2009313192

☞ Questions to be answered by the pupils in classes.

How is any number of figures multiplied by a *single* figure? Repeat each figure in the multiplicand by the figure of the multiplier. How, by more than *one* figure? Repeat the multiplicand as many times as there are units in the multiplier. Multiply 42 by 3. How many figures in the multiplicand, multiplier, product? Multiply 21 by 13. How many figures in the multiplier? How many lines of figures to be *added* to obtain the product? *Ans.* Two. If by three figures, how many? If by four? Five? Six? &c.

§ **9.** When there is a cipher or ciphers intermixed with the significant figures.

Rule :—Set down whole numbers in the place of a cipher, and not carry to the next product.

Note 1. When there is but *one* cipher, set down whole numbers.

Note 2. When there are *two* or more ciphers, set down *whole* numbers under the *first* cipher; then say 0 is 0, &c.

Note 3. When there is a cipher or ciphers, and none to carry, set down a cipher or ciphers in the product.

Ex. Mult. 607807064 by 2 Mult. 907640308 by 7

OPERATIONS.

607807064	907640308
2	7
1215614128	6353482156

Note 2. **Ex.** Mult. 70086003 by 6 Mult. 7005009 by 5

OPERATIONS.

70086003	7005009
6	5
420516018	35025045

Note 3. Ex. Mult. 1020302102 by 3 Mult. 4003002004 by 2.

OPERATIONS.

1020302102	4003002004
3	2
3060906306	8006004008

Exercises for the Slate.

1.	2.	3.
70208307	30708236	8073083
8	38	523
561666456	1166912968	4222222409

4.	5.	6.
80091009	700613008	400573006
79	68	5678
6327189711	47641684544	2274453528068

7.	8.	9.
230203021	302030212	40302031
12	212	313
2762436252	64030404944	12614535703

☞ Questions to be answered by the pupils in classes.

What must be done when there are ciphers in the multiplicand? Set down the whole product figure, and not carry to the next product. When there is but one cipher what do? See Note 1. When two or more? See Note 2. What does the 3d Note say? Mult. 102 by 3. Mult. 705 by 5. Mult. 2006 by 3.

§10. When there is a cipher or ciphers intermixed with the figures in the multiplier.

Rule:—Multiply by the significant figures only. Omit the ciphers; and for every cipher remove the figure of the multiplier one place towards the left hand. Place the first figure of each product directly under its multiplier.

Ex. Mult. 47238127 by 3001 Mult. 382789 by 20403

OPERATIONS.

47238127	382789
3001	20403
47238127	1148367
141714381	1531156
141761619127	765578
	7810043967

Exercises for the Slate.

1.	2.	3.
387628	374683	473172
7203	30203	30608
2792084484	11316550649	14482848576

☞ **Questions to be answered by the pupils in classes.**

What must be done when there are ciphers in the multiplier? Omit them and put the first figure after the cipher, one place back towards the left hand. Mult. 121 by 102. How many lines will there be to add together to show the whole product? Two. If by 1002, how many? Two. Why? Because the ciphers are omitted.

§ 11. When there is a cipher or ciphers both in the multiplicand and multipler.

Rule:—Proceed as directed in § 9 and 10, relative to ciphers in the multiplicand and multiplier. Set first figure of the product directly under the multiplying figure—add up the products and it will produce the correct answer.

Ex. Mult. 7030217 by 2603. Mult. 807906 by 3057.

OPERATIONS.

```
   7030217          807906
      2603            3057
   -------         -------
   21090651        5655342
  42181302        4039530
 14060434        2423718
 -----------     ----------
 18299654851     2469768642
```

Exercises for the Slate.

```
      1.             2.              3.
   9807906         306709          670809
      9035          40809            9017
 -----------   ------------     ----------
 88614430710   125164875810     6048684753
```

☞ **Questions to be answered by the pupils in classes.**

When ciphers are in both the multiplicand and multiplier? Set down whole numbers for the ciphers in the multiplicand, and omit the cipher or ciphers in the multiplier. Give an example! Mult. 407 by 103. Mult. 103 by 102.

§ 12. When a cipher or ciphers terminate the multiplier.

Rule:—Omit the cipher or ciphers and perform the operation by the figures only; then bring the cipher or ciphers down and join it or them to the product, on the right hand. Bring the significant figures of the multiplier out perpendicular with those of the multiplicand.

Ex. Mult. 723284 by 320. Mult. 23697 by 7300.

OPERATIONS.

```
   723284          23697
      320           7300
  -------        -------
  1446568         71091
 2169852        165879
 ---------      ---------
 231450880      172988100
```

Exercises for the Slate.

```
      1.              2.              3.
   7475683         367812          43679
     35200          18700         826000
 ------------   ------------   ------------
 263144041600   6878084400     36078854000
```

Questions to be answered by the pupils in classes.
Suppose it were required to multiply a number by 20, or 300, how must the ures be placed? The figures ought to stand directly under the multiplicand; nit using the ciphers and join them to the product of the significant figures. ult. 14 by 20. Mult. 15 by 30. How many figures must there be joined to e product? One. If by 200 how many? If by 4000 how many?

§ **13.** When a cipher or ciphers terminate the multipli-
and.

RULE:—Multiply the significant figures together; then join the product as many ciphers as are annexed to the multipli-
and.

Ex. Mult. 874371820 by 32. Mult. 7897652000 by 72.

OPERATIONS.

874371820	7897652000
32	72
1748743640	15795304
262311546	55283564
27979898240	568630944000

Exercises for the Slate.

1.	2.	3.
78347300	876941000	3876740000
42	75	123
3290586600	65770575000	476839020000

☞ Questions to be answered by the pupils in classes.
When ciphers terminate the multiplicand, what is done with them? Join hem to the total product; after using the figures in the multiplier. Give an xample! Mult. 410 by 13. Mult. 5100 by 81.

§ **14.** When ciphers terminate both the multiplicand and multiplier.

RULE:—Perform the work only with the significant figures, hen find the number of ciphers both in the multiplicand and multiplier, and join them to the product.

Ex. Mult. 73824000 by 3600. Mult. 7438000 by 2600.

OPERATIONS.

73824000	7438000
3600	2600
442944	44628
221472	14876
265766400000	19338800000

Exercises for the Slate.

1.	2.	3.
7328000	38273000	3893262000
43200	73200	531000
316569600000	2801583600000	2067322122000000

Questions to be answered by the pupils in classes.

When ciphers terminate both the multiplicand and multiplier, what is t done? Multiply by the figures, and annex the same number of ciphers t product that there are in both the factors. Give an example! Mult. 41 30. Mult. 310 by 60.

§ 15. How to multiply any number by 10, 100, 10 10000, &c.

Rule:—Annex the *ciphers* of the multiplier to the figures the *multiplicand*, and the work is performed.

Ex. Mult. 578 by 10; 347 by 100; 879 by 1000; 976 10000; 762 by 100000.

OPERATIONS.

5780; 34700; 879000; 9760000; 76200000.

Exercises for the Slate.

7856 by 10. 8763 by 100. 7623 by 1000. 8793 by 10000

Note. There are a few useful contractions in multiplying, which cannot be give this place, for they include the rules of *Subtraction* and *Division*. There are a many ingenious contractions given by several arithmeticians, such as multiplying by 1 to 19; 101 to 119; 111 to 1111, and by 66, 777, &c.—but the methods are more intric and puzzling than useful, they are therefore omitted.

N. B. As it respects *composite numbers*, they are confined to but a few number and besides this, no person ever inquires what the composite number of a sum is, wh he wishes to find the amount of any article or commodity at a given price per yard bushel; these *facts* preclude the necessity of examples of composite numbers.

☞ Questions to be answered by the pupils in classes.

Multiply 231 by 2. What case is it? Where there is none to carry; and i is multiplying a number of figures by a *single* figure.* Mult. 87 by 3. Wha case is it? One of carrying. Mult. 423 by 4, by 7. What case is it? Whe *one number* is to be multiplied by *two* figures. Mult. 731 by 13. What ca is it? Where there must be an addition of products. Mult. 1002 by 3. Wha case is it? Of ciphers in the multiplicand. Mult. 1413 by 1003. What case this? Where ciphers are intermixed with figures in the multiplier. Mult. 40 by 103. What case is this? Where ciphers are intermixed in both multiplica and multiplier. Mult. 187 by 200. What case is it? Of ciphers terminatin the multiplier. Mult. 730 by 20. What case is this? Where ciphers terminate both factors. Mult. 4700 by 3. What case? Of ciphers terminating th multiplicand. Mult. 81 by 10. What case? Where annexing a cipher wil perform the operation.

A General Rule for multiplying any number of Figures.

Multiply each figure of the multiplier into each figure of th multiplicand separately, taking care when multiplying by unit to make the first figure of the result stand in the units' place; and when multiplying by tens, to make the first figure stand i the tens' place; and when multiplying by hundreds, to mak the first figure stand in the hundreds' place, &c. and then add the several products together.

Examples involving the principles of the foregoing Rules.

Obs. The following examples are intended to discov whether the pupil understands every part of Multiplication t perfection.

Mult. 213121 by 3 *Ans.* 639363

59876 by 6 *Ans.* 359256

---

* Let the teacher ask the pupil what should be done in each case.

Mult. 333333 by 77	*Ans.* 25666641
327819 by 10	*Ans.* 3278190
3471 by 666	*Ans.* 2311686
479618 by 100	*Ans.* 47961800
121800 by 8100	*Ans.* 986580000
2141732 by 3200	*Ans.* 6853542400
121700 by 121	*Ans.* 14725700
138172 by 1000	*Ans.* 138172000
4031041 by 2503	*Ans.* 10089695623
40002 by 6	*Ans.* 240012
1080206 by 7	*Ans.* 7561442
137241 by 40201	*Ans.* 5517225441
1006004 by 6	*Ans.* 6036024
7000005 by 9	*Ans.* 63000045
7901375 by 30000	*Ans.* 237041250000
2040302 by 2	*Ans.* 4080604
12318 by 7004	*Ans.* 86275272
5476208 by 3942	*Ans.* 21587211936
530674 by 45007	*Ans.* 23884044718
80970 by 4	*Ans.* 323880
10709 by 36	*Ans.* 385524
21700 by 27	*Ans.* 585900
360500 by 6	*Ans.* 2163000
27680709 by 40700609	*Ans.* 1126621713851781
504030201 by 102030405	*Ans.* 51426405540261405

THEORETICAL PRINCIPLES.

1. Any given number may be easily added to itself a given number of times, by repeating that number as many times as there are units in the repeating number, which operation is called *Multiplication*. The adding, of the same number to itself a certain number of times, is *Multiplication*.

2. The number to be repeated is generally the larger of the two given numbers, and is placed first in the operation, called the *Multiplicand*, yet this is immaterial, for the result will be the same, in either case. The number which shows how many times the other number is repeated, is called the *multiplier*, and is placed below the *multiplicand*. The sum produced by the repetition of the multiplicand, is the result of the operation, and called the *product*. The two numbers, which are producers of the product, are called *Factors*.

3. We begin to multiply at units' place, except in case of ciphers. As many figures as there are in the multiplier, so many several products there must be in the operation, before adding them together. Care must be taken in setting them down in their proper place. The first figure in each product, is ever of the same numerical denomination as its multiplier; units produce units; tens, tens; therefore the figures of each product, must be thrown back into the place of their own rank, in order to give to each product its proper value.

4. When the multiplicand or multiplier is terminated by a cipher, it is joined to the product, and the multiplying begins with the first significant figure; when intermixed in the multiplicand, its product is a cipher, which is written down when there is none to carry; and when found in the multiplier, it is nothing—and is neglected, and the next product begins with a higher order.

QUESTIONS ON MULTIPLICATION.

1. What is meant by the term multiply? *A.* It is to repeat a sum a given number of times. What is multiplication? What is the design of this rule? Give an example!

2. What number is placed first in the operation? What is immaterial? What number shows how many times the other is repeated? Where placed? What is the result of the repetition? What is meant by multiplicand? By multiplier? By product? By factors? How are they placed in an operation? Propose an example with a multiplicand, multiplier and product?

3. Where do we begin to multiply? What exception? How many several products will there be in an operation? What is necessary in setting them down? How are the figures of each product placed? Why?

4. What is done with ciphers when terminating a number? When intermixed with figures? What is its value in the multiplier? Where does the next product begin?

CHAPTER III.

§ 16. THE second primary principle in Arithmetic is

SUBTRACTION.

The design of this rule is to find the difference between two numbers, and to see what remains after the less number has been taken from the greater. It is the converse of addition; *that* puts numbers together, THIS takes one number from another.

Subtraction has two numbers given or implied to find a third, which is always less than the greater, and may be more than the second. In setting down sums to be subtracted, place the largest number first; then the smaller, and what is left of the larger number will be the Remainder. Units must be placed under units, tens under tens, &c.

From 387943872 Take 273431461.

OPERATION.	EXPLANATION.
387943872 273431461 114512411	To find the difference between the numbers here given to be subtracted,

RULE:—Begin by saying, 1 from 2 leaves 1; 6 from 7, 1; 4 from 8, 4; 1 from 3, 2; 3 from 4, 1; 4 from 9, 5; 3 from 7, 4; 7 from 8, 1; then 2 from 3, 1. The operation is completed. In like manner are the other examples in this § performed.

Exercises for the Slate.

1.	2.	3.
From 9786489	From 54786328	From 8673456
Take 7565327	Take 33454117	Take 3432324
2221162	21332211	5241132

4.	5.	6.
From 9988776	From 7894687	From 8789838
Take 1654623	Take 6733354	Take 3573524
8334153	1161333	5216314

☞ Questions to be answered by the pupils in classes.

What is the second principle in arithmetic? Subtraction. How many rows of figures are there given in this rule? Two. How many sought or found in the answer? One. What is the name given to the third row of figures? Difference or remainder.

§ **17.** The following examples and explanations are calculated to develop to the learner, the nature of the 0, and teach the pupil the theory of subtracting.

From 14727428	From 1776485	From 87956847
Take 12516200†	Take 1060475‡	Take 87015315
*2211228	716010	**941532
283098070	4874096	878308
171032030‖	3702036	476108
112066040	1172060	402200
1360876304	9)(487863078	86)(4589
1360526102	327542063	3272
*** 350202	9 160321015	86 1317

EXPLANATIONS.

†Naught from a significant figure is *that figure.*

*** When the figure or figures on the left hand of both numbers are alike, omit writing down the 0s.

‡ Equals taken from equals leave nothing—set down 0.

‖ Naught from naught is 0.

)(A figure in the upper line having no figure under it, is brought down and joined to the remainder.

The operations now performed may be reduced to the following

Rule:—The less number is always to be taken from the greater. Place the less number under the greater, (in the same order as in Addition,) draw a line under them, begin at the right hand, take each figure in the lower line from the figure under which it stands. The figures remaining will show the difference, or how much the greater number exceeds the less.

Exercises for the Slate.

From 34789376 Take 1438101	33351275 Rem.	From 79743078 Take 2323072*
From 20631278 Take 31234	20600044 Rem.	From 4927894 Take 3616372*
From 87604389 Take 53602343	34002046 Rem.	From 863424 Take 41323*

---

* For remainders see Key.

☞ Questions to be answered by the pupils in classes.

Is a cipher mentioned in subtracting? When in the lower line it is not? Why? Naught from a figure is what? 0. Why? Eight from eight leaves what? A cipher. What is this called? Equals from equals. A cipher taken from a figure leaves how many? That figure. Take a cipher from a cipher, what will remain? Nothing. Why?

DEFINITION OF TERMS IN SUBTRACTION.

Subtraction, the taking of one number or quantity from another.

Minuend, the larger number, and number from which the smaller is to be taken.

Subtrahend, the less number, which is to be taken from a larger.

Remainder, a part of the Minuend, and the sum that is left after subtracting.

Borrowing, taking a unit from a higher order and adding it to a lower, that we may subtract units of the same order from each other.

Carry, adding one to a figure in the subtrahend, to pay for the *ten* borrowed in the Minuend.

Obs. The words *excess*, *remainder*, and *difference*, are synonymous terms.

§ 18. When one or more ciphers come between the significant figures, and located in the minuend.

Rule:—Call the ciphers 10: add *one* to the next figure below the *first* cipher, then subtract: Add *one* to the first figure after the last cipher, in the lower line—then subtract, &c.

EXAMPLES AND OPERATIONS.

1.	2.	3.	4.
87300¶26	8730000¶32	7630000¶26	23040¶79
76246 13	362376 21	341243 14	21623 62
11054 13	510624 11	421757 12	1417 17

EXPLANATIONS.

¶ 1st. Ex. Say 6 from 10 leaves 4: then add 1 to 4, and say 5 from 10 leaves 5. Now add 1 to 2 and say, 3 from 3 leaves 0.

2d. Ex. Say 6 from 10 leaves 4: then add 1 to 7, and say 8 from 10 leaves 2: 4 from 10 leaves 6. Now add 1 to 2, and say 3 from 3 leaves 0.

3d. Ex. Say 3 from 10 leaves 7: then add 1 to 4, and say 5 from 10 leaves 5. Now say 3 from 10 leaves 7: then add 1 to 1, say 2 from 3 leaves 1.

4th. Ex. Again say 3 from 10 leaves 7: then add 1 to 2, and say 3 from 4, 1. Then say 6 from 10 leaves 4: now add 1 to 1, and say 2 from 3, 1.

Exercises for the Slate.

From 97030470 Take 32624316	64403154 Rem.	607008 342872	264136 Rem.
From 72042084 Take 51831462	20210622 Rem.	880497 774292	106205 Rem.

☞ Questions to be answered by the pupils in classes.
What is the value of ciphers in the minuend in subtracting? They are regarded as tens. When so valued, what is the operation called? Borrowing ten. How is this borrowing paid? By adding 1 to the next order. Why? It is adding equals to both numbers. Ten in units' place is equal to what in *tens'* place. Ans. One. Why? Because our notation increases in a tenfold ratio. Subt. 51 from 100. Subt. 87 from 200.

§ **19.** Instances frequently occur where many of the figures in the second line, or smaller number, exceed those of the first, or larger number.

Rule:—Prefix one to the upper figure when less than the figure under it; then add the *one* prefixed to the next figure on the left hand in the subtrahend, before subtracting it from the figure of the minuend.

Ex. Subtract 742307 from 938213.

OPERATION.

9 ·38 ·2 1 ·3
7 42 3 0 7
1 9*59 *0 6

EXPLANATION.

7 cannot be taken from 3; but suppose the · in the upper line to be 1. then 1 3. becomes 13: Now say 7 from 13 leaves 6; 1 to be added or carried to the 0 which makes 1, 1 from 1, 0; go to the next column, call the · 1 as before; then say 3 from 12, 9; carry 1 to the next figure, say 3 from 8. 5; go to the next, say 4 from 13. 9; 1 carried to 7 makes 8, 8 from 9 leaves 1; and the operation is performed.

The above operation is called borrowing 10, and carrying 1 to pay it.

ILLUSTRATION.

Seven units cannot be taken from 3; therefore add 10 to 3, which makes 13 units, (i. e. 1 ten and 3 units,) 7 from 13 leaves 6. Ten was added to the minuend, so there must be its equivalent added to the subtrahend, *which is* 1: for 1 is in ten's order, and is the same as 10 units—therefore the 0 becomes 1 ten. 1 ten from one leaves 0. 3 is in hundreds' place; so is 2:—3 hun. cannot be taken from 2—but it can from 12 hun.—3 hun. from 12 hun., 9 hundred. Now 2 is in thousands' place; and as 10 hun. make 1 thou. we must pay what was borro'd by adding 1, which is equal to ten, to 2, which makes it 3 thou.; 3 from 8 thou. leaves 5 thousand. Thus through all the orders.

---

* The remainder in Subtraction can never be over 9.

Exercises for the Slate.

1.
9 ·73 ·893 ·2
2 82 961 4
6 90 931 8

2.
87 ·324 ·2
85 412 5
1 911 7

3.
9 ·78 ·632 ·18
1 83 711 83
7 94 920 35

4.
98 ·7 ·63 ·8 ·2
32 9 72 9 6
65 7 90 8 6

5.
8 ·73 ·82 ·7
6 82 91 8
1 90 90 9

6.
9 ·1 ·02 ·03 ·7 ·1
7 2 91 91 9 4
1 8 10 11 7 7

☞Now let the pupil be interrogated as dictated on page 21.

Recapitulating the remarks, &c. in the two preceding §s, the RULE to be observed in performing subtraction may be given thus,

Place the less number under the greater, so that the units of the same order may be in the same column, and draw a line under them; beginning at the right, take successively each figure of the lower number from the one in the same column of the upper; if this cannot be done, prefix unity, i. e. increase the upper figure by ten units, counting the next significant figure, in the lower number, more by unity, and if ciphers come between, regard them as 10s.

Examples will now be presented which include every variety in subtraction.

1.
4507086
739007*

2.
510700930
170910742

3.
601191005
11010930

4.
214200040
107400760

5.
10000
999

6.
400500600700800900
320110230450670089

7.
600198056
350009278

8.
82795
69839

9.
12000988
11998986

10
1009052
1008987

11.
10000
1001

12.
1010101
101010

13.
10000000
1000101

14.
95000000
240000

---

* For Remainders see Key, page 230.

15.	16.	17.	18.
75643[*]	7000000	5500623	78645
9000	986432	2006935	46543

19.	20.	21.
1752895342	87461	5555555
561922471	18672	4187766

22.	23.	24.
11110001	3787634	878678941
1101119	1478121	95968978

25.	26.
32786532146	102367423179810
10679361234	87912845067032

§ 20. How to place numbers in order to be subtracted.

RULE :—Begin at the left hand, place the figures under each other as in addition. (Consult § 3, page 32.) Write units under units, tens under tens, &c.

From 786348 Take 43173.

OPERATION.

786348
43173
743175 *Rem.*

Examples for Practice.

From 7697834 Take 4387543 Rem. 3310291
From 9387463 Take 3794367 Rem. 5593096
From 7869438 Take 4358432 Rem. 3511006

From 89763 Take 734; Take 674, and add the remainders.

OPERATION.

89763	89763	89029 } *Rem.*
734	674	89089 } added.
89029	89089	178118 *Ans.*

Examples for Practice.

From	2378	Take	1103	Take	2163	*Ans.* 1490
	8586		1341		7355	*Ans.* 8476
	9685		587		1675	*Ans.* 17108
	1001		973		87	*Ans.* 942
	97387		8691		3	*Ans.* 186080
	36987		1721		100	*Ans.* 72153
	93871		1347		1111	*Ans.* 185284
	87638		1238		2312	*Ans.* 171726

---

[*] For remainders see Key. page 230.

From	38721	Take	1031	Take	639	*Ans.* 75772
	97894		732		327	*Ans.* 194729

From 90 take 30, from 40 take 10;
Subtract 6 from 60, and what remains then?

Ans. 144

THEORETICAL PRINCIPLES.

1. Any given number may be diminished by taking another from it, which operation is called *Subtraction.* And to find the difference between two numbers, is *Subtraction:* and consists in finding a *number*, which if added to the less, will just equal the larger. The number, from which subtraction is made, is understood to be the *whole*, and the number to be subtracted is supposed to be a part of *that whole;* consequently, if that part be taken from the whole, the *remainder* will be the other part, proving "that the whole is equal to all its parts."

2. The number to be subtracted, must be larger than the *number* subtracted, or it would be a *negative* quantity. The arrangement of the orders, must be the same as in addition, units under units, &c.—though the operation is limited to only two given numbers.

3. When each and all the figures in the minuend exceed those of the subtrahend, subtraction can be made by beginning at the left hand; [*see* § 16. *Ex.* 1 to 7.] indeed, it can be when that is not the case, if units of one order be taken from the same order. But for uniformity, in rules, we begin to subtract at units' place. If an order be wanting in the minuend, the deficiency is supplied by taking *one* from the next higher order, calling the cipher *ten*, and adding *one* to the next figure, in the subtrahend, which makes the next significant figure *one* less than its nominal value. Showing that we borrowed a unit of a higher order, and paid it by diminishing the figure in the subtrahend 1, (of the same order,)—consequently, the figure from which we borrowed becomes *one less*, and the figure to which the *one* is added, *ten greater*. Because, if two numbers be equally increased, their difference is the same; for 2 from 6 is 4, so is 12 from 16—both increased by 10. The result would be the same if the cipher were called 9, and the figure below it diminished by 1, and the carrying made to the next figure. Ciphers in the subtrahend do not diminish or increase the value of the figures in the minuend.

4. When one or more figures in the subtrahend exceed that of the minuend, it is called *ten*, and the figure in the upper line is added to the remainder—or prefix one to the upper figure and subtract from the amount, or take the upper figure from the lower; then take that remainder from ten, and set down the result; in either case, the next figure to be subtracted must be diminished *one* before taking it from the upper figure. The carrying will be paid before the end of the operation; but it must be continued when commenced, till the upper figure exceeds the lower. This operation is borrowing in *subtraction*, and is the converse of carrying in addition.

5. Prefixing *one*, or adding 10 to the upper figure, is taking one from the next left hand figure, of a higher order, which leaves it

one less than its nominal value; and adding *one* to the figure in the subtrahend, of the same order, pays it. This shows that the subtraction of a smaller sum from a larger is always possible.

QUESTIONS ON SUBTRACTION.

1. How can a number be diminished? How is the difference between two numbers found? What is the operation called? What sum is understood to be the whole? What a part of the whole? What will the remainder be? What does this prove?

2. What is a negative quantity? Do they occur in subtraction? Must units be placed under units? The operation is confined to how many numbers?

3. Can subtraction be performed by beginning at the left hand? Can it be when that is not the case? Why begin on the right hand? When is one taken from a higher order in the minuend? When is the cipher called ten? What does adding one to the next figure in the subtrahend do? What do we borrow a unit from, when subtracting? How is it paid? What becomes one less? What becomes ten greater? What if two numbers be equally increased? What if the cipher be called *nine?* What must be done? What of ciphers in the subtrahend?

4. When the upper figure is less than the lower, what methods of subtracting are there? What must be done with the next subtrahend figure? When will the carrying be paid? What is this operation called? It is the converse of what operation?

5. When we prefix one to the upper figure how many is taken from the next higher order? What does adding one to the figure in the subtrahend do? Is the subtraction of a less sum from a greater, in all cases possible?

CHAPTER IV.

§ **21.** The operation of Subtraction may be performed by a very short and quick Rule, called

DIVISION.

In this rule there are two numbers given to find a third. The larger number is first written down, then the smaller at the left thereof: then the inquiry is made how many times the smaller number is contained in the larger, which is found by making answer to the question, what figure in the multiplication table multiplied into the figure on the left hand, will make a number just equal to the larger number. This figure being found, and multiplied therewith will perform the operation.

Division is either single, or composed of a number of figures. Single, when the dividing number consists of one figure only, and the divided number of two or more figures.

§ **22.** Division is a rule by which we discover how often one number is contained in another.

EXAMPLES.	OPERATIONS.	EXPLANATIONS.
Divide 8 by 4	4)8(2 8	What figure multiplied by 4 will make 8? 2, for 2 times 4 are 8.
Divide 9 by 3	3)9(3 9	3 in 9 how many times? 3 times, for 3 times 3 are 9.
Divide 6 by 2	2)6(3 6	6 contains 2 how many times? 3 times, for 3 times 2 are 6.
Divide 4 by 2	2)4(2 4	How many times 2 in 4? 2 times, for 2 times 2 are 4.
Divide 6 by 3	3)6)2 6	6 is how many times 3? 2 times, for twice 3 are 6.

Exercises for the Slate.

2 in 8 how many times? *Ans.* 4; Perform the operation on the slate.

If 9 be divided by 3 what is the *Ans.* 3; How done on the slate.

8 is how many times 4? *Ans.* 2; Show the operation on the slate.

9 contains 3 how many times? *Ans.* 3; Exhibit the operation on the slate.

Divide 8 by 4. *Ans.* 2 Divide 9 by 3 *Ans.* 3 Divide 4 by 2. *Ans.* 2
Divide 6 by 3. *Ans.* 2 Divide 8 by 4. *Ans.* 2 Divide 8 by 2. *Ans.* 4
Divide 4 by 2. *Ans.* 2 Divide 6 by 2. *Ans.* 3 Divide 9 by 3. *Ans.* 3

OBS. The operation is essentially the same if the divided number be increased to two figures, if they can be measured by the dividing number.

EXAMPLES.	OPERATIONS.	EXPLANATIONS.
Divide 72 by 8	8)72(9 72	What figure multiplied by 8 will just produce 72? 9, for 8 times 9 are 72.
Divide 48 by 6	6)48(8 48	6 in 48 how many times? 8 times, for 6 times 8 make 48.
Divide 56 by 8	8)56(7 56	8 is one factor and 56 is the product of another factor, which is to be found; and is 7, for 8 times 7 are 56.

Exercises for the Slate.

Divide 81 by 9 *Ans.* 9

Divide 63 by 7 *Ans.* 9

Divide 45 by 9 *Ans.* 5

Find what two figures multiplied together will make 63. *Ans.* 9 and 7

54 is produced by multiplying two figures into each other. What are they? *Ans.* 6 and 9

4 is one factor of 32, what is the other? *Ans.* 8, for 4 times 8 are 32

§ 23. Division is finding how many times one number is contained in another, and what remains.

EXAMPLES.	OPERATIONS.	EXPLANATIONS.
Divide 39 by 6	6)39(6 36 3 *rem.*	39 is not a number that can be produced by multiplying two figures together, therefore the nearest figure is sought.
Divide 67 by 8	8)67(8 64 3 *rem.*	67 is not a composite number, i. e. made by the multiplication of two or more numbers into each other, for this reason there must be a remainder.
Divide 58 by 7	7)58(8 56 2 *rem.*	7 times 8 are 56, which is the nearest possible product to 58, therefore 8 is the nearest factor that could be chosen to make 58.

Exercises for the Slate.

Divide 50 by 9, i. e. find the times 9 is contained in 50, and what remains.

Divide 67 by 9, find the answer and what is left of the divided No.

Divide 38 by 4, see how many times 4 is contained in 38, and what is over.

Divide 87 by 9, perform the operation and find what is remaining at the close of the operation, the number is called the *Remainder.*

☞ Division is finding how many times one number may be taken or subtracted from another.

EXAMPLE 1.

How many times 6 in 24 or 6 can be taken from 24 how many times? *Ans.* 4 *times.*

OPERATION.

	24
1.	6
	18
2.	6
	12
3.	6
	6
4.	6
Ans.	0

OR.

6) 24 (4
24
00

Exercises for the Slate.

How many times 6 in 30? 9 can be taken from 63 how many times?

How many times 7 in 49? 7 is contained in 42 how many times?

EXAMPLE 2. **OPERATION.**

5 in 25 how many times, or 5 can be taken from 25 how many times? *Ans.* 5 *times.*

```
   | 25
1. |  5
   | --
   | 20
2. |  5
   | --
   | 15
3. |  5
   | --
   | 10
4. |  5
   | --
   |  5
5. |  5
   | --
   |  0
```

OR.

```
5) 25 (5
   25
   --
   00
```

Exercises for the Slate.

How many times 8 in 72? 8 in 56 how many times?

§ **24.** Explanation of the terms Divisor, Dividend, Dividual, Quotient and Remainder.

In division there are two numbers given to find a third.

1. The number given to be separated or divided, which is generally the largest of the two given numbers; and is placed at the right of the dividing number, is called *The Dividend.**

2. The separating number, which is placed on the left of the divided number, and is a less number; is called *The Divisor.*‡

3. The answer, or number which shows how often the dividing number is contained in the number divided, or the number of parts into which the *dividend* is then separated; is called *The Quotient.*‖

4. The number that is made after subtracting the sum produced from multiplying the divisor and quotient figure together, with a figure of the dividend annexed thereto, is called *The Dividual.*†

5. At the close of the operation, there is sometimes an uncertain number, (part of the divided number,) which is less than the divisor; and is called *The Remainder.*¶

These terms may be represented to the pupil's better understanding as follows:

REPRESENTATION OF PARTS.	NAMED.
‡7) 864* (123‖	864 *the Dividend.*
7	7, 16. and 24 *the Dividuals.*
†16.	7 *the Divisor.*
14	123 *the Quotient.*
†24.	and 3 *the Remainder.*
21	
3¶	

DEFINITION OF TERMS IN DIVISION.

Dividend, the number to be measured or separated into parts by another given number.

Divisor, the number by which the dividend is to be measured or separated into equal parts.

Quotient, the number that shows how many times the divisor is contained in the dividend.

Dividuals, numbers in division, being a part of the dividend, together with the subtracted products.

Remainder, the number that is left at the end of some operations, in division, which must be less than the divisor, and of the same kind with the dividend.

From the preceding operations, explanation, definitions, &c. the following RULES may be presented.

1. When the divisor consists of one figure only, and the dividend of one or two,

RULE:—Find a figure, and place it in the quotient, which when multiplied by the divisor will just produce the dividend, place the product directly under the dividend; then substract it therefrom, if ciphers be the result the operation is correct.

2. When the divisor is a single figure, and the dividend consists of a number,

RULE:—Take but one figure of the dividend for a dividual at a time. Seek how many times the divisor can be had in it, place the number of times in the quotient; multiply the divisor by the quotient figure, subtract the product from the dividual; then bring down the next figure and annex it to the remainder for another dividual, so continue to seek, multiply, subtract, and bring down, till no figure shall remain unused.

EXAMPLES AND OPERATIONS

```
3)87342(29114      3)93469(31156      4)7898(1974
  6····              9····              4···
  27·                3·                 38·
  27                 3                  36
   3·                 4·                 29·
   3                  3                  28
    4·                16·                 18·
    3                 15                  16
    12·                19·                 2 Rem.
    12                 18
    00                  1  Rem.
```

EXPLANATION of the first example which will virtually illustrate the whole.

Say, 3 in 8 2 times, place it at the right of the curve: then say 2 times 3 are 6, place it under 8—then say 6 from 8 leaves 2: then bring down 7 and join it to 2, which becomes 27: then say 3 in 27 9 times, 9 times 3 are 27—27 from 27—naught 00; then bring down 3, and say 3 in 3, 1; place the one as before; say once 3 is 3; 3 from 3, naught 0; then bring down 4, say, 3 in 4, 1; then say once 3 is 3 3 from 4, 1; bring down 2 and join it to 1, which becomes 12 3 in 12, 4 times; 4 times 3 are 12—12 from 12, naught 0—the operation is now completed.

Exercises for the Slate.

2)71314	*Ans.*	35657	2)6789274	*Ans.*	3394637
3)423438	*Ans.*	141146	3)37485765	*Ans.*	12495255
4)5678356	*Ans.*	1419589	4)4545688	*Ans.*	1136422
5)687895	*Ans.*	137579	6)6786792	*Ans.*	1131132

§ **25.** Examples in Single division where the first figure of the dividend is less than the divisor.

Rule:—Take two of the first figures in the dividend for a dividual; then seek how many times the divisor is contained in it: place the number of times in the quotient, multiply the divisor by the figure, subtract the product from the dividend: then bring down the next figure and annex it to the remainder for a dividual, so continue to do, till all the figures in the dividend shall have been brought down and used.

☞ Never place a cipher in the quotient the first time you seek.

EXAMPLES AND OPERATIONS.

4)13·87464(346866	7)16·32274(233182
12 · · · · ·	14 · · · · ·
18·	23·
16	21
27·	22·
24	21
34·	12.
32	7
26·	57·
24	56
24·	14·
24	14
00	00

Note. The dots (·) show that the figures above them must be brought down and joined to the dividual.

Exercises for the Slate.

Divide 254624 by 8	Divide 157492 by 4
Quot. 31828	Quot. 39373

Divide 3746892 by 6
Quot. 624482
Divide 134724 by 6
Quot. 22454
Divide 284864 by 4
Quot. 71216

Divide 192798 by 9
Quot. 21422
Divide 417865 by 5
Quot. 83573
Divide 389746 by 7
Quot. 55678

☞ Questions to be answered by the pupils in classes.
2 in 127, how many times? *Ans.* Say, 2 in 12, 6 times, 7 over. 2 in 180? say, 2 in 18, 9 times, 0 over. 3 in 241? say, 3 in 24, 8 times, 1 over. 3 in 271? say, 3 in 27, 9 times, 1 over. 6 in 184? say, 6 in 18, 3 times, 4 over. 4 in 280? say, 4 in 28, 7 times, 0 over. 7 in 496? say, 7 in 49, 7 times, 6 over. 5 in 404? say, 5 in 40, 8 times, 4 over. 8 in 642? say, 8 in 64, 8 times, 2 over. 6 in 540? say, 6 in 54, 9 times, 0 over. 4 in 361? say, 4 in 36, 9 times, 1 over.

What must be done with what is over? *Ans.* It must be brought down and joined to the dividual, each time after subtracting.

§ **26.** Examples of long division will now be presented with two, three, or more figures in the divisor.

Ex. Divide 7890123 by 426. *Ans.* 18521

OPERATION.

```
426)789·0123(18521
    426 ····
    ----
    3630·
    3408
    -----
     2221·
     2130
     -----
       912·
       852
       ----
       603·
       426
       ---
Rem.   177
```

EXPLANATION.

There are 3 figures in the divisor, point off 3 in the dividend for a dividual:—seek how many times 426 is contained in 789, (1) then multiply 426 by 1, place 426 under 789 and subtract it therefrom; there remains 363; then bring down the 0, and annex it to 363; making it 3630 for a dividual; then seek how often 426 is contained in 3630, (8 times); then multiply 426 by 8, which produces 3408; place it under 3630, and subtract 3408 therefrom—the remainder will be 222; bring down the 1 and annex it to 222; making it 2221 for a dividual. Continue the operation till you have used all the figures in the dividend, and obtained a remainder of 177, which is less than the divisor, 426.

From the preceding example and explanation we derive this

Rule:—Point off as many places in the dividend as there are places in the divisor for the *first* dividual. Seek how many times the divisor is contained in it, place the number of times in the quotient; multiply each figure of the divisor by the figure placed in the quotient; subtract the product arising from the multiplying of the divisor by the quotient figure from the dividual; then bring down *but one* figure out of the Dividend and annex it to the remainder for a dividual: (☞ which is all that ever must be brought down out of the dividend,) *then* seek how many times the divisor may be contained in the dividual, place the number in the quotient—multiply the divisor by the quotient figure, and subtract the product therefrom; so go on with the

operation till the last subtraction will be ciphers, or a number less than the divisor.

☞ **Questions to be answered by the pupils in classes.**

1. What is single division? It is dividing a number by a single figure. How do you divide a number in single division? By finding a quotient figure, which will just produce the dividend. How is that figure found? By multiplication.

2. How do you divide a large number by one figure? Take one figure at a time, out of the dividend, for a dividual. How many figures are joined to the remainder each time? One. What does this make? A new dividual each time. How do you know when the operation is correct? If the last product be as the last dividual, it is right.

3. When the first figure of the dividend is less than the divisor, what must be done? Take two figures for a dividual. How many figures must be brought down after subtracting? One; and it must be annexed to the remainder, for a dividual. One thing must *never be done*, what is it? Never place a 0 in the quotient, because the divisor is larger than the same number of figures in the dividend. What must be done? Take one or more figures, or a sufficient number so that it will go.

4. What part of a division sum is the dividend? It is the number to be divided. What is the divisor? It is the number by which the dividend is divided. The quotient is what? It is the number of times that the divisor is contained in the dividend. What makes the dividual? It is made by joining one figure to what is left after subtracting. The remainder is what? It is a number less than the divisor, and a part of the dividend which sometimes remains at the close of an operation.

§ **27.** Rules to determine how many times the divisor will go in the dividend and dividuals.

1. When the *first* figure of the divisor is contained in the first figure of the *dividend or dividual*,

Rule:—Find how often the *first* figure of the Divisor is contained in the *first figure of the Dividend* or *Dividuals*, the answer is the true quotient figure.

EXAMPLES AND OPERATION.

```
21230)89399·861(4211
      84920 ···
      -----
       44798·
       42460
       -----
        23386·
        21230
        -----
         21561·
         21230
         -----
           331
```

EXPLANATION.

2 in 8, [first figures of the divisor and dividend] 4 *times*, first quotient figure. 2 in 4, [first figure of the dividual] 2 *times*, 2d quotient figure. 2 in 2, [2d dividual] 1, 3d quotient figure. Again, 2 in 2, [3d dividual] 1, last quotient figure.

Exercises for the Slate.

Divide 26483·21 by 23645
Quot. 112 Rem. 81
Divide 47984·56 by 43162
Quot. 111 Rem. 7474
Divide 97872·34 by 23811
Quot. 411 Rem. 913

Divide 367848·3219 by 303212
Quot. 12131 Rem. 218447
Divide 89783·128 by 42321
Quot. 2121 Rem. 20287
Divide 56912·46 by 51232
Quot. 111 Rem. 4494

☞ **Questions to be answered by the pupils in classes.**
How many figures must be pointed off in the dividend? As many as there are in the divisor. What is to be done when the second figure of the *dividend or dividual* exceeds the second of the divisor? Ans. See, if the first figure of the divisor will go in the *first* figure of the dividend, if it will, it will be the true quotient figure. Give an example! Divide 26 by 21. Divide 47 by 23.

2. When the *first* figure of the *dividend* is less than that of the divisor.

Rule :—Find how many times the first figure of the divisor is contained in the *first two* figures of the *dividend* or *dividuals*, the number will be the correct quotient figure.

EXAMPLES AND OPERATION.

```
54256)126549·3252(23324
      108512····
       180373·
       162768
        176052·
        162768
         132845·
         108512
          243332·
          217024
           26308
```

EXPLANATION.

5 in 12, [5 first divisor figure; 12 first two dividend figures] 2 *times*. 5 in 18, [5 divisor figure; 18 first two dividual figures] 3 *times*. Then 5 in 17, [next dividual figures] 3 *times*. Now, 5 in 13, [next dividual figures] 2 *times*. Again, 5 in 24 [last dividual] 4 *times*.

Exercises for the Slate.

Divide 119184·6 by 31273
Quot. 38 Rem. 3472
Div. 317148·675 by 41321
Quot. 7675 Rem. 10000

Divide 1139717·4324 by 512321
Quot. 22246 Rem. 81358
Div. 1335331·234364 by 602134
Quot. 2217664 Rem 339388

☞ **Questions to be answered by the pupils in classes.**
What is to be done when the first figure of the divisor exceeds the first of the dividend or dividual? Take the *first two*, and inquire how many times the *first* figure of the divisor will go in them, the No. will be the right quotient figure. Give an example! 2 in 16; 6 in 18; 9 in 27.

3. When the *second* figure of the *dividend* exceeds the FIRST figure of the *Divisor*,

Rule :—Multiply the *second* figure of the *divisor* by the number of times that the *first figure of the divisor appears* to go in the *first* figure of the *dividend or dividual*; then diminish the figure so found (on account of carrying) 1, 2 or 3, and it will give the true quotient figure each time.

EXAMPLES AND OPERATIONS.

```
27235)63273·0626(23232
      54470 ····
      ------
       88030·
       81705
       ------
        63256·
        54470
        ------
         87862·
         81705
         ------
          61576·
          54470
          ------
           7106
```

EXPLANATION.

2 in 6 [first divisor and dividend figures] 3 times; now 3 times 7 [2d divisor figure] are 21; 2 to carry—3 times 2 [first divisor figure] are 6, and 2 carried are 8, which is 2 more than 6: [first figure of the dividend] therefore, 2 is the first quotient figure. Now, 2 in 8, [first dividual figure] 4 times; then 4 times 7, [2d divisor figure] are 28. 2 to carry; 4 times 2, [first divisor figure] are 8 and 2 carried are 10, which is 2 more than 8, [first dividual figure]; therefore, 3 is the true quotient figure. 2 in 6 [next dividual figure] 3 times; then 3 times 7 are 21; 2 to carry—3 times 2 are 6 and 2 are 8, which is 2 more than 6, [first dividual figure]. 2 is the next quotient figure. 2 in 8 [next dividual figure] 4 times, 4 times 7 are 28—2 to carry; then 4 times 2 are 8, and 2 carried are 10, which is 2 more than 8; [3d dividual], 3 next quotient figure. Then 2 in 6 [last dividual] 3 times; now 3 times 1 is 3, which is 2 more than 1, [2d dividual figure]; therefore, 2 is the last quotient figure.

Exercises for the Slate.

Divide 304·828 by 131 Quot. 2326 Rem. 122	Divide 68132·782 by 12472 Quot. 5462 Rem. 10718
Divide 8804 872 by 26487 Quot. 332 Rem. 11188	Divide 88075·784 by 27325 Quot. 3223 Rem. 7309
Divide 622·7134 by 267 Quot. 23322 Rem. 160	Div. 87816·65452 by 26345 Quot. 333333 Rem. 7567

☞ Questions to be answered by the pupils in classes.

What is to be done, when the second figure of the dividend exceeds the *first* figure of the *divisor?* Ans. See how many times the *first* figure of the divisor is contained in the first of the dividend or dividual, and lessen the No. one or two. Give an example! 28 in 62; 37 in 92.

Remark. The greatest difficulty, in division, is to know how to proceed in every new case that may occur.

§ **28.** The previous section and the following examples include every variety in long division.

N. B. Each example should be wrought at length by the pupil—the book being closed.

I. With a remainder, which must be less than the divisor.

Rule:—Divide, and obtain a remainder, which will be less than the divisor.

OPERATION.

```
9)28·974(3219
  27 ···
  --
   19.
   18
   --
    17.
     9
    --
    84.
    81
    --
     3 Rem.
```

II. With one or more ciphers at the right hand of the divisor and dividend.

RULE:—Cut off the same number of ciphers in the dividend as in the divisor; divide by the significant figures, and then annex the remaining ciphers to the remainder.

OPERATION.

```
55|0)5290|0(96
      495 ..
      ---
       340.
       330
       ---.
        100 Rem.
```

III. With ciphers in the divisor, but *none* in the dividend.

RULE:—Cutoff as many figures of the dividend as there are ciphers in the divisor; and annex them to the remainder.

OPERATION.

```
67|000)86348|634(1288
       67     ...
       --
       193
       134
       ---
        594
        536
        ---
         588
         536
         ---...
          52 634 Rem.
```

IV. With ciphers in the quotient when the divisor will not go in the dividend.

RULE:—Whenever a figure is brought down and annexed to the dividual, and the dividual remains still less than the divisor, a cipher must be placed in the quotient, and another figure brought down; and if the dividual then be less than the divisor, there must be another figure brought down, and so on, *till* the dividual becomes just equal to or exceeds the divisor.

OPERATION.

```
54)147·42271(273005
   108     ..
   ---
    394
    378
    ---
     162
     162
     ---..
       271
       270
       ---
         1 Rem.
```

V. After subtracting a product from a dividual, and there remains nothing; yet there is a cipher or ciphers attached to the dividend.

RULE:—Place the ciphers attached to the dividend at the right hand of the quotient figures and the operation is completed.

OPERATION.

```
335)586·2500(17500
    335  ..  ..
    ---
    2512
    2345
    ----
     1675
     1675
     ----
     0000
```

VI. With ciphers in the dividend, but none in the divisor or quotient.

RULE:—Bring down the ciphers, one at a time out of the dividend, and annex them to the dividual in the same manner as if they were figures.

OPERATION.

```
52)46200(888
   416..
   ---
    460.
    416
    ---
     440.
     416
     ---
      24 Rem.
```

VII. With a cipher on the left hand of the dividual.

Rule:—Ciphers on the left hand of *whole numbers*, are of no value.

OPERATION.

```
8)16·08·016(20102
  16·····
  --
   08.
    8.
   --
    016.
     16
     --
     00
```

VIII. When the 2d figure of the divisor exceeds that of the dividend.

Rule:—Point off one more figure in the dividend, than there are of the divisor.

OPERATION.

```
467)4593·6(98
    4203 ·
    ----
     3906·
     3736
     ----
     170 Rem.
```

IX. The greatest number of times a divisor can be contained in any dividual.

Rule:—The divisor is never contained in a dividual more than 9 times.

OPERATION.

```
326)325674(999
    2934
    ----
     3227
     2934
     ----
      2934
      2934
      ----
      0000
```

☞ Questions to be answered by the pupils in classes.

The remainder in division must be less than what? The divisor. When there is a cipher or ciphers in the divisor and dividend, what must be done? Cut them off. When there are ciphers in the divisor, and none in the dividend, what may be done? Cut off as many figures in the dividend as there are ciphers in the divisor. When the divisor will not go in a dividual, what must be done? Place a cipher in the quotient, and annex another figure to the dividual. When there are ciphers on the right hand of the dividend, what must be done? Bring them down and join them to the remainder. Are ciphers of any value on the left hand of the dividual? No. What is the reason that the divisor can never go more than 9 times in a dividual? Because if the divisor be multiplied by 10, it will become greater than the dividual.

Examples including the principles of the preceding operations.

		Divisor.	*Quotient.*	*Rem.*
I.	Divide 211499 by	8	26437	3
	Divide 318727 by	5	63745	2
II.	Divide 9622800 by	1320	7290	000
	Divide 84981700 by	970	87610	000
III.	Divide 654654 by	4500	145	2154
	Divide 149596478 by	12000	12466	4478
IV.	Divide 1212288 by	24	50512	00
	Divide 27072329 by	47	576007	00
V.	Divide 14968800 by	324	46200	00
	Divide 21156900 by	327	64700	00
VI.	Divide 76800 by	52	1476	48
	Divide 576000 by	145	3972	60
VII.	Divide 240364 by	12	20030	4
	Divide 46092 by	23	2004	00

VIII.	Divide 3782681 by	3871	977	714
	Divide 276932 by	287	964	264
IX.	Divide 4679532 by	468	9999	
	Divide 8450541 by	8459	999	

PROMISCUOUS EXAMPLES.

Divide 1406373 by 108
Quot. 13021 Rem. 105

Divide 146200 by 430
Quot. 340 Rem. 0

Divide 4944000 by 960
Quot. 5150 Rem. 000

Divide 987654321 by 123456
Quot. 8000 Rem. 6321

Divide 42952 by 160
Quot. 268 Rem. 72

Divide 30000 by 365
Quot. 82 Rem. 70

Divide 78901 by 32
Quot. 2465 Rem. 21

Divide 215059670 by 125
Quot. 1720477 Rem. 45

Divide 49561776 by 5137
Quot. 9648 Rem. 0000

Divide 1191467 by 57
Quot. 20902 Rem. 53

Divide 1765391 by 180
Quot. 9807 Rem. 131

Divide 11200 by 2240
Quot. 5 Rem. 00

A General Rule for working Division.

§ **29.** 1. Point off for the first period as many places in the dividend as there are figures in the divisor, except when the first figure in the dividend is less than the first figure in the divisor; then point off one figure more for the first period.

2. Seek how many times the divisor is contained in the first period, place the number of times in the quotient.

3. Multiply the divisor by the quotient figure;

4. Subtract the product from the figures pointed off;

5. Bring down the next figure from the dividend, and annex it to the remainder. (This makes the first dividual.)

6. Then seek how many times the divisor is contained in the dividual, ☞ which can never be over 9 times, and place the number of times in the quotient, &c.

A General Rule to determine how many times the Divisor is contained in the Dividend and Dividuals.

1. Inquire whether the first figure of the divisor can be contained in the first figure of the dividend; if it can, the number of times will give the first quotient figure—provided *the* 2d *and* 3d *figures of the dividend exceed* those of the divisor. If they do not, the quotient figure must be diminished 1 or 2.

2. Inquire whether the first figure of the divisor will go in the *first two* figures of the dividend, if so, the number of times will be the quotient figure—provided *the* 3*d and* 4*th figures of the dividend exceed* the 2d and 3d of the divisor. If they do not, the quotient figure must be diminished 1 or 2.

Exercises for the Slate.

Divide 1198·714* by 396
Quot. 3027 Rem. 22

Divide 987·651192 by 132
Quot. 7482206 Rem. 000

---

* The dots (·) show the number of figures in the first period.

Divide 159·84 by 48
Quot. 333 Rem. 00
Divide 2326·174 by 600
Quot. 3876 Rem. 574
Divide 712·364 by 462
Quot. 1606 Rem. 392
Divide 64·95436 by 52
Quot. 121912 Rem. 12
Divide 217·33985 by 91
Quot. 238835 Rem. 00

Divide 1234·56789 by 365
Quot. 338237 Rem. 284
Divide 313·26744 by 114
Quot. 274796 Rem. 00
Divide 2674·236 by 634
Quot. 4218 Rem. 24
Divide 999·936 by 144
Quot. 6944 Rem. 000
Divide 4798·9536925 by 735
Quot. 65291886 Rem. 715

§ 30. Examples with the amount of the quotients arising from two operations added into one. The direct figures to each quotient are in the Key, page 230.

Let the pupil be required to find a correct Answer without consulting the Key for each quotient figure.

Ex. Divide 874632 by 31, by 42.

OPERATION.

31)874632(28213
29 Rem.

42)874632(20824
24 Rem.

28213 } Quot.
20824 } figures.
49037 *Amt. of.*

Exercises for the Slate.

2 Div. 473482 by 46 by 36 — Amt. 23445 Rem. 4. 10
3 Div. 873462 by 41 by 83 — Amt. 31826 Rem. 39. 53
4 Div. 8787238 by 432 by 137 — Amt. 84480 Rem. 358. 58
5 Div. 3787384 by 212 by 612 — Amt. 24053 Rem 4. 328
6 Div. 97978436 by 521 by 621 — Amt. 345833 Rem. 218. 161
7 Div. 74863284 by 632 by 712 — Amt. 223599 Rem. 356. 44
8 Div. 9784632 by 631 by 821 — Amt. 27423 Rem. 346. 775

☞ Questions to be answered by the pupils in classes.

What is the first step in division? To write down the dividend and divisor. Name the other steps! Point off: seek: multiply: subtract; and bring down. What is meant by divisor? By dividend? By quotient? By remainder? By dividual? What rules are used in dividing? Multiplication, and Subtraction.

When we say, 3 is contained in 20, 6 times, and 2 over, which of these numbers is the divisor? Which the dividend? Which the quotient? Which the remainder? Propose an example that has a divisor, dividend, quotient and remainder?

SHORT DIVISION.

§ 31. This is a contraction of long division.

Examples for Operation.

Divide 7642 by 2 Divide 7695 by 2 Divide 61231280 by 5

OPERATIONS.

2)7642
3821

2)7695
3847 1 Rem.

5)61231280
12246256

EXPLANATIONS.

1 Ex. Here one half of 7 is 3; set down 3 and carry the 1 that remains; then half of 16 is 8; set down 8, and half of 4 is 2, and half of 2 is 1; so the quotient is 3821.

2 Ex. 2 in 7, 3 times and 1 over: then 2 in 16, 8; then 2 in 9, 4; and 1 over: then 2 in 15, 7; and 1 over.

3 Ex. 5 in 6, once, and 1 over, set down 1: then 5 in 11, twice, and 1 over, set down 2: then 5 in 12, twice and 2 over, set down 2: then 5 in 23, 4 times, and 3 over, set down 4: then 5 in 31, 6 times, and 1 over, set down 6: then 5 in 12, twice and 2 over, set down 2: 5 in 28, 5 times and 3 over; set down 5: then 5 in 30, 6 times.

A general Rule for Short Division.

Inquire how often the divisor is contained in the first left hand figure of the dividend, and if that be too small to contain it, take the first two; set that number down for the first left hand figure of the quotient—next, multiply the divisor by the quotient figure, and subtract the product from the figure, or figures, taken in the dividend, and to the remainder annex the next figure of the dividend for a new dividual (performing this operation in your mind only) and extend the same process to each remaining figure of the dividend successively, setting down only the quotient.

Exercises for the Slate.

				Quotients.	Rems.
Divide	7687267	by	2	3843633	1
	8719875	by	3	2906625	0
	1760019	by	5	352003	4
	576973	by	7	82424	5
	867918	by	9	96435	3
	8965462	by	6	1494243	4
	2687146	by	4	671786	2

§ **32**. The following examples, in addition, subtraction, multiplication and division, have incorrect answers annexed, that the learner may prove them to be so, and by that means obtain a correct answer to each example.

ADDITION is proved by subtracting *the total less top line* from The SUM TOTAL.

SUBTRACTION is proved by adding the Remainder and *Subtrahend* together.

MULTIPLICATION is proved by dividing the *Product* by the *Multiplier*: The *Multiplicand* will answer to the Quotient.

DIVISION is proved by multiplying the quotient by the divisor: The dividend will answer to the Product.

☞ It is very absurd to require *proof* of a new beginner, before he has been made acquainted with the means of obtaining it. The rudiments of Arithmetic should first be given.

ADDITION.

7969797986	7377367	472837672	563218
8658138987	9788472	364778738	987647
3791297346	7323689	973844998	478295
18085900986	11544225	1488138175	1685821

SUBTRACTION.

From	9600740998	From	6000998803	From	9780342000
Take	3798392602	Take	1654037645	Take	3366782
Rem.	5691458286		5445850268		9686086328

MULTIPLICATION.

Multiply 372347832 by 6 *Ans.* 1901953669
Multiply 372347832 by 7 *Ans.* 2327324891
Multiply 673483723 by 5 *Ans.* 1034184882

DIVISION.

213)9798639(*Quot.* 47003 472)7634600(*Quot.* 16275
312)7784712(*Quot.* 24851 36)83808(*Quot.* 2327
103)8874686(*Quot.* 86262 56)82432(*Quot.* 1471

ABBREVIATIONS.

To multiply by 9, 99, 999, &c. in one line. *Method*—Annex to the multiplicand as many ciphers as there are 9's in the multiplier; then subtract the multiplicand from the new one; the remainder will be the answer.

Ex. Mult. 472341 by 999

OPERATION.
472341000
472341
Ans. 471868659

To divide by 10, 100, 1000, 10000, &c. *Method*—Cut off the same number of figures in the dividend, as there are ciphers in the divisor.

OPERATIONS.

Ex. Divide 73214 by 10 *Quot.* 7321 | 4 *Rem.*
Divide 87234 by 100 *Quot.* 872 | 34 *Rem.*

Summary view of Addition, Subtraction, Multiplication and Division.

1. Add together the numbers, 473, 464, and 364.
   What is the sum total of, 647, 432, 5678 and 43.
   What is the amount of, 567, 483, 756 and 89.
   20 and 5 and 341 are how many ?
2. Subtract 632 from 1847. Take 8121 from 9871.
   What is the difference between 464 and 502?
   How much larger is 912 than 732 ?
   How much smaller is 872 than 1047?
   How much must be added to 976 to make 1473?
   How much must be taken from 2483 to leave 527?
   Take 4732 from 9783, what will remain ?
3. Multiply 473 by 81. Find the product of 56 by 4.
   How many are 5 times 328 ?
   856 is the multiplicand, 57 the multiplier, what is the product?
   Repeat 432, 5 times.
   781 and 32 are factors, what is their product?
4. Divide 4731 by 612. What is the quotient?
   How many times is 4 contained in 3684?
   9746832 is how many times 82571 ?
   436 can be taken from 8673 how many times ?
   Suppose 4518 to be a dividend and 5 the divisor; what is the quotient?

Signs and Abbreviations used in Arithmetic.

$+$ signifies *plus* or *added to.* Ex. $8 + 11 + 14 = 33$.
$-$ signifies *minus* or *lessened by.* Ex. $7 - 5 = 2$.
$\times$ signifies *multiplied by.* Ex. $6 \times 8 = 48$.
$\div$ signifies *divided by.* Ex. $8 \div 4 = 2$.
$=$ signifies *equals.* Ex. 2 and $3 = 5$.

SUMMARY OF DEFINITIONS.

In the course of the last four chapters, you have practiced four kinds of operations on numbers: viz. Addition, Subtraction, Multiplication, and Division.

Addition is the operation by which two or more numbers are united in one.

Subtraction is the operation by which the difference between two numbers is found.

The larger number is the *minuend*, the smaller number is the *subtrahend.*

Multiplication is repeating one number as often as there are units in another number.

The *multiplicand* is the number to be repeated; the *multiplier* is the number which shows *how often* the multiplicand is to be repeated; the *factors* are both the multiplier and multiplicand; and the *product* is the number obtained by multiplying.

Division is the operation by which we find how many times one number contains another,— or, by which we divide one given number into as many equal parts, as there are units in another given number.

The *dividend* is the number to be divided. The *divisor* is the number by which you divide. The *quotient* is the answer obtained by dividing.

THEORETICAL PRINCIPLES.

1. One number may be subtracted from another as many times as it is contained therein, which operation is called *Division.* To find how many times one number is contained in another, and what remains, is *division;* and *Division* teaches to decrease the greater of two numbers, given, as often as there are units in the less number.

2. In division we inquire how often one number, called the *divisor*, may be subtracted from another, called the *dividend.* The number of times is shown by the third number, called the *quotient.* The fourth number shows what is left of the dividend, after dividing, and is called the *remainder.*

3. We resolve the dividend into parts and find how many times the divisor is contained in each of these parts, which operation begins contrary to addition, subtraction, or multiplication, viz. at the left hand, or higher order, and decreases the dividend by a repeated subtraction of each product arising from the divisor when multiplied into the quotient figure. When a figure of the dividend is annexed to the remainder, for a dividual, it is so many *tens* of a lower order.

4. The quotient figure must be such as being multiplied into the divisor, will give a product equal to such a part of the dividend, as is then taken for a dividual, or leave a remainder less than the divisor.

5. When the divisor will not go in a dividual, a cipher is placed in the quotient and another figure annexed to it. When there is no remainder, the quotient is the absolute and perfect answer to an operation. With a given divisor the greater the dividend the greater the quotient. And with a given dividend the greater the divisor the less the quotient.

6. *Division* is a concise method of performing many subtractions of the same number. *Multiplication* is repeating the same number a given number of times; therefore *division* is the converse of *multiplication.* The divisor multiplied by the quotient produces the dividend, and the divisor and quotient answer to the Factors in multiplication; therefore multiplication is the converse of *division*, the opposite—divide the dividend by the divisor gives the quotient, or divide the dividend by the quotient gives the divisor; therefore division is the converse of multiplication, and multiplication of division.

7. Addition is finding the *amount* of several numbers. Subtraction is finding the *difference* between two numbers; therefore addition is the converse of subtraction, and subtraction of addition.

QUESTIONS ON DIVISION.

1. To divide means what? What does it do? What is proved by it? What does it teach?

2. How many numbers are given in division? How many found? Name them.

3. What is resolved into parts in division? What is contained in each of these parts? Wherein does division differ from addition, and subtraction? What part of the dividend is decreased, and how much when dividing? What order is the figure which is annexed to the remainder to make a dividual.

4. The quotient figure must produce a number equal to what? The number must be less than what? Do all sums have a remainder?

5. Why is a cipher placed in the quotient? What does the quotient show when there is no remainder? How is the magnitude of the divisor and quotient determined?

6. Division is the converse of what rule? Illustrate it? Multiplication of what? Addition of what? Subtraction of what? Would subtraction perform operations in division? What is the object of division then?

SUPPLEMENT TO ADDITION.

§ **33.** The pupil has been taught the theory of arithmetic in the preceding pages. It is now proposed to make an application of the principles therein developed, and apply them to the solution of such questions as will occur in the transaction of business.

Place 211. 43131. 102 and 34723, so as to amount to 78167 when added.

OPERATION.

```
  211
43131
  102
34723
-----
78167
```

Add the following sums 76389 576. 8799. 532, together.

OPERATION.

```
76389
  576
 8799
  532
-----
86296
```

From the above operations we deduce the following general rule for placing integers or whole numbers in order to be added.

Rule:—Place the figures directly under each other, (beginning at the right hand) in such a manner as when set down will form a perpendicular column on the right hand. This will be placing units under units, tens under tens, hundreds under hundreds &c.

Examples for Practice.

1. How should the following numbers be placed, 10327. 120. 12. 102. and 9. so as to amount to 10570, when added?

2. In what order must 1327. 40. 31. 735. 8100. be arranged so as to make 10233 when added?

3. If you add 102. 3723. 41. 56. 109. and 302. together what will be their amount? *Ans.* 4333

4. Take 370. 8000. 301. 5. 327. and make the amount 9003

§ 34. **Practical questions: such as will occur in the transaction of business. Rules are given when new CASES occur, so that the learner need not apply to the teacher for assistance.**

QUESTIONS FOR SOLUTION.

What is the sum of 3765 bushels; 990 bushels; and of 127 bushels, when added together? *Ans.* 4882 *bush.*

RULE:—Place units of one order under each other, then add them as in the 33d section.

OPERATION.

```
3765
 990
 127
----
4882 Ans.
```

1. A merchant bought at one time, 543 barrels of beef; at another 407; at another 152; and at another, 414. How many did he buy in all? *Ans.* 1516 *barrels.*

2. Suppose you sold 47 yards of cloth to one man; 37 to another; and 50 to another; find the number of yards sold! *Ans.* 134 *yds.*

3. Suppose you bought a carriage for 325 dols., and wished to sell it at 25 dols. advance: how much must it be sold for? *Ans.* 350 *dols.*

4. A drover bought sheep as follows, of A. 37; of B. 86; of C. 102: how many sheep in all? *Ans.* 225

5. There are four numbers, the first of which is 532; the second 895; the third 240; and the fourth as much as the other three: what is the sum of them all? *Ans.* 3334

6. A gentleman purchased a farm for 8257 dols.; paid 953 dols. for having it fenced; and 300 dols. for having a barn built upon it: how much must he sell it for, in order to gain 100 dols? *Ans.* 9610 *dols.*

7. A man spent 30 years in the United States; 5 years in France; 12 years in Italy; 7 years in Germany; 4 years in the Netherlands; and afterwards returned to the United States, where he lived 26 years: how old was he at his death? *Ans.* 84 *years.*

8. From the creation of the world to the flood, was 1656 years; thence, to the building of Solomon's temple, 1344 years; thence, to the birth of our Savior, 1004; thence, to the present time, 1833: how old is the world? *Ans.* 5837 *years.*

9. A trader bought four pieces of cloth, the first piece contained 86 yards; the second, 55 yards; the third, 87 yards; and the fourth, 91 yards: what was the number of yards bought? *Ans.* 319

10. A farmer sold 150 bushels of corn; and then had 60 bushels left: how much corn had he at first? *Ans.* 210 *bush.*

11. The protestant missionary societies in different parts of

the world have 211 stations; 388 missionaries and 325 teachers: what is the total number of missionaries and teachers? *Ans.* 713

12. A man paid 16 dols. to A., 9 dols. to B., 7 dols. to C., 10 dols. to D., 6 dols. to E., 4 dols. to F., and had 8 dollars left: how many had he at first? *Ans.* 60 *dols.*

13. ¶ A farmer sold 3 oxen for 45 dols.; 2 horses for 103 dols.; 7 cows for 21 dols.; and 8 sheep for 10 dollars: how much did he receive for all of them? *Ans.* 179 *dols.*

RULE:—Set down the *price* of the animals not the animals themselves. Or set down the price of an article sold, and not the thing *itself*, &c.

14. B. bought 212 barrels of flour for 1060 dols.; 320 barrels of sugar for 6400 dols.; and 66 barrels of cider for 180 dols.: find the amount of money expended! *Ans.* 7640 *dols.*

15. ¶ A grocer bought 560 bushels of rye for 530 dols.; 387 bushels of corn for 237 dols.; 287 bushels of potatoes for 75 dols.: find the amount of money and articles sold! *Ans.* 842 *dols.* 1234 *bush.*

RULE:—Set down the price of the commodity sold, find the amount of the same—then set down the quantity of bushels, pounds, yards, &c. and find the amount; their sums will be the respective answers.

16. J. K. bought of G. B. merchandise as follows—9 yards of silk for 21 dols.; 12 yards of black broadcloth for 30 dols.; 10 yards of shalloon for 3 dols.; 10 yards satin for 16 dols.; and 12 yards of blue broadcloth for 32 dols.: find the number of yards sold, and the price of the whole! *Ans.* 53 *yds.* 102 *dols.*

17. A drover paid 300 dols. for 100 sheep; 525 dols. for 150 sheep; and 1000 dols. for 250 sheep. How many did he buy? What did the whole cost? *Ans. Bought* 500; *cost* 1825 *dols.*

18. ¶ The exports from the United States in the year 1819, were as follows, viz. produce of the sea 2024000 dols.; of the forest 4858000 dols.; of agriculture 41502000 dols.; manufactures 2578000 dols.; uncertain 630000 dols.: what was the whole amount? *Ans.* 51592000 *dols.*

RULE:—To find the amount of several sums where dates are given, disregard the time mentioned.

19. The coinage at the mint of the United States during the year 1830 consists of 8357191 pieces of coin, of which 643105 dols. in gold coin; 245400 dols. in silver coin; 1715 dols. in copper: what was the amount of money coined? *Ans.* 890220 *dols.*

20. In 1828 the Universities and Colleges in the U. S. amounted to 39, there were 3482 Academical Students, and 1184

Medical Students. In the Theological Institutions there were 542 students; how many students in all? *Ans.* 5208

21. The total amount of expenditures for the Armory at Springfield, Mass. from 1795 to 1829 is as follows, viz. for lands buildings, workshops, coal houses, arsenal, &c. 23785611 dols.; pay of officers, workmen, &c. 370055975 dols.; for miscellaneous purposes, 3700280 dols.: find the total sum!

Ans. 397541866 *dols.*

22. ¶ Letters were invented by Memnon, an Egyptian, 1822 years B. C.: how many years since the invention? (date 1831.) *Ans.* 3653

RULE:—To find the number of years since an event that happened before the Christian era, add the current year to that date and it will give the time sought.

23. Danus brought from Egypt the first ship that ever appeared in Greece, in the year 1485, B. C.: that event happened how many years ago? (date 1831.) *Ans.* 3316

24. Iron was found, 1406 B. C. from the accidental burning of Mount Ida: how many years since its discovery? (date 1831.) *Ans.* 3237

25. Socrates the philosopher was born at Athens in Greece 470 B. C.—he was put to death by the Athenians on a false charge of being an atheist 400 B. C.: how many years since up to 1831? *Ans.* 2231

26. Julius Cesar, was murdered in the senate house 44 B. C. and Louis the XVI. king of France was beheaded, A. D. 1793: the distance of time between the events is required.

Ans. 1837 *years.*

§ **35.** Questions will now be presented showing the nature and use of Subtraction.

What is the difference between 453 yards bought and 232 sold? *Ans.* 221

RULE:—Place the larger number first, then the less directly under it and subtract, the remainder will be the answer.

OPERATION.

$$\begin{array}{r} 453 \\ 232 \\ \hline 221 \end{array}\ \textit{Ans.}$$

QUESTIONS FOR SOLUTION.

1. Find the difference between 756 gallons received, and 444 delivered! *Ans.* 312 *dif.*

2. Bought 74000 yards of cloth, sold 29460: how many yards unsold? *Ans.* 44540

3. Sold 210 acres of land from a farm of 537: how many acres were there remaining? *Ans.* 327

4. Bought merchandise for 487 dols., sold the same for 537 dols.: how much did I gain by the bargain? *Ans.* 50 *dols.*

5. G. bought rye of C. F. for 863 dols., but was obliged to dispose of it (being damaged while in his hands) for 23 dols. less than cost: how much did he get for it? *Ans.* 840 *dols.*

6. A merchant has 1387 dols. worth of goods, and owes 378 dols.: how much is he worth? *Ans.* 1009 *dols.*

7. If a farm be valued at 6587 dols. and the owner thereof should be indebted to A. B. & Co. of a neighboring city 1000 dols.: what would he be worth if his debts were paid? *Ans.* 5587 *dols.*

8 ¶ A. bought a chaise for 215 dols. and paid in part a wagon worth 37 dols. and the rest in momey: how much money did he pay? *Ans.* 178 *dols.*

RULE:—Subtract from the original price of an article the price of the commodity proposed to be given in part pay for the same, the remainder will be what is due in money.

9. A grocer sold a farmer melasses, sugar, coffee, &c. to the amount of 50 dols. The farmer paid in grain, butter, cheese, and pork to the amount of 36 dols., and the balance in money: how much did the grocer receive in cash? *Ans.* 14 *dols.*

10. ¶ Bought 378 acres of land for 4536 dols. sold 125 acres for 1975 dols.: how many dols. were regained, and how many acres unsold? *Ans. reg.* 2561 *dols.* 253 *acres unsold.*

RULE:—Find the difference between the money expended and received; then between the quantity bought and sold, the result will be the correct answer.

11. A grocer buys 560 bushels of rye for 531 dols. and sells 201 bushels for 401 dols.: how many bushels will he have left, and what will they cost him? *Ans.* 359 *bush. cost* 130 *dols.*

12. ¶ A. borrowed of B. 437 dols.; A. paid B. at several times, 100 dols; 50 dols; 25 dols.; and at another time 30 dols.: what is still due B.? *Ans.* 232 *dols.*

RULE:—When the sum borrowed or lent is in one sum and at ONE time, but the payment in different sums, and at sundry times; find the total of the payments, and subtract their amount from the sum lent or borrowed. When both the sum borrowed and payments are in different sums and at sundry times, find the amount of both, and subtract the less from the greater, the difference will be the answer.

13. D. borrowed of H. at several times 35 dols, 50 dols. 100 dols., and 4 dols.—H. lent D. at sundry times 30 dols. 12 dols. 10 dols., and 25 dols.: which is indebted D. to H., or H. to D., and how much? *Ans. due H.* 112 *dols.*

14. A. on a settlement, found B. indebted to him the following sums; for a lot of lumber 25 dols.; for a yoke of oxen 65

dols.; for a horse 80 dols., and for the use of a plow 2 dols. B.'s charge against A. was 35 dols.: which was indebted?
Ans. B. to A. 137 *dols.*

15. ¶ Silver was first coined at Rome A. M. 3935; in America A. M. 5334: how many years between the two dates?
Ans. 1399

RULE:—The time from one date to another is found by subtracting the former from the latter. The age of an individual by subtracting the year of their birth from that of their death.

16. The second temple of Jerusalem was finished A. M. 3489, under Darius. It was besieged, and the Jewish nation was destroyed and scattered by Titus A. M. 4074: how many years elapsed between the events? *Ans.* 585

17. Robert Pollok, was born at Muirhous near Glasgow 1798; died at Devonshire A. D. 1827: what was his age?
Ans. 29 *years.*

18. Flavius Josephus, the famous historian of the Jews, was born at Jerusalem A. D. 37, and died 93 A. D.: what was his age? *Ans.* 56 *years.*

19. ¶ In 1830 the national debt of the United States was 48565406 dols.; in 1831 it was 39123191 dols.: how much was paid in 1 year? *Ans.* 9442215 *dols.*

RULE:—Disregard the time.

20. The number of children between the ages of 4 and 16, in the school districts of Connecticut, in 1828, was 84889; in 1829, 85482: how many more children were there in 1829 than in 1828? *Ans.* 593

§ **36.** Questions will now be presented showing the nature and utility of Multiplication.

What will 378 hundred of beef amount to, at 5 dols. per hundred? *Ans.* 1890 *dols.*

RULE:—Multiply the price by the quantity, the product will be the answer. Or make the larger number the multiplicand, the smaller the multiplier, and the product will be the amount.

OPERATION.

378 *quantity,*
5 *price,*
Ans. 1890 *dols.*

QUESTIONS FOR SOLUTION.

1. Suppose you sold 48 cords of wood at 4 dols. a cord: what would it amount. *Ans.* 192 *dols.*

2. What cost 9 tuns of Russia Iron, at 112 dols. per tun?
Ans. 1008 *dols.*

3. What cost 6 tuns of plaster Paris, at 61 dols. a tun? *Ans.* 366 *dols.*

4. If one tun of hay cost 8 dols. what will be the cost of 50 tuns? *Ans.* 400 *dols.*

5. A man bought a quarter of beef, weighing 237 lbs. at 7 cents per lb.: how much did it cost? *Ans.* 1659 *cts.*

6. A merchant bought 84 pieces of linen, at 16 dols. a piece; to how much did they amount? *Ans.* 1344 *dols.*

7. A farmer purchased a farm containing 534 acres, at 51 dols. per acre: how much did he pay for it? *Ans.* 27234 *dols.*

8. A drover bought 253 cows, and paid 18 dols. per head: how much did the whole cost? *Ans.* 4554 *dols.*

9. A brewer sold 325 barrels of ale, at 12 dols. per barrel: to how much did they amount? *Ans.* 3900 *dols.*

10. What would be the amount of 375 hogsheads of sugar at 35 dols. per hogshead? *Ans.* 13125 *dols.*

11. How much wheat will 36 persons consume in 2 years if they use 12 bushels each week? *Ans.* 1248 *bush.*

12. A farmer employed 128 men to finish a piece of fencing; he gave each 17 dols.: how much did the fencing cost? *Ans.* 2176 *dols.*

13. It is calculated that 13500000 tuns of water descend at the falls of Niagara every hour: how much at that rate falls every day? *Ans.* 324000000 *tuns.*

NOTE. There are 24 hours in one day.

14. ¶ A. bought 12 spoons at 80 cts.; 12 at 310 cts.; and 11 knives and forks at 31 cts. each: what did they cost? *Ans.* 5021 *cts.*

RULE:—Find the amount of each article by multiplying the price by the quantity—Add the several products into one sum, this will be the total amount.

15. Suppose a lady bought 4 yards of silk at 75 cts. per yard; 3 yards of cotton at 32 cts. per yard; 2 yards of brown Holland at 44 cts. per yard; 5 yards of black lace at 96 cts. per yard; and 4 skains of sewing silk at 5 cts. a skain; to what did her bill amount? *Ans.* 984 *cts.*

16. If a man earn 3 dols. a week: to what will it amount in 3 years and 7 weeks? *Ans.* 489 *dols.*

NOTE. There are 52 weeks in one year.

17. ¶ There are three numbers; the first is 3721: the other 4 times as much; the third 1823; how much is the fourth? *Ans.* 20428

RULE:—Repeat the number mentioned in the question as many times as dictated by the language; then add to that the sum mentioned in the question. The amount will be the answer.

18. A. raised on his farm 56 bushels of wheat; twice as

many oats; 24 bushels of peas: three times that quantity of rye; 5 times the quantity of barley that he did of wheat: what was the number of bushels raised? *Ans.* 544

19. ¶ If 112 pounds of sugar cost 1050 cts. and selling it at 11 cts. per pound: what is the net gain? *Ans.* 182 *cts.*

RULE:—Multiply the commodity by the selling price; then subtract the first cost from that amount and it will give the net gain.

20. If a piece of cloth, containing 31 yards cost 187 dols., what is gained by selling it at 725 cts. per yard? *Ans.* 3775 *cts.*

21. If a chest of tea, containing 110 pounds be sold at 113 cts. per pound; what is the gain, the first cost being 9675 cts.? *Ans.* 2755 *cts.*

22. ¶ Bought a piece of broadcloth containing 53 yards at 465 cts. per yard, and sold it at 500 cts. a yard: what was the profit on the whole? *Ans.* 1855 *cts.*

RULE:—First find what the whole commodity cost at the purchase price; then find what the same amounts to at the selling price subtract the first cost from the amount; this will give the net gain.

23. Bought 658 pounds of cheese at 9 cts. per pound, and sold it at 11 cts. a pound: what was the gain upon the whole? *Ans.* 1316 *cts.*

21. A merchant bought 100 yards of cloth at 300 cts. per yard and sold it at 250 cts. a yard: what was lost by the sale? *Ans.* 50 *dols.*

§ **37.** Questions showing the necessity and utility of Division will now be presented.

If 1 yard of cloth cost 5 cts.: how many yards can be bought for 7500 cts.? *Ans.* 1500 *yards.*

RULE:—Make the price of one yard, pound, bushel, &c. the divisor; then place the amount of money proposed to be expended for the dividend; or place the less sum for a divisor, and the larger for the dividend, the quotient will be the thing sought.

OPERATION.

5)7500

1500 *Ans.*

QUESTIONS FOR SOLUTION.

1. How many bushels of rye could be bought for 441 dols. if 1 bushel cost 63 cts.? *Ans.* 700

2. If 3025 dols. were to be divided between 5 persons: what will each individual's share be? *Ans.* 605 *dols.*

3. At 5 dols. a barrel: how many barrels of fish may be bought for 455 dols.? *Ans.* 91

4. If 1 chest of tea cost 50 dols.: I wish to know how many I can have for 1350 dols.? *Ans.* 27

5. How much beef can be bought for 168 dols.: if 100 weight cost 8 dols.? *Ans.* 21 *hun.*

6. At 8 dols. a cord: how many cords of wood may be bought for 856 dols.? *Ans.* 107

7. At 4 dols. a barrel: how many barrels of flour may be bought for 328 dols.? *Ans.* 82

8. If 107 barrels of beef cost 321 dols.: what is the cost of 1 barrel? *Ans.* 3 *dols.*

9. A gentleman erected a house in 30 days which cost 1325 dols. There were 5 persons employed in doing it: what did each receive? *Ans.* 265 *dols.*

10. A laborer engaged to work 8 months for 96 dols., how much did he receive per month? *Ans.* 12 *dols.*

11. A flour mill with 3 run of stones, ground in 6 days 630 bushels of wheat: how much is that per day? *Ans.* 105 *bush.*

12. A tobacconist has 4368 pounds of tobacco which he wishes to pack into boxes containing 56 pounds each: how many boxes must be made to contain it? *Ans.* 78

13. ¶ A farmer died and bequeathed his estate to 2 of his sons. The property consisted of 349 dols. in cash; a house and furniture worth 504 dols.; a farm valued at 600 dols.; stock worth 230 dols., and produce worth 125 dols.: how much did each receive? *Ans.* 904 *dols.*

RULE:—Add the several sums of money, &c. together; then divide their amount by the number of parts, or different individuals among whom it is to be distributed. The quotient will be the answer.

14. Two merchants dissolved copartnership. The stock in trade was valued at 1237 dols.; Book accounts, 2745 dols.; value out in Promissory Notes 561 dols.; cash on hand 1255 dols.: what ought each to receive if divided equally among them? *Ans.* 2899 *dols.*

15. A merchant bought 8 hogsheads of brandy, for 950 dols.; he paid for freight 100 dols.; for duties and other charges 50 dols., and he wishes to gain 100 dols.: how must he sell it per hogshead? *Ans.* 150 *dols.*

16. ¶ A legacy of 5727 dols. was left to 3 persons, and was to be divided equally among them. But immediately after the death of the testator, they lost by fire, a house valued at 1230 dols.: what ought each to receive of the remaining legacy? *Ans.* 1499 *dols.*

RULE:—First find by subtraction what the entire gain or loss is, then divide the remainder by the number mentioned in the question the quotient will be the answer.

17. Two merchants had gained by trade 17365 dols.—out of which they lost by bad debts, miscalculation and fire 1241 dols.

The remainder they divided among themselves: what was the portion of each? *Ans.* 8062 *dols.*

18. A man dies leaving an estate of 71600 dols., there are demands against the estate amounting to 39876 dols. the residue is to be divided among 7 sons: what will each receive?

Ans. 4532 *dols.*

19. Bought 30 hogsheads of melasses, for 600 dols. paid duties 20 dols.; for freight 40 dols.; for porterage 6 dols., and for insurance 30 dols.: if I sell them at 26 dols. per hogshead how much shall I gain?

Ans. 84 *dols.*

20. ¶ A man bought 5 pieces of cloth, at 44 dols. each; 976 pairs of shoes, at 3 dols. a pair; 700 pieces of calico, at 5 dols. each; he divided the whole amount among 8 persons: what did each receive?

Ans. 831 *dols.*

21. A man going a journey, traveled 21 days; 45 miles per day. He agreed to pay 60 cts. for every 5 miles: how far did he travel, and what was the cost of the journey?

Ans. 945 *miles—cost* 11340 *cts.*

22. A farmer sold 435 bushels of corn at 56 cts. per bushel; 365 of rye, at 75 cts. per bushel; he wished to divide the amount equally among 3 creditors: how much did each receive?

Ans. 17245 *cts.*

23. Bought 4 bales of cloth, each containing 6 pieces, and each piece 27 yards, at 54 dols. per piece: what is the value of the whole, and the rate per yard?

Ans. whole 1296 *dols. at* 2 *dols. per yard.*

¶ Divide 25 dols. between two persons and give one 7 dols. more than the other. *Ans.* 9 *dols.* and 16 *dols.*

RULE:—From the whole sum take out what one has *more* than the other; then divide the remainder equally, to which add the sum or sums subtracted.

PROOF. 9 dollars and 16 dollars make 25 dollars, the sum to be divided; and 16 dols. is 7 dollars more than 9 dollars

OPERATION.

25 9
7 7
2)18(9 16
18

Again—Three persons drew a prize of 20000 dols.; the second is to receive 600 dols. more than the first; the third 800 more than the second: what is each person's share?

The *first* 6000 *dols.*, 2d. 6600 *dols.*, 3d. 7400 *dols*

OPERATION.

whole sum 20000 2d. *rec'd.* 1200

what 2*d*. 3*d*. *has* 2000 3d. *rec'd.* 800 *more.*

3)18000(6000 1*st*. *which is* 2000

Proof 6000 *dols.* 18000 600

6600 *dols.* 6600 2*d*. *share.*

7400 *dols.* 800

equals 20000 the sum to be div'd. 7400 3*d*. *share.*

N. B. To solve this question by double position, (as done by Daboll and others) takes 128 *figures*, exclusive of the answer.

24. A gentleman divided an estate of 15000 dols. between his two sons, giving the elder 2500 dols. more than the younger: what was the shares of each?

Ans. { *Elder* 8750 *dols.*
{ *Younger* 6250 *dols.*

25. Divide 1200 acres of land among A., B., and C.; so that B. may have 100 more than A.; and C. 64 more than B.

Ans. A. 312 B. 412 C. 476

Note. Find the sum 264 to subt. from 1200.

26. Divide 1000 dols. among A., B., and C.; give A, 129 dols. more than B.; and B. 178 *less* than C.

Ans. B's. 231 *dols.*, A's. 360 *dols.*, C's. 409 *dols.*

Note. Find the sum 307 to subt. from 1000.

¶ Divide 117 dols. among A., B., and C.; give B. twice as much as A., and C. three times as much as B.: what will each receive? *Ans.* A. 13 *dols.*, B. 26 *dols.*, C. 78 *dols.*

Rule:—First, find the number of simple shares, then divide the sum by them; the quotient will be the first one's share; then multiply the first one's share by so many as the second has shares more than the first, the product will be the second one's share: proceed thus for every succeeding share.

A. has 1 simple share.
B. 2 simple shares.
C. 6 simple shares.
9 No. of simple shares.

OPERATION.

9)117

13 A's. *share.*

2

26 B's. *share.*

3

78 C's. *share.*

Proof. Add all the shares into one; if it be equal to the dividend, the work is right.

13 A's.
26 B's.
78 C's.
117 *dollars; the sum to be div'd.*

27. Divide 380 dols. among A., B., C., and D.; give B. three times as much as A.; C. four times as much as B.; and D. five times as much as C.: what will each have?

Ans. A. 5 *dols.*, B. 15 *dols.*, C. 60 *dols.*, D. 300 *dols.*

76 shares.

28. Divide 4114 dols. among A., B., C., and D.; give B.

four times as much as A.; C. three times as much as B.; and D. ten times as much as all the others: what will each have?

Ans. A 22 *dols.*, B. 88 *dols.*, C. 264 *dols.*, D. 3740 *dols.*

187 shares.

29. Divide 333 dols. among A., B., and C.; give B. ten times as much as A., and C. ten times as much as B.: what will each receive? *Ans.* A. 3 *dols.*, B. 30 *dols.*, C. 300 *dols.*

111 shares.

30. Two men bought a lottery ticket in partnership, A. paid 9 dols. towards it, B. 7 dols.; the ticket drew a prize of 2000 dols.: how much did each receive?

16 shares. *Ans.* A. 1125 *dols.*, B. 875 *dols.*

31. The age of A. is double that of B., the age of B. triple that of C., and the sum of all their ages 140: what is the age of each? *Ans.* A's. 84 B's. 42 C's. 14

10 shares.

General Principles of the Application of the Four Rules of Arithmetic.

1. Under the head of *Addition* will come all such questions, where *quantities of the same kind* are to be counted together. It is of course impossible, to add *quantities of different kinds* together under any denomination than as mere *things*; and this remark, simple as it is, may escape notice in certain cases.

That all this applies equally to *Subtraction*, is evident from the principle, that it is only the converse operation of Addition.

2. The application of *Multiplication* occurs in every case where one of the quantities occurs repeatedly, this repetition being as often as indicated by another number, which forms the multiplier; such is the case, for instance, in all purchases, at a certain rate for the adopted unit of the things bought, or sold; and in general, whenever the same thing or quantity is repeatedly taken.

3. *Division* applies to all cases where any quantity of things is to be divided among an equal number of persons, or into an equal number of lots or parts; the quotient will give the share of each person, or the quantity of things in each lot or part. It will therefore apply to find the price of a single piece of a thing, of which a large number has been purchased for a certain price.

The *amount*, or sum total in ADDITION, will be of the same denomination, as the numbers that comprise the example.

The *remainder*, in SUBTRACTION, will be of the same kind, or denomination, as the minuend and subtrahend.

The *product*, in MULTIPLICATION, will be of the same kind as the *multiplicand*, for the multiplier is always a *number*. The multiplicand is a quantity, or the price. We repeat the quantity or price according to the amount purchased.

From the nature of DIVISION, it is evident, that the value of the quotient depends both on the divisor and the dividend.

When the divisor is a NUMBER, then the quotient will be a QUANTITY of the same kind as the dividend.

When the DIVISOR is a quantity, the same as the dividend, then the *quotient* will be a number.

PRACTICAL APPLICATION OF THE FOUR RULES OF ARITHMETIC.

Teacher. Having the quantity of several commodities or numbers of things given, how is the amount found?

Pupil. Ans. By adding them together. *T*. Give an example! [*Let the pupil do this orally.*]

T. What questions will come under the head, *Addition? Pupil, see page* 82. Can we add things of a different kind together? Why not? *P. see page* 82.

T. When the purchase price of an article is given, and a certain advance on the whole is given; how can the *selling* price be found?

P. By adding the purchase price to the gain proposed.

T. Give an example!

T. When the quantity, sold and unsold is given, how is the *total* found?

P. By adding all into one sum. *T*. Give an example!

P. Ans. Sold 30 bushels of rye; had 70 left; how many bushels had I at first?

T. In every example, for solution there is something known, and something required; what is known and what required in Ex. 6, page 72. [*The pupil consults the ex. and answers,* thus—

P. The price of the farm, fencing, and barn is known; the price of sale to gain 100 dollars, is required.

T. When the number of articles, &c. are given, and the amount paid for them is required; what is the method of solution?

P. Find the amount of *money* paid for the whole, and disregard the number of articles.

T. Give an example!

T. What method is taken when the amount of *money* expended, and the number of *articles*, are required?

P. Add the several sums of money together for one *Ans*.; then the number of articles named for the other.

T. Give an example!

T. When the amount of several things is required, and the date is given, what is the method of operation?

P. The time is not added in with other things. *T*. Give an example!

T. How is the space of time up to the present found, when the event happened before the Christian era?

P. By adding the current year to the date of the event. *T*. Give an example!

T. When the purchase and selling price are given, how is the gain found?

P. By subtracting the purchase price from the selling price. *T*. Give an example!

T. Can we subtract unlike quantities from each other? Why not? (*see page* 82.)

T. When an article is sold for less than cost; how is the loss found?

P. Take the selling price from what the article cost; the remainder will show what the loss is.

T. When exchange is made in property, and the difference in value is to be paid in money; how is the sum of *money* found?

P. By taking the price of the article given in exchange, from the price of the article *exchanged*, the difference will be the sum paid.

T. Give an example!

T. When commodities or sums of money are borrowed, in several parcels, and paid at different times; how is the difference in *amount* found?

P. By adding the several sums borrowed, together, for one total; then the sums paid for the other total; and then take the less from the greater.

T. Give an example!

T. How is the time from one date to that of another found?

P. Take the former from the latter. *T.* Give an example!

T. How is the age of an individual found?

P. Take the year of his birth from that of his death.

T. Give an example!

T. When you have the price of one pound, yard or bushel given; how is the price of a quantity found?

P. Repeat the *price* of one yard, &c., as many times as there are number of yards, pounds, &c. mentioned.

T. Give an example! What will the product be? *P.* Money.

T. Why? (*see page* 82.)

T. What questions will occur in multiplication? What forms the multiplier? Does this rule apply in cases of purchase? What is repeatedly taken? *P.* [*see page* 82.]

T. When the price of several articles is given per yard, &c. and their total amount is required: how is it found?

P. Multiply the price of each article by the amount bought; and add their products into one sum. *T.* Give an example!

T. How is the net gain on an article found, when the cost of the whole is given, and rate per pound, &c. of sale given?

P. Find what the commodity amounts to at the selling price; subtract the first cost from the same, the remainder will show the net gain.

T. Give an example!

T. When the price of a number of yards &c. is given; how is the price of one found? *P.* Divide the amount paid for the whole by the quanity.

T. Give an example!

T. When any number of things are to be divided among several individuals, how is the operation performed?

P. Divide the whole sum by the number of individuals who are to receive it. *T.* Give an example!

T. What questions will occur in division! *P.* (*see page* 82.)

T. Why will the quotient be of the same denomination as the divisor? *P.* (*see page* 82.)

T. Propose a question performed by addition and division!

P. A. had 500 dollars of personal property, and 1000 dollars of real estate, and divided it among his two sons.

T. Propose a question performed by subtraction and division!

P. B. had property to the value of 5173 dollars; but lost by misfortune 200 dollars, and then divided the remainder among 3 persons.

T. Propose a question solved by multiplication and division!

P. Sold 71 yards of cloth at 5 dollars per yard; 31 acres of land at 10 dollars per acre: divide the amount equally among 3 persons.

Rem. It would be a profitable exercise for the pupil to tell the teacher how each example is solved in the "*recapitulation*" and miscellaneous examples in *Federal Money*.

RECAPITULATION.

⁂ For Answers see Key, page 230.

1. Maps and Globes were invented by Anaximander, 600 B. C.: how long have they been in use up to 1831?

2. Gen. La Fayette was born in June 1757. He joined the army under Washington when but 19 years of age. He visited the U. S. in 1824; and returned to France in 1825: how many years since? (date 1831.)

3. A man bought 20 casks of wine, containing 2459 gallons, and sold 14 casks containing 1682 gallons: how many casks, and how many gallons were left?

4. Plato a Grecian philosopher died at Athens 34 B. C.: how many years since his death, date 1831?

5. Bought 3 yards of cambric at 2 dols. a yard, 7 yards of silk for 3 dols. per yard, 5 yards of ribin for 4 dols.: to what did the whole amount?

6. It appears that the United States in 1828 imported silk to the amount of 8463563 dols. of which 1274461 were exported: France annually imports to the amount of 20000000 dols.: what is the amount of silk imported?

7. A trader, that has 48 dols. wishes to buy all the boots he can pay for, at 5 dols. a pair, and then lay out the remainder of his money in shoes, at 1 dollar a pair: how many pairs of boots, and of shoes, must he buy?

8. The number of checks, stripes, and ginghams manufactured in the State prison at Windsor (Vt.) annually, is 300000 yards: how many is that per day?

9. The iron made in the United States in 1828, amounts to about 30000000 dols. and employed 35000 men as colliers, wagoners, founders, forgers, &c.: how many dollars is that to each man?

10. A butcher bought 4 fat oxen: the first ox slaughtered weighed 1045 pounds; the second 20 pounds more than the first; the third 40 pounds more than the second, and the fourth 55 pounds more than the first: what is the weight of the four oxen?

11. A trader bought 25 barrels of flour, paying 7 dols. a barrel for 11 barrels of it, and 9 dols. a barrel for the remainder: what did the whole cost?

12. The number of votes given in favor of the Federal constitution of Connecticut in 1818, was 13918, and against it 12364; of how many votes did the majority consist?

13. George Washington was President of the United States from 1789 to 1797; John Adams from 1797 to 1801; Thomas Jefferson from 1801 to 1809; James Madison from 1809 to 1817; James Monroe from 1817 to 1825; John Q. Adams

from 1825 to 1829; Gen. Andrew Jackson from 1829 to 1837: how many years were they one after another Presidents of the United States?

14. The saving already effected by temperate movements in the article of distilled spirits, amounts, on the lowest estimate, to more than 2000000 dols. a year: how much would that be in 5 years?

15. It has been shown that 40000 die annually in the U. S. in consequence of hard drinking: how many is that per day?

16. A merchant had 650 barrels of flour, sold 95 barrels to one man; 38 to another; and 225 to another; how many barrels had he left?

17. A man has a farm containing 200 acres, which is divided into plow, pasture, meadow, and wood-land, viz. 50 acres of wood, 25 of meadow, 60 of pasture: how many acres were there left for plow-land?

18. A grocer had, according to his inventory, 317 lbs. of sugar; 561 lbs. of coffee; 451 lbs. of tea; 15 lbs. of pepper; 3 oz. of mace; 152 lbs. of rice; 17 gallons of oil; He sold since, 283 lbs. of sugar; 341 lbs. of coffee; 349 lbs. of tea; 11 lbs. of pepper; 2 oz. of mace; 5 gallons of oil; 121 lbs. of rice: how much is there left of each kind?

19. There are 16 bags of coffee, weighing each 120 pounds, and 8 bags weighing each 343 pounds: what is the weight of the whole?

20. A merchant bought 963 barrels of flour; on weighing them, he finds their average weight 202 lbs. and that the barrels average 7 lbs. each: how many pounds of flour is there in the whole?

21. A man had six bags of hops; the first weighed 14 lbs.; the others 20 pounds each: how many pounds in all?

22. Four men bought 200 acres of land each, for 1500 dols.; the first paid 1 dol. an acre; the second twice as much: the third three times as much as the first: what did the fourth pay?

23. La Fayette arrived in America in the Spring of 1777: how many years have elapsed since?

24. A man, at his decease, left 12426 dols.; 1000 of which to be given to his niece; and the remainder to be divided equally among his two nephews: what is the share of each?

25. Required the amount of years from the following events up to the year 1833, viz. The Arabic figures, introduced from Arabia into Europe by the Saracens, A. D. 991; linen, first made in England 1253; spectacles invented by a monk of Pisa, 1299; gunpowder first used in Europe 1330; Algebra introduced into Europe from Arabia, 1412; printing invented, 1440; made public by the plundering, &c. of the city of Mentz, 1462;

spinning-wheels invented, 1530; telescopes invented, 1590; Decimal Fractions invented, 1602; thermometers invented, and satellites of Jupiter discovered, 1614; circulation of the blood discovered by Hervey, 1619; barometers invented, 1643; and air-pumps, 1654.

26. Three merchants make up a stock of 506 dols. B. puts in twice as much as A., and C. four times as much as B.: how much did each of them put in?*

27. B. asked C. how much his horse cost; C. answered, that if he cost him three times as much as he did, and 15 dols. more, he would stand him in 300 dols.; what was the price of the horse?

28. A. B. and C. would divide 100 dols. between them, so that B. may have 3 dols. more than A., and C. 4 dols. more than B.: I demand how much each man must have?

29. Three persons bought goods at Hartford, which cost 600 dols. The first person was to have a third part more than the second, and the third a fourth part more than the first. what was each man's share?

30. Four persons purchased a farm for 4755 dols.; of which B. paid three times as much as A.; C. paid as much as A. and B.; and D. paid as much as C. and B.: what did each pay?

31. A. bought a chaise, horse, and harness for 360 dols. The horse cost twice as much as the harness; and the chaise cost twice as much as the harness and horse together: what was the price of each?

FEDERAL MONEY.

Federal money is the national currency of the United States. Its several denominations are the MILL, the CENT, the DOLLAR, and the EAGLE.

Federal Money is added in the same manner as whole numbers; care being taken that dollars be placed under dollars, cents under cents, and mills under mills, and carry as in addition of whole numbers. As it takes one hundred cents to make a dollar, there must be but two places for cents. Ten mills make a cent, therefore there must be but *one* place for mills. When there are no cents inserted, *two* ciphers must occupy their place: when no mills, a cipher may occupy the vacancy.

Dollars are separated from cents by a colon (:) cents from mills by an inverted comma ('). To designate what part of a dollar, cents and mills are, the following characters are used; $ for Dollars; cts. for cents; *m.* for mills, according to the following TABLE.

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* The rules to solve the 6 last examples, are given at Page 80 and 81.

$	*cts.*	*m.*		EXPRESSED.
1	09	5	*Read one Doll. nine cts. five mills.*	$1: 09‘5,
2	08	3	“ two dols. eight cts. three mills.	2: 08‘3
3	07	2	“ three dols. seven cts. two mills.	3: 07‘2
10	04	0	“ ten dols. four cents no mills.	10: 04
11	00	0	“ eleven dols. no cents no mills.	11,
12	00	7	“ twelve dols. no cents seven mills.	12: 00‘7
13	11	8	“ thirteen dols. eleven cts. eight mills.	13: 11‘8

ADDITION.

Rule:—Place dollars under dollars, cents under cents, mills under mills; keep the separtrix directly under each other, add as in whole numbers, and point off in the sum total, as many places for dollars, cents, and mills as there are in any of the given numbers.

Questions for Exercise.

1. Express in figures, three dollars, seven cents, six mills. Seventy dollars, eighty cents, one mill. Ninety-nine dollars, ninety cents, four mills. Twenty-two dollars and five cents, two mills. Thirty dollars, ten cents, one mill. Twenty-one dollars and two cents, five mills. One dollar and nine cents, eight mills. Eight dollars and two cents, five mills.
Ans. when added, $256: 08‘2

2. Express eleven dollars, no cents, one mill. Twelve dollars, no cents, five mills. Eighty-five dollars and three cents, one mill. One hundred dollars and one cent, no mills. Thirty-seven dollars and one cent, two mills; and Eighty-five dollars, twenty-one cents, three mills. *Ans. when added* $330: 27‘2

3. Express in figures, One hundred dollars, and five cents, eight mills. Two hundred twenty-one dollars, forty cents, one mill. Three hundred and five dollars. Four hundred thirty dollars. Five hundred and six dollars, thirty-one cents, one mill. *Ans. when added* $1562: 77

Examples for Practice.

1. F. received of A., B., and C. the following sums, viz.: A. paid him $140: 50; B. $500: 58; C. $1000; can you tell how much F. received from A., B., C.?
Ans. $1641: 08

2. A merchant settling his accounts, found that he owed A. $80; B. $120; C. 150, and D. 480; how much did he owe in all? *Ans.* $830

3. Bought a quantity of goods for $125: 10; paid for truckage $8; for freight $9: 20; for duties $5: 40; and my expenses were $8: 50; what did the goods cost me?
Ans. $156: 20

4. A man commenced trade with three thousand two hundred fifty dollars; after trading for some time, he found he had

gained two hundred thirty-seven dollars; how much had he then? *Ans.* $3487

5. A man has four notes, viz.: Eighty-nine dollars and five cents. Sixty-five dollars and seven cents. Twenty-five dollars, ten cents. Four dollars and six cents; what is the sum of all the notes? *Ans.* $183 : 28

SUBTRACTION.

The operations in this rule are the same as in whole numbers. Care being taken to place dollars under dollars, cents under cents, mills under mills, according to the following

TABLE.

	$	*cts.*	*m.*		$	*cts.*	*m.*		$	*cts*	*m.*
From	72	82	9	F.	6	00	0	F.	5	00	0
Take	31	72	3	T.	0	00	9	T.	0	37	9
From	3	00	8	"	0	37	8	"	0	00	9
Take	0	04	1	"	0	09	1	"	0	00	2
From	4	09	1	"	0	75	1	"	8	85	9
Take	0	87	0	"	0	04	0	"	8	85	8

Rule:—When dollars and cents are to be taken from dollars, annex *two ciphers* to the dollars: when dollars, cents and mills are to be taken from dollars, annex *three ciphers* to the dollars, and when cents and mills are to be taken from cents, annex *one* cipher to the cents. Point off the same as in addition of Federal money.

Questions for Exercise.

1. Write down in order to be subtracted, seventy-five cents, three mills: under which place sixty-seven cents, one mill. *Ans. when sub.* $0 : 8‘2

2. Eighty cents seven mills: under which place sixty-five cents one mill. *Ans. when sub.* $0 : 15‘6

3. Write down thirty seven dollars eighty cents, three mills: under which place twenty dollars, five cents, three mills. *Ans. when sub.* $17 : 75

4. Write down fifty dollars, eighty-five cents: under which place five dollars, five cents, and one mill. *Ans. when sub.* $45 : 79‘9

5. Write down thirty-one dollars, thirty-five cents, eight mills: under which place nine dollars, eleven cents, one mill. *Ans. when sub.* $22 : 24‘7

6. Write down eighty-seven dollars, one cent, three mills: under which place seventy-five dollars, sixty-five cents, one mill. *Ans. when sub.* $11 : 36‘2

7. If one mill be taken from a thousand dollars; what will remain? (*See Key.*)

8. From forty dollars, take forty cents and five mills. (*See Key.*)

9. Subtract one thousand dollars and ninety-five cents; from two thousand one hundred and two dollars. (*See Key.*)

10. If one cent be taken from ten thousand dollars; what will then remain? (*See Key.*)

11. From forty-one dollars, eighty cents; take one dollar nine cents. (*See Key.*)

Examples for Practice.

1. Bought a farm for $12585 : 50; sold it for $12695 : 75‘5; how much was gained by the sale? *Ans.* $110 : 25‘5

2. A man has property to the amount of $34764; but there are demands against him to the amount of $14297; how many dollars will be left after the payment of his debts? *Ans.* $20467

3. A man in business bought, during the year, goods to the amount of $106409, and sold to the amount of $59879; what amount remained unsold? *Ans.* $46530

MULTIPLICATION.

The operations in this rule are performed as in whole numbers; the multiplier being written under the multiplicand as in whole numbers.

RULE:—Place the parts as in whole numbers. Then multiply the multiplicand by the multiplier. Point off as many places in the product for dollars, cents, and mills, as there are in the multiplicand.

Exercises for the Slate.

Multiply $45 : 75‘5 by $136 *Ans.* $6222 : 68
$896 : 85‘6 by $2 : 37 *Ans.* $212554 : 87‘2
$378 : 95‘8 by $3 : 63 *Ans.* $327040 : 75‘4

Examples for Practice.

1. Bought 3878 bushels of corn for 85 cts. per bushel; to what did it amount? *Ans.* $3296:30

2. How much will 12 yards of cloth amount to at $5 : 37 per yard? *Ans.* $64 : 44

3. G. bought 12 cords of wood at $5 : 25 a cord; to what did it amount? *Ans.* $63

4. What will 31496 feet of boards amount to at $8 per M. *Ans.* $251 : 96‘8

5. If a man receive $1 : 25 a day; what must he receive for 48 days labor? *Ans.* $60

6. What is the value of 275 acres of land, at $15 : 50 per acre? *Ans.* $4262 : 50

7. What is the value of 112 pounds of Hyson-skin tea, at $2: 12‘5 per pound? *Ans.* $238

DIVISION.

The performance of operations in this rule is similar to that of whole numbers, care must be taken in pointing off the quotient figures.

Rule:—If the dividend be dollars, cents, and mills, work as in whole numbers. Point off *one* figure at the right hand for mills; *two* for cents; and the remaining will be dollars.

If the dividend be dollars only, then annex *two* ciphers for cents, and one for mills: then point off as above directed.

Exercises for the Slate.

Divide $624 by 4 *Ans.* $156
$8781 : 857 by 7 *Ans.* $1254 : 55'1
$3686 : 975 by 1 : 39 *Ans.* $26 : 525

Examples for Practice.

1. Lent $4738 : 88 to 8 men; how much must each pay to amount to the sum borrowed? *Ans.* $592 : 36

2. 37 men lost by fire, storm, &c. $87436 : 92; how much did each man lose upon an average? *Ans.* $2363 : 16

3. A. bought 24 yards of cloth for $47 : 87'5; what did it cost per yard? *Ans.* $1 : 99'4

4. G. sold his farm of 300 acres, for $3870; what was that an acre? *Ans.* $12 : 90

5. At 75 cts. per bushel; how many bushels of rye may be bought for $141? *Ans.* 188 *bush.*

QUESTIONS ON FEDERAL MONEY.

How do you write down Federal money for adding? Dollars under dollars, cents under cents, and mills under mills. After your numbers are written down, how do you proceed in adding? The same as in whole numbers. Why? Because one in a superior column, is equal to ten in the next inferior column. In federal money what are the figures called at the left hand of the separatrix? Dollars. What are the next two at the right called? Cents. What is the next at the right called? Mills. If a denomination be wanting what do you write in its place? A cipher or ciphers to fill the vacancy.

How do you place sums in Federal Money for subtracting? Dimes under dimes, dollars under dollars, cents under cents, mills under mills. What are integers in federal money? Dollars. What are dimes? Tenths. What are cents? Hundredths. What are mills? Thousandths. What is the decimal expression called when dimes, cents and mills are taken together? Thousandths of a dollar. How do you subtract? The same as in whole numbers. Why? Because ten in an inferior column or denomination, is equal to one in the next superior. Why do we call dimes and cents, cents only? We call them all cents, because the left hand figure in cents expresses tens in cents, which is the same as dimes. When your subtrahend contains denominations which are not named in the minuend, what do you do? Supply the vacant denominations of the minuend with ciphers, then place the subtrahend under, so that cents may occupy the place of cents, and mills the place of mills. How do you point off in subtraction? Place the separatrix in the remain. directly below the separatrix in the given numbers. Why is federal money decimals? Because dimes, cents and mills form a decimal expression, of which a dollar is the integer. How do you multiply federal money? The same as in whole numbers. Why? Because they increase in a tenfold proportion, the same as in whole numbers. How do you point off in the product? As far from the right, as the separatrix is in the multiplicand. How do you proceed in dividing? The same as in whole numbers. In dividing dollars, if you have a remainder, what do you do, after all the figures are brought down from the dividend? Join two ciphers to the remainder, for cents, and proceed as before; and if something still remain, annex another cipher to the remainder, for mills, after which again divide as before. What will the two figures in the quotient next to dollars express? Cents. What does the one next to cents express? Mills. Why do we divide the same as in whole numbers? Because they increase in a tenfold proportion.

MISCELLANEOUS EXAMPLES AND CONNECTED OPERATIONS.

1. B. pays rent $200 a year; he pays his tailor $64 : 73; his shoemaker $18; his wood man $43 : 18; his butcher $87 : 40; and his baker $59; his trade brings him in $556; does he gain or lose? *Ans. gains* $83 : 69

2. C. bought 120 bushels of wheat at $1 : 82 a bushel and sold it for $2 : 25; what did he gain in all? *Ans.* 51 : 60

3. A miller bought a quantity of corn, for which he paid $480; he afterwards sold it for $580 : 68'5; what did he gain by the sale? *Ans.* 100 : 68'5

4. A merchant borrowed $600, for which he gave his note, he paid at one time $240, at another $150; how much then remained due? *Ans.* $210

5. The Governor of Maine has a salary of $1500 a year; the Governor of New Hampshire, $1200; the Governor of Vermont $750; the Governor of Mass. $3666 : 67; the Governor of R. I. $400; the Governor of Connecticut $1100; how much do they all receive? *Ans.* $8616 : 67

6. If a cow yields 20 quarts of milk in a day for 240 days; and 25 quarts make 1 pound of butter; how many pounds of butter will be obtained in the season, and to what does it amount, at : 12'5 per pound? *Ans.* 192 *lbs. and value,* $24

7. A man's property is worth $8560, but he has debts to the amount of $3500; what will remain after paying them? *Ans.* $5060

8. B. lent H. $1240 for forty days, at the expiration of the time, H. paid him $984; how much was still due? *Ans.* $256

9. A merchant bought wheat in Vermont for the New York market, 5526 bushels at $1 : 50 per bushel, expecting to sell it at $2 : 50, but finding the market fast rising, he stored it in the city with directions to his factor not to sell it under $3; the price arose to $2 : 95, and then suddenly fell to 95 cts.; what did the

merchant lose in his wheat, exclusive of the expense of transportation, storage, wasteage, lying out of his property, &c.
Ans. $3039 : 30

10. Suppose a gentleman has an estate of $600; and he pays a land tax of 140; for repairs $94 : 17'5; what is his net gain per annum? *Ans.* $365 : 82'5

11. C. bought 210 reams of paper for $55 : 62'5; and sold it again for $60 : 37'5; what did he gain by the bargain?
Ans. $4 : 75

12. A laborer received during 8 months in the year, $30 per month; and during the other months, only $4. Wishing to be steadily employed, he agreed to serve one year, for $240; how much less does he receive per month, than he did before?
$1 : 33'3

13. The President of the U. States has a salary of $25000; Vice President $5000; Secretary of State $6000; of the Treasury $6000; of War $6000; of the Navy $6000; the Post Master General $6000; and Attorney General $3500; how much do they all receive? *Ans.* $63500

14. A shoemaker paid $1 : 58 a-piece for 10 calf-skins, and 22 cts. a pound for 3 sides of sole leather, each side weighing 35 pounds. From this stock he made 48 pairs of shoes, which he sold at $1 : 75 a pair; what did he receive for his work?
Ans. $45 : 10

15. The expenses of paving a street, 500 feet in length, amount to $1000; the amount is to be distributed among the owners, each having a lot of 25 feet; how much will each owner have to pay? *Ans.* $50

16. A merchant bought 109 bales of calico, for the total amount of $12232; he finds that 40 bales contain each 30 pieces, 50 contain each 25 pieces, and the rest contain 32 pieces each; how much did each piece cost? *Ans.* $4

17. A man owing $379; paid at one time $47; at another $23; at another $84; and at another $143; how much did he still owe? *Ans.* $82

18. A farmer purchased 15 sheep—he sold 8 of them at $4 a-piece, and the remainder at $3 a-piece; and then found that he had gained $7; how much did he give for the sheep?
Ans. $46

19. Suppose butter to be worth 12 cts. a pound, and tea 42 cts. a pound; how many pounds of butter must be given for 2 pounds of tea? *Ans.* 7 *lbs.*

20. A merchant bought 56 bales of cotton goods: 15 of them held 21 pieces, 29 held 28 pieces, and the rest 25 pieces each; for each piece he pays $3; how much must he pay for the whole? *Ans.* $4281

21. A farmer went to the city with 8 barrels of cider, which he sold at $4 a barrel. He then purchased 3 hogsheads of salt at $3 per hogshead, and paid a debt of $12; how many dollars had he to carry home? *Ans.* $11

22. A father, having six sons, leaves a property of $76590 to be shared equally among them; how much will each receive? *Ans.* $12765

23. A farmer sold wheat to his merchant to the amount of $342 : 75 at the rate of 75 cts. per bushel; what was the number of bushels sold? *Ans.* 457 *bush.*

24. But the merchant failing before payment, was able to remit only 64 cts. upon a dollar; how much did the farmer lose, and what were his avails? *Ans. Avails,* $219 : 36
Loss, $123 : 39

25. A farmer sold 2 cows at $23 a-piece, and 9 sheep at $5 a-piece; he received in payment, 3 plows at $8 a-piece, and the rest in money; how much money did he receive? *Ans.* $67

26. A grocer, making an inventory, finds he has in cash $17 : 52; in various liquors to the amount of $215 : 17; in soap, candles, and such articles, $92 : 54; in spices, $107 : 32; in salt fish and similar provisions, $49 : 62; and in various small articles besides the furniture of his store, in all $57 : 84; what is the whole amount of his stock? *Ans.* $540 : 01

27. Divide $400 among A. B. and C.; give B. three times as much as A. and C. four times as much as B.; what will each have? (*See page* 81, ¶ *Divide.*)
16 shares. *Ans.* A. $25 B. $75 C. $300

28. Divide $2666 among A. B. C. and D.; give B. twelve times as much as A.; C. ten times as much as B., and D. ten times as much as C.; what will each have? (*See page* 81 ¶ *Divide.*)
1333 shares. *Ans.* A. $2 B. $24 C. $240 D. $2400

29. A trader failed in business, owing $11000, and having only $5000 to divide among his creditors; how much did he pay on a debt of $95 : 20? *Ans.* $43 : 26'8

REDUCTION OF FEDERAL MONEY.

1. When dollars are reduced to cents,*

RULE :—Set down the dollars, and annex two ciphers.

Dem. Because one dollar is equal to 100 cents.

EXAMPLES.

In $672 how many cents? *Ans.* 67200
In $45 how many cents? *Ans.* 4500
In $10 how many cents? *Ans.* 1000

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* The teacher can convert the above, &c. into questions, by saying—How are dollars reduced to cents? &c.

2. When dollars are reduced to mills,
RULE :—Set down the dollars and annex three ciphers.
Dem. Because one dollar is equal to 1000 mills.

EXAMPLES.

In $496 how many mills ? *Ans.* 496000
In $84 how many mills ? *Ans.* 84000
In $50 how many mills ? *Ans.* 50000

3. When dollars and cents are reduced to cents,
RULE :—Take away the separatrix, or set them down as one whole number.

EXAMPLES.

In $986 : 88 how many cents ? *Ans.* 98688
In $99 : 29 how many cents ? *Ans.* 9929
In $1 : 10 how many cents ? *Ans.* 110

4. When dollars and cents are reduced to mills,
RULE :—Take away the separatrix, and annex a cipher.

EXAMPLES.

In $494 : 19 how many mills ? *Ans.* 494190
In $654 : 29 how many mills ? *Ans.* 654290
In $57 : 01 how many mills ? *Ans.* 57010

5. When dollars, cents, and mills, are reduced to mills,
RULE :—Take away both the separatrix.

EXAMPLES.

In $256 : 59'2 how many mills ? *Ans.* 256592
In $46 : 60'2 how many mills ? *Ans.* 46602
In $3 : 91'6 how many mills ? *Ans.* 3916

6. When cents and mills are reduced to mills,
RULE :—Take away the saparatrix.

EXAMPLES.

In : 99'9 how many mills ? *Ans.* 999
In : 84'2 how many mills ? *Ans.* 842
In : 21'1 how many mills ? *Ans.* 211

7. When cents are reduced to mills,
RULE :—Annex a cipher.

EXAMPLES.

In 33 cents, how many mills ? *Ans.* 330
In 50 cents, how many mills ? *Ans.* 500
In 2 cents, how many mills ? *Ans.* 20

8. When mills are reduced to cents,
RULE :—Point off the right hand figure for mills ; the next two for cents, and the remaining left hand figures will be dollars.

EXAMPLES.

In 167 mills, how many cents? *Ans.* 16'7
In 489 mills, how many cents? *Ans.* 48'9
In 32 mills, how many cents? *Ans.* 3'2

9. When mills are reduced to dollars,

Rule:—Point off the right hand figure for mills, the next two for cents, and the remaining left hand figures will be dollars.

EXAMPLES.

In 672889 mills, how many dollars? *Ans.* 672 : 88'9
In 888623 mills, how many dollars? *Ans.* 888 : 62'3
In 6899 mills, how many dollars? *Ans.* 6 : 89'9

10. When cents are reduced to dollars,

Rule:—Point off the two right hand figures for cents, and the left hand figures will be dollars.

EXAMPLES.

In 79369 cents, how many dollars? *Ans.* 793 : 69
In 8456 cents, how many dollars? *Ans.* 84 : 56
In 945 cents, how many dollars? *Ans.* 9 : 45

QUESTIONS ON REDUCTION.

What is reduction? Changing one denomination to another. By changing denominations, do you alter the value? No, because if we change from the superior to an inferior denomination, the inferior must express as many more as shall equal the superior in value. How do you reduce dollars to cents? By annexing two ciphers to dollars the product will express cents. Why should that reduce dollars to cents? Because we must have 100 times as many cents as dollars to express the same in value. If your given sum contains dollars and cents; how may you reduce them to cents? By taking away the separatrix, and joining the cents to the dollars. Why should that reduce the whole to cents? Because the dollars when united to the cents express hundreds in cents, and the cents count the same as before. How do you reduce cents to mills? By annexing a cipher to the cents. Why should that reduce cents to mills? It is repeating the cents 10 times, and we must have ten times the number of mills, to equal the cents in value.

MULTIPLICATION AND DIVISION.

Examples like the following have generally been placed in the Rule of Three or Proportion: but very improperly, as will appear evident from investigation, viz.

1. Knowing the price of a unit of any kind, that is, of *one* yard, pound, &c. to obtain the price of a quantity.

Ex. If one yard of cloth cost 4 dollars, what will 25 yards cost? *Ans.* $100

Rule:—Multiply the price of unity by the number expressing the quantity.

SOLUTION.

25 *the quantity,*
4 *price of one yard*; *4 is the unit.*
Ans. $100

ILLUSTRATION.

It is evident that when we have the price of one yard, pound, &c. and wish to find what several yards, pounds, &c. cost at the same price, we need only multiply the price of one yard, pound, &c. by the quantity. Now 25 yards of cloth at 4 dollars a yard must cost 25 times as much as one yard—that is, how many is 4 times 25, or what is the product of 25 multiplied by 4. *Ans.* $100 the price of 25 yards of cloth at 4 dollars per yard.

QUESTIONS FOR SOLUTION.

1. If one pound of sugar cost 14 cts.; what cost 48 pounds? *Ans.* $6 : 72

2. If one tun of hay cost $8; what will be the cost of 50 tuns? *Ans.* $400

3. If one piece of silk cost 27 cts.; how much will 50 pieces cost? *Ans.* $13 : 50

4. If 1 bushel of corn cost 75 cts.; what will 64 bushels cost? *Ans.* $48

5. If I pay $2 : 25 for 1 week's board; what will be the bill for 30 weeks? *Ans.* $67 : 50

2. Having the price of a quantity given, to find the price of unity or one.

Ex. If 6 yards of cloth cost $42; what is the cost of 1 yard? *Ans.* $7

RULE:—Divide the price of the quantity by the quantity *itself*, the quotient will give the price of unity or one.

SOLUTION.

The quantity, 6 yds. 6)42 *dols. price of the quantity,*
Ans. 7 *dols. price of* 1 *yard.*

ILLUSTRATION.

In the former examples, the price of *one* yard was given, and the price of 25 required—*here* the price of *one* is required, and the whole number of yards is given, for that reason, it is the converse of that operation, viz. *division:* therefore we divide the price of 6 yards by 6, the quotient shows what 1 yard cost. For it is evident that *one* yard costs only a sixth part as much as six yards, viz. 7 dollars.

QUESTIONS FOR SOLUTION.

1. If 17 yards of cloth cost $136; how much is it per yard? *Ans.* $8

2. If 27 barrels of pork cost $675; how much is that per barrel? *Ans.* $25

3. If 37 acres of land rent for $74; what is that an acre? *Ans.* **$2**

4. If 8 bushels of corn cost $3:60; how much is that per bushel? *Ans.* 45 *cts.*

5. If 35 bushels of wheat cost $42; how much is it per bushel? *Ans.* $1:20

6. If 35 sheep cost $78:75; what are they a head? *Ans.* $2:25

7. A laborer engaged to work 8 months for $96: how much did he receive per month? *Ans* $12

3. How to obtain the *cost* of an unknown quantity, by having the price of *one* quantity given.

Ex. If 13 yards of cloth cost $130; what will 18 yards cost? *Ans.* $180

Rule:—First, find the price of unity; then multiply the price of unity by the quantity; the product will be the answer.

SOLUTION.

13)130

10 *price of unity,*

18 1 *yard.*

80

10

*$180 *price of* 18 *yards.*

ILLUSTRATION.

It is plain, that if we knew the price of one yard, we might repeat this price 18 times, and the result would be the price of 18 yards. Now since 13 yards cost 130 dollars, one yard must cost the thirteenth part of 130 dollars, which is 10 dollars, and multiplying this answer by 18, we have 180 dollars, which is the cost of 18 yards at the rate of 13 yards for 130 dollars.

Obs. The operation last performed is called

THE RULE OF ANALYSIS,

and is a method by which we discover unknown truths from those that are given or assumed, or it is a method by which we obtain other relative numbers or quantities from those that are known or given.

☞ There are two methods of finding the price of unity, viz. by *Division* and by *Multiplication.*

1. By Division, as in the last operation.

QUESTIONS FOR SOLUTION.

1. If 4 yards of cloth cost $8; what will 16 yards cost? *Ans.* $32

2. If 18 barrels of flour cost $162; what cost 12 barrels? *Ans.* $108

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* The general method of solving questions by Analysis has been by fractions, thus: taking the first example, $\frac{130}{13} + 10 \times 18 = 180$. But this method is unnecessary and need never be resorted to.

3. If 3 cords of wood cost $4 : 35; what will be the cost of 30 cords? *Ans.* $43 : 50

4. If I pay $106 : 50 for 12 yards of cloth; what must I pay for 4 yards? *Ans.* $35 : 50

5. If 13 gal. of melasses cost $6 : 50; what cost 25? *Ans.* $12 : 50

6. If 15 lbs. of sugar cost $1 : 87'5; what cost 29? *Ans.* $3 : 62'5

7. If 21 lbs. of coffee cost $5 : 25; what cost 57? *Ans.* $14 : 25

Examples which can be solved by *dividing twice*, without multiplying at all.

Ex. If $25 will buy 50 pieces of Nankin; how many pieces will $2 : 50 buy? *Ans.* 5 *pieces.*

SOLUTION.

50)25 : 00
price of 1 piece, : 50)250
Ans. 5 *pieces.*

ILLUSTRATION.

As many times as 50 is contained in $25, (price of the whole) so much is the cost per piece; and, as many times as 50 cts. (the price of one piece) is contained in $2 : 50 (the sum to be expended) so many pieces $2 : 50 will buy.

QUESTIONS FOR SOLUTION.

1. If $22 : 50 will buy 15 yards of cloth; how many yards will $6 pay for? *Ans.* 4 *yards.*

2. If a man can earn $64 in 4 months; how long must he work to earn $304? *Ans.* 19 *months.*

3. A merchant bought 795 yards of cloth for $107 : 50, he has still $427 : 50 which he wishes to lay out in cloth, at the former price; how many yards can he purchase? *Ans.* 3161 *yards.*

2. Unity or one, found by *multiplication.*

Ex. If 18 men can perform a piece of work in 45 days; how many days would it take 1 man to do it? *Ans.* 810 *days.*

SOLUTION.

45
18
360
45
Ans. 810 *days.*

ILLUSTRATION.

45 days is the time that 18 men can do the work: Now one man cannot do the work of 18 men in the same time:—only the eighteenth part of it, therefore 1 man will be 18 times longer, than 18 men, in doing the same work: so 45 days is repeated 18 times.

QUESTIONS FOR SOLUTION.

1. If 3 men can dig a ditch in 16 days; how long will it take 1 man? *Ans.* 48 *days.*

2. If 60 men can do a piece of work in 79 days; how many days will it take 1 man to do it? *Ans.* 4740 *days.*

3. If 27 men can do a piece of work in 300 days, how many men will it take to do it in one day? *Ans.* 8100 *men.*

3. After finding unity or one by multiplication, we divide the answer to obtain the other relative number.

Ex. If 100 men can do a piece of work in 12 days; how many men will it take to do it in 3 days? *Ans.* 400

SOLUTION.

100
12
200
100
3)1200
Ans. 400 *men.*

ILLUSTRATION.

12 days is the time that 100 men can do the work: Now it will take 12 times as many men to do it in 1 day, which is 1200 days; and one-third as many to do it in 3 days, as in 1; therefore, that time is divided by 3, which gives the number of men required.

QUESTIONS FOR SOLUTION.

1. If 12 men build a wall in 20 days; how many will do it in 8 days? *Ans.* 30 *men.*

2. 17 men can perform a piece of work in 25 days; in how many days would 5 men perform the same work? *Ans.* 85 *days.*

3. 10 men can build a ship in 120 days; how many men will be necessary to do it in 30 days? *Ans.* 40 *men.*

4. To find how much of one commodity must be given for another, having unity or one given.

Ex. How many yards of cloth, at $2 a yard, may be bought for 4 barrels of cider, at $3 a barrel? *Ans.* 6 *yards.*

SOLUTION.

3
4
2)12
Ans. 6

ILLUSTRATION.

If one barrel cost $3, 4 will cost 3 times as much, or $12. As each yard is worth $2, you must give as many yards as there are $2 in 12 dols.

QUESTIONS FOR SOLUTION.

1. If you buy 48 bushels of coal for 12 cts. per bushel, and pay for it with cheese at 10 cts. per lb.; how many pounds do you give? *Ans.* $57\frac{6}{10}$ *lbs.*

2. How many pairs of gloves, at 25 cts. per pair, will pay for 36 dozen and 8 pairs of stockings, at 75 cts. per pair? *Ans.* 1320

3. What will be the price of 6 dozen of eggs, at 2 cts. for 3 eggs? *Ans.* 48 *cts.*

5. When, from the nature of the example, the operation should begin with division; yet, if the numbers be unreducible, we may multiply first and then divide.

Ex. If a man can cut 46 cords of wood in 14 days; how many cords can he cut in 60 days? *Ans.* $197\frac{2}{14}$

SOLUTION.	ILLUSTRATION.
46 60 14)2760 *Ans.* $197\frac{2}{14}$	We multiply 46 cords by 60 days, the time in which the man is to work; the product is 14 times too much, therefore, we divide it by 14, which gives the required number.

QUESTIONS FOR SOLUTION.

1. If 34 barrels of flour be made from 147 bushels of wheat; how much wheat will make 9 barrels of flour? *Ans.* $38\frac{31}{34}$ *bush.*

2. If 16 men can fell 209 trees in a day; how many trees can 35 men fell in the same time? *Ans.* $457\frac{3}{16}$

6. All questions can be solved by *Analysis*, however complicated, without the formality of a statement, as in the Rule of Three. This will be shown by the solution of the following

EXAMPLES.

1. If 56 lbs. of bread be sufficient for 7 men 14 days; how much bread will serve 21 men 3 days? *Ans.* 36 *lbs.*

2. If a man travel 273 miles in 13 days, travelling only 7 hours in a day; how many miles will he travel in 12 days, if he travel 10 hours in a day? *Ans.* 360 *miles.*

3. If 8 men can build 36 rods of wall in 4 days; how many rods can 20 men build in 16 days? *Ans.* 360 *rods.*

SOLUTIONS.

Ex. 1st. By dividing *first.*		Ex. 2d.	Ex. 3d.
7)56	*Multiplying first.*	13)273(21	36
8	56	12	20
21	21	42	8)720
14)168(12	56	21	90
14 3	112	7)252	16
28 36 *Ans.*	7)1176	36	540
28	168	10	90
	3	*Ans.* 360	4)1440
	14)504		*Ans.* 360
	Ans. 36		

ILLUSTRATION. 1st. Ex.

1. We divide by 7, to find how much *one* man would eat in 14 days, and multiply by 21, to find how much 21 men would eat. Having found how much 21 men would eat in 14 days, we divide the quantity (168 lbs.) by 14, to find how much they would eat in *one* day, and then multiply by 3, to find how much they would eat in 3 days.

In the second operation the 56 is multiplied by the 21 and the answer divided by 7. This gives the same answer as 56 divided by 7 and the quotient multiplied by 21, *viz.* 168 lbs.; then that *answer* is multiplied by 3 and divided by 14, which is the same as 168 lbs. divided by 14 and the *Quotient* multiplied by 3. This shows that we can *multiply* first and then divide.

ILLUSTRATION. 2d. Ex.

We divide by 13, to find how much the man would travel in *one* day, at the rate of 7 hours per day.

We multiply by the 12, to find how much he would travel in 12 days, at the same rate.

We divide by 7 to find how much he would travel in *one* hour, and multiply by 10 to find how much he would travel in 10 hours.

ILLUSTRATION. 3d. Ex.

36 cannot be divided by 8 without a remainder; therefore we multiply 36 by 20, and divide by 8. This shows what 20 men can do in 4 days: then multiplying the answer by 16 gives a product 4 times too much—and dividing it by 4 gives what 20 men can do in 16 days.

QUESTIONS FOR SOLUTION.

1. If 2 men can mow 16 acres in 4 days; how many acres can 5 men mow in 3 days? *Ans.* 30 *acres.*

2. If 12 men can earn $240 in 8 days; what sum will 21 men earn in 15 days? *Ans.* $787 : 50

3. If 20 laborers will clear 60 acres of woodland in 2 months; how many acres will 35 laborers clear in 6 months? *Ans.* 315 *acres.*

4. If 67½ acres be plowed by 6 men in 15 days; how many acres can 18 men plow in 60 days? *Ans.* 810 *acres.*

OBS. There are 160 rods in *one* acre; find how many rods 1 man can plow in 1 day first, &c.

5. If 2 men can build 12 rods of wall in 6 days; how many rods can be built by 8 men in 24 days? *Ans.* 192 *rods.*

Direction. First find what one man can do in 1 day.

6. If 8 men receive $40 for 5 days' labor; how much ought 32 men to recieve for 24 days' labor? *Ans.* $768

Direction. First find what 1 man received per day.

A general Rule to solve Questions by Analysis. Change each term of supposition to a unit. Then repeat or separate into parts each corresponding term of demand. The last answer is the term required.

CORRESPONDENT EXAMPLES,

embracing all the principles of the preceding operations.

1. If 23 men can build a wall in 32 days; how many men would it require to do it in one day? *Ans.* 736 *men.*

2. If 2 men in three days will earn $15 how much will 7 men earn in the same time? *Ans.* $52 : 50

3. How many sheep, at $4 a head, can a butcher, who has $747 buy; and how many dollars will he have remaining? *Ans.* 186 *sheep* $3 *Rem.*

4. How many barrels of flour can I purchase for $748; at the rate of $5 : 50 per barrel? *Ans.* 136 *bar.*

5. E. and F. purchased 245 acres of land, for $2600, E. paid $1200 of the money, and F. paid the remainder. How much land should each one have? *Ans.* E. 113 *acres* F. 132 *nearly.*

6. If a field will feed 7 horses 8 weeks; how long will it feed 28 horses? *Ans.* 2 *weeks.*

7. If a man who has 40 acres of land, be taxed $10; what would be the tax on a town containing 23620 acres? *Ans.* $5905

8. The annual wages of a man being $100 to be paid in land at $6 per acre: how many acres will he receive after 3 years and 7 months? *Ans.* 59,7 *acres.*

9. Suppose D. owes F. $798, and D. has but $638 : 40 to pay the demand; how much will F. receive on the dollar? *Ans.* 80 *cts.*

10. If a field will pasture 6 cows 91 days, how long will it pasture 21 cows? *Ans.* 26 *days.*

11. If a farmer has only hay sufficient to keep his 625 sheep 2 months; how many of them must he sell to keep the rest 5 months? *Ans.* 375

12. If 7 men can reap 84 acres of wheat in 12 days; how many men can reap 100 acres in 5 days? *Ans.* 20 *men.*

13. If 14 men, in 15 days build 16 rods of wall; how many men must be added, to do it in 2 days? *Ans.* 105 *men.*

14. A ship has sailed 24 miles in 4 hours; how long will it take her to sail 150 at the same rate? *Ans.* 25 *hours.*

15. It is worth as much to pasture 1 cow, as 5 sheep. If I pay $1 a month for pasturing a cow; what must I pay for pasturing 35 sheep, 7 months? *Ans.* $49

16. If the tuition of 3 boys, for 6 months, be $40 : 20; what will that of 60 boys amount to for 4½ years? *Ans.* $7236

17. If 1500 yards of cloth cost $7500; what would be the price of one yard? *Ans.* $5

18. What cost 215 pounds of bread, at 90 cts. for 14 pounds? *Ans.* $13 : 82

19. If 13 yards of cloth cost $39; how many yards of the same may be bought for $156? *Ans.* 52 *yds.*

20. A merchant gaining $7500 in 6 years with a capital of $18000; what would he gain at the same rate in 14 years? *Ans.* $17500

21. If 4 reapers receive $11 : 04 for 3 days' work; how many men may be hired 16 days for $103 : 04? *Ans.* 7 *men.*

22. If 48 men can build a wall in 24 days; how many men can do the same in 192 days? *Ans.* 6 *men.*

23. If $5 will buy 2 yards; what will $15 buy? *Ans.* 6 *yds.*

24. A man failing in trade finds that he can pay only $2 on every $3, of his debts; how much can he pay on a debt of $25? *Ans.* $16 . 66'6

25. If 20 men perform a piece of work in 30 days; how many men will it take to accomplish a piece 4 times as large in 4 days? *Ans.* 600 *men.*

26. If 8 men can do a piece of work in 12 days: in how many days will 16 men do it? *Ans.* 6 *days.*

27. If 20 bushels of wheat are sufficient for 8 persons 5 months; how many will be sufficient for 4 persons 12 months? *Ans.* 24 *bush.*

28. Bought a piece of cloth for $48 : 24, at the rate of $2 : 38 for every 2 yards; what was the number of yds. *Ans.* 40 *yds.* 2 *qrs.* 0 *na* +

29. A man failing in trade, gives up his property, amounting to $1020 : 20 to his creditors, to whom he owes the following sums, viz. to A. $1600 : 60; to B. $500 : to C. $750:20; to D. $1000; to E. $230; how much does he pay on a dollar, what does he pay each creditor; and what was the loss to the whole.

Ans. $0 : 25 on a dollar. To A. $400 : 15; B. $125; C. $187 : 55; D. $250; E. $57 : 50. TOTAL LOSS $3060 : 60

30. If 12 oxen in 5 days plow 11 acres; how many oxen would plow 33 acres in 18 days? *Ans.* 10 *oxen.*

QUESTIONS ON ANALYSIS.

1. When the price of a unit is given, how can the price of a quantity be found? Give an example. What is a unit in the example you gave? Which is the quantity? Which the price? Why should the rule mentioned give the price of the quantity? Because it is repeating the price of one thing, as many times as the quantity contains ones of the same kind. *V. p.* 96.

2. When the price of a quantity is given, how is the price of unity found? Give an example. Which is the price of the quantity in the example you gave? What number is the quantity? What does the quotient show? Why should dividing as you named, show the price of unity? Because we find how many times the price of a unit is contained in the quantity. *V. p.* 97.

3. Having the price of a quantity given, how can the price of another quantity be found? Give an example. What is the operation called? What is Analysis? Name a question solved by this rule! What is given or known in the question you proposed? What is found? What is required? How is that found? By what two rules are such questions solved? How many methods are there of finding the price of a unit? Give an example of dividing first, then multiplying. Of dividing twice without multiplying at all. Of multiplying only once. By multiplying first, then dividing the product, &c. *V. p.* 100.

4. How can you find how much of one commodity must be given for

another, having the price of a unit given? Give an example. Why multiply? Why divide the product? What does the quotient show? *V. p.* 100.

5. When must a question be multiplied first, instead of dividing first? Give an example of unreducible numbers. Name a question which must be multiplied first instead of dividing first. What does the product show? What is the name of the divisor? Of the quotient? Why? *V. p.* 100.

6. By what rule have you solved the above-named examples? Will that rule solve all questions in the Rule of Three? What is its object and design? Give the Rule of Analysis, at page 102.

DECIMALS.

A fraction is part of a whole number and arises from division. The remainder in division is a fraction. A decimal is distinguished from a whole number by a comma or point prefixed thus ,5 and signifies 5 tenths of an integer: as will appear from the following table.

,5	signifies	5 tenth parts.
,05		5 hundredth parts.
,25		25 hundredth parts.
,255		255 thousandth parts.
5,5		5 and 5 tenths.
5,005		5 and 5 thousandths,
,0001		1 ten thousandth.
,122		122 thousandths.
,1222		1222 ten thousandths.
,0122		122 ten thousandths.
,0012		12 ten thousandths.
1,0002		1 and 2 ten thousandths.

As whole numbers increase by a ten-fold proportion from the units' place to the left hand, so decimals decrease in the same proportion, from unity to the right hand, as may be seen by the following table.

Integers,							*Decimals,*					
Millions.	C's of Thousands,	X's of Thousands,	Thousands,	Hundreds,	Tens,		Tenth parts,	Hundredths,	Thousandths,	Ten thousandths,	100 Thousandths,	Millions of parts.
7	6	5	4	3	2	1	2	3	4	5	6	7
						Unit.						
Seventh place.	Sixth place,	Fifth place,	Fourth place,	Third place,	Second place,		Primes,	Seconds,	Thirds,	Fourths,	Fifths,	Sixths,
Integers, or whole increasing numbers.							Decimals, or Fractional decreasing numbers.					

The order of places in whole numbers is from the right hand to the left; but in decimals it is from the left hand to the right. So in this decimal ,456, the figure 4 stands in the first place, and is 4 primes, or 4 tenths of an integer; and 5 the second figure is 5 seconds, or five hundredth parts of an integer, &c.

Ciphers before integers, and after decimals, are of no value, but after integers and before decimals, they have their value; in integers they increase, but in decimals they decrease the value of the figures joined to them. For 4, and 04, and 004, in whole numbers is still but 4; but in decimals, 4 by having a comma prefixed (,4) is decreased from 4 integers to $\frac{4}{10}$ of an integer, and ,04 to 4 hundredth parts of an integer, &c. Again, in whole numbers, 30 is thirty, and 300 is three hundred; but in decimals, ,30 or ,300 is still but $\frac{3}{10}$ of an integer.

DECIMAL ADDITION.

Rule:—Place figures of the same order under each other: place a separatrix between the orders of units and tenths; then proceed in every respect as in Simple Addition. Point off as many places from the total sum, for decimals, as there are decimals in the greater number added.

Ex. What is the sum of 234,406 4,6490 13234 2,2 3650 ,4002 999,4699?

OPERATION.

234,406
4,6490
13,234
2,2
3650,4002
999,4699
4904,3591

EXPLANATION.

9 ten thousandths added to 2, are 11 ten thousandths; which is 1 *ten thousandth*, to be written under that order; and 1 of the order of *thousandths*, to be carried to that order.

1 thousandth carried to 9, is 10, and 4 are 14, and 9 are 23, and 6 are 29 thousandths; which is 9 *thousandths*, to be written under that order, and 2 *hundredths*, to be carried to the next order.

Thus through the other orders, observing to *place a separatrix between the orders of units and tenths.*

Exercises for the Slate.

Add 4,003 54,9 3,21 6,7203 *Ans.* 68,8333
Add 409,903 107,7842 6,1043 10,2974 *Ans.* 534,0889
Add 427, 603,04 210,15 3,364 ,021 *Ans.* 1243,575

DECIMAL SUBTRACTION.

Rule:—Place the numbers as directed in addition of decimals; then subtract as in whole numbers, and from the remainder point off so many places for decimals, as there are decimals in the greater number.

Ex. Subtract 2,56 from 24,329.

OPERATION.

24,329
2,56
21,769

EXPLANATION.

Nothing from 9 thousandths, and 9 remains to be set down. 6 hundredths cannot be taken from 2 hundredths; we therefore add 10 to the minuend, which makes 12. 6 taken from 12 leaves 6. As 10 was added to the minuend, an equal quantity must be added to the subtrahend. 1 of the order of tenths is the same as 10 hundredths, we therefore add 1 to the 5 tenths, making it 6 tenths. 6 tenths cannot be taken from 3 tenths, we therefore add 10 to the minuend, which makes 13. 6 taken from 13, leaves 7. As 10 was added to the minuend, an equal amount must be added to the subtrahend. 1 of the order of units is the same as 10 tenths, we therefore add 1 to the 2 units, making it 3 units.

Exercises for the Slate.

From 304,567 take 158,632 *Ans.* 145,935
From 215,003 take 1,1034 *Ans.* 213,8996
From 1 take ,9993 *Ans.* 0,0007
From 68,8333 take ,00042 *Ans.* 68,83288

DECIMAL MULTIPLICATION.

Rule:—Multiply the same as in whole numbers, and point off from the right hand of the product as many figures for decimals as there are decimal places in both the factors; if there are not figures enough in the product, supply the deficiency by annexing ciphers to the left hand of the product.

Ex. Multiply 20,8 by ,5

OPERATION. **EXPLANATION.**

```
 20,8
   ,5
-----
10,40
```

We multiply by 5 tenths, which is equal to $\frac{1}{2}$, consequently we are taking one half of the multiplicand; because when our multiplicand is 1, we take the multiplicand once; when it is 2, we take the multiplicand twice, &c.; when it is $\frac{1}{4}$ we take one fourth part of the multiplicand; when $\frac{1}{3}$, we take 1 third part of the multiplicand: So it will be perceived that multiplying by a pure decimal, is only taking a part of the multiplicand, as plainly appears from the example, for our multiplier is only the half of a unit, and our product is only one half of the multiplicand.

Obs. Multiplying a whole number increases it; but in decimals it diminishes it, for the value of the multiplier is less than one. The product of a number multiplied by 1 is exactly the same as the multiplicand; now a less multiplier must produce a less product than a greater multiplier.

Multiplying by a pure decimal produces a product less than the multiplicand, and the product bears the same proportion to the multiplicand that the multiplier bears to a unit. The multiplication of any numbers by a decimal fraction, serves to diminish the value of that number, by as much as the fraction falls short of unity. Consequently, multiplying one fraction by another, must produce a fraction smaller than either of the factors. Hence you may observe, that the number of decimal figures in any product must be equal to the number of decimal figures in both the factors.

Exercises for the Slate.

Multiply 12,836 by ,354	*Ans.*	4,543944
Multiply ,3785 by ,003	*Ans.*	,0011355
Multiply 8,50 by 83,7	*Ans.*	711,450
What is the product of ,00037 and ,0685 ?		,000025345

DECIMAL DIVISION.

Rule :—Proceed as in Division of the whole numbers; then point off as many places of the quotient for decimals, as the dividend has decimal places more than the divisor.

Note 1. If there happen to be not so many places in the quotient as are required, prefix a sufficient number of ciphers to make up the defect.

Note 2. When the decimal places of the divisor are more than those of the dividend, this defect must be supplied by annexing ciphers.*

Note 3. When there is a remainder, ciphers may be annexed to it, which render it capable of being further divided, and the succeeding figures in the quotient are decimals; which by annexing a cipher, or ciphers, to every succeeding remainder, may be continued at pleasure.

Note 4. When the dividend is a whole number, and the divisor a decimal, annex as many ciphers to the dividend as there are decimal places in the divisor, the quotient figures will be whole numbers till all the annexed ciphers are brought down and divided; then, if there be a remainder, annex a cipher or ciphers to it and divide; the remaining figures in the quotient will be decimals.

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* When the dividend consists of an integer only, or when it is a mixed number, if the decimal places of the divisor be more than those of the dividend, supply the defect by annexing ciphers to the integer, or decimal; the quotient figures will be whole numbers, till all the annexed ciphers are brought down and divided; then, if there be a remainder, annex a cipher, or ciphers, and divide; the remaining figures in the quotient will be decimals.

N. B. Four or five places of decimals are generally sufficient.

Note 5. When the dividend is a decimal, and the divisor a whole number, divide as in whole numbers till every figure of the dividend is brought down and divided; and if there are not as many places in the quotient as there are decimals in the dividend, supply the defect by prefixing a cipher or ciphers; and if there be a remainder, ciphers may be annexed, and the quotient carried on still further.

Note 6. When the dividend is a decimal and the divisor a whole number, if the divisor is not contained in the dividend, place a cipher in the quotient in the first place; then, annex a cipher to the dividend, and if the divisor is not contained in the dividend after *one* cipher is annexed, place another cipher in the quotient and annex another to the dividend; thus proceed till the dividend can be divided, and if there be a remainder, a cipher or ciphers may be annexed, and the quotient carried on still further.

EXAMPLE.

```
2,75)23,30625(8,475 quotient.
     2200
     ----
      1306
      1100
      ----
       2062
       1925
       ----
        1375
        1375
```

Obs. In this example, there being three places of decimals in the dividend *more* than in the divisor, point off the three right hand figures of the quotient, viz. 475 for decimals, according to the rule.

Exercises for the Slate.

Divide 7,735 by 3,25	*Ans.*	2,38
Divide 18,3578 by ,748	*Ans.*	24,542+
Divide ,68543 by 12,5	*Ans.*	,0548+
Divide 753 by ,578	*Ans.*	1302,768+
Divide 1,87 by 3,5	*Ans.*	,5342+
Divide ,55736 by 48	*Ans.*	,01161+
Divide 8,38 by ,666	*Ans.*	12,582+
Divide 19 by ,333	*Ans.*	57,057+
Divide ,00075 by 0025	*Ans.*	,3

REDUCTION OF DECIMALS.

I. To reduce a decimal to its lowest terms.

Rule:—When there are ciphers at the right hand of a decimal,* cast them off; the decimal will then be in its lowest terms.

OPERATION.

$$\text{Reduce}\left\{\begin{matrix},756000\\,250000\\,7000\\,5000\end{matrix}\right\}\text{to their lowest terms.}\left\{\begin{matrix},756\\,25\\,7\\,5\end{matrix}\right.$$

---

* All decimals except those which have ciphers at the right hand are naturally in their lowest terms.

II. To reduce decimals of different denominators to those having the same denominator.

RULE:—Annex so many ciphers to the *least*, as shall make the number of places equal to the *greatest;* the decimals will then have one common denominator.

Reduce ,8 ,75 ,25 ,378 ,9875 ,037 to a common denominator.
Ans. ,8000 ,7500 ,2500 ,3780 ,9875 ,0370

CORRESPONDENT EXAMPLES.

1. What is the amount of one, and five tenths; forty-five, and three hundred and forty-nine thousandths; and sixteen hundredths? *Ans.* 47,009

2. From two thousand, and sixteen hundredths; take one thousand and four, and four millionths. *Ans.* 996,159996

3. From twenty-four thousand nine hundred and nine and one tenth, take fourteen thousand and twenty-nine thousandths. *Ans.* 10909,071

4. Add three hundredths, five tenths, forty-five hundredths, eleven thousandths, three ten-thousandths, and four millionths together. *Ans.* ,991304

5. Take eighty-five and seven hundred and thirty-seven thousandths from one hundred. *Ans.* 14,263

6. How many yards of cloth in 3 pieces, each piece containing 20,75 yards? *Ans.* 62,25 *yds.*

7. At $5 : 47 per yard; what cost 8,3 yards of cloth? *Ans.* $45,401

8. Divide three thousandths by four hundredths. *Ans.*,075

9. Multiply twenty-nine and three tenths, by seventeen. *Ans.* 498,1

10. Multiply twenty-seven thousandths, by four hundredths *Ans.* ,00108

11. Multiply two thousand and four, and two tenths by twenty-seven. *Ans.* 54113,4

12. If ,6 of a barrel of flour cost $5; what is that per barrel? *Ans.* $8 : 33‘3+

13. A man bought cloth at several times as follows, viz. 20,3 yards; 41,5 yards; 21,7 yards; 9,025 yards and 5 yards; how much did he buy in all? *Ans.* 97,525 *yds.*

14. What cost 6,5 yards of cloth, at $2 : 35‘2 per yard? *Ans.* $15 : 28‘8

15. What will 20,75 yards cost at $3 : 50 per yard? *Ans.* $72 : 62‘5

16. From ninety yards and 3 tenths, take 47 yards and 8 tenths. *Ans.* 42,5 *yds.*

17. If you divide 116,5 barrels of flour equally among 5 men, how many barrels will each have? *Ans.* 23,3 *bar.*

18. A. bought 13,51; 5,625; 3,1625 cords of wood, and burnt 11,5; and sold 5,125 cords; what was left? *Ans.* 5,6725

19. B. bought 136,375 bushels of wheat, for $119 : 85'5 and sold 49,25 bushels for $57 : 35'5; what had he left in wheat, and what was its cost? *Ans.* 87,125 *bush. cost* $62 : 50

VULGAR FRACTIONS.

This term is given to parts of unity or whole numbers in contradistinction to integer an entire thing or quantity. Fractions originate in division as will appear from these two examples. Divide 13 by 3; 3 is contained in it 4 times and one over, this is a fraction: again, let 26 be divided by 3, the answer will be 8 times and 2 over. To express these remainders, the divisor and remainder are written thus :—$_3$ and $\frac{2}{3}$; read, one third, two thirds. The smaller number is the remainder and shows what part of unity is meant. The larger number is the divisor and shows into how many parts it is divided. The upper figure of the fraction is called the numerator; and the lower the denominator. Fractions are divided into proper and improper fractions, and mixed numbers. Examples of each; —$\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{8}$, are proper fractions :—$\frac{6}{3}$, $\frac{8}{7}$, $\frac{12}{}$, are improper fractions, A mixed number, is a fraction joined to a whole number, thus: —12$\frac{1}{3}$, 36$\frac{3}{8}$, 47$\frac{3}{7}$. It often happens that such numbers are to be used as multipliers or divisors.

ADDITION OF FRACTIONS.

This is collecting in one sum, broken numbers.

Rule :—Add the numerators into one sum, and divide by the denominator; set the remainder under the column.

2. If the columns be composed of mixed numbers, carry the quotient arising from the fractions to the column of whole numbers.

Ex. What is the sum of $\frac{3}{12}$ and $\frac{5}{12}$ and $\frac{8}{12}$?

OPERATION.

$$\begin{array}{r} 3 \\ 5 \\ 8 \\ \hline 12)16(1\frac{4}{12}\ \textit{Ans.} \\ 12 \\ \hline 4\ \textit{the fraction.} \end{array}$$

Exercises for the Slate.

1. Add $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$ and $\frac{1}{4}$ of a dollar together. *Ans.* 1\frac{3}{4}$
2. Add $\frac{2}{3}$, $\frac{2}{3}$, $\frac{1}{3}$ and $\frac{2}{3}$ of a yard together. *Ans.* 2$\frac{1}{3}$ *yds.*
3. Add $\frac{5}{8}$, $\frac{7}{8}$, $\frac{6}{8}$, and $\frac{3}{8}$ of a bushel together. *Ans.* 2$\frac{5}{8}$ *bush.*
4. Add $\frac{2}{8}$, $\frac{4}{8}$, $\frac{1}{8}$, and $\frac{7}{8}$ of a dollar together. *Ans.* 1\frac{6}{8}$

5. Add $\frac{2}{15}$ $\frac{4}{15}$ $\frac{6}{15}$ and $\frac{5}{15}$ of a pound together.
Ans. $1\frac{2}{15}$ *lbs.*

Note. When $\frac{1}{2}$ is used for $\frac{2}{4}$, in the same column with fourths, it must be regarded as $\frac{2}{4}$. Examples: $\frac{3}{4}$, $\frac{2}{4}$, $\frac{1}{2}$, $\frac{1}{2}$. *Ans.* $2\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{2}{4}$ $\frac{1}{2}$. *Ans.* $2\frac{1}{4}$

Exercises for the Slate.

$16\frac{7}{8}$	$13\frac{6}{10}$	$17\frac{8}{15}$	$42\frac{56}{135}$
$18\frac{6}{8}$	$14\frac{8}{10}$	$21\frac{14}{15}$	$36\frac{95}{135}$
$19\frac{5}{8}$	$15\frac{7}{10}$	$14\frac{7}{15}$	$41\frac{87}{135}$
$55\frac{2}{8}$	$44\frac{1}{10}$	$53\frac{14}{15}$	$120\frac{103}{135}$

Examples for Practice.

1. What is the amount of $8\frac{1}{2}$ yards, $6\frac{1}{2}$ yards, $2\frac{1}{2}$ yards, $4\frac{1}{2}$ yards, and $9\frac{1}{2}$ yards? *Ans.* $31\frac{1}{2}$

2. Find the amount of $12\frac{1}{3}$ of a bushel, $8\frac{2}{3}$ of a bushel, $13\frac{1}{3}$ of bushel, and $25\frac{2}{3}$ of a bushel? *Ans.* 60

3. How many whole dollars in $6\frac{2}{4}$, $8\frac{3}{4}$, $19\frac{3}{4}$ and $6\frac{1}{4}$ dollars? *Ans.* $41\frac{1}{4}$

4. What is the amount of $21\frac{1}{8}$ of an acre, $6\frac{3}{8}$, $23\frac{5}{8}$, and $\frac{3}{8}$? *Ans.* 52

5. A merchant sold $\frac{12}{18}$ of a ship to one man, and $\frac{3}{18}$ to another: what part of the ship did he sell? *Ans.* $\frac{15}{18}$

SUBTRACTION OF FRACTIONS.

This is taking one broken number from another.

1. To subtract one fraction from another.

Rule:—Subtract the numerators of the fraction from each other, write the remainder over the denominator.

Ex. From $\frac{7}{8}$ take $\frac{3}{8}$

OPERATION.

$$\begin{array}{l} 7 \\ \underline{3} \\ 4 \quad \textit{written } \frac{4}{8} \textit{ Ans.} \end{array}$$

Exercises for the Slate.

From $\frac{3}{4}$ take $\frac{1}{4}$ *Ans.* $\frac{2}{4}$
From $\frac{4}{5}$ take $\frac{1}{5}$ *Ans.* $\frac{3}{5}$
From $\frac{8}{9}$ take $\frac{3}{9}$ *Ans.* $\frac{5}{9}$
From $\frac{2}{3}$ take $\frac{1}{3}$ *Ans.* $\frac{1}{3}$
From $\frac{5}{6}$ take $\frac{4}{6}$ *Ans.* $\frac{1}{6}$
From $\frac{4}{7}$ take $\frac{3}{7}$ *Ans.* $\frac{1}{7}$

2. To subtract a fraction or mixed number from a whole number.

Rule:—Invert the fraction: then subtract the denominator from the numerator, which will give the fractional remainder; for the unit borrowed, take one from the whole number. In mixed numbers *add* 1 to the integral part, before subtracting it from the whole number.

Exs. From 25 take $\frac{6}{8}$	Take $7\frac{3}{5}$ from 14

OPERATIONS.

$\frac{8}{6}$ *fraction inverted.*	$\frac{5}{3}$ *fraction inverted.*
8	5
6 *fraction subt.*	3 *fraction subt.*
2 *fractional rem.*	2 *fractional rem.*
written $\frac{2}{8}$.	*written* $\frac{2}{5}$.
25 *the integer.*	14 *the integer.*
1 *subt.*	8 *subt.*
$24\frac{2}{8}$ *Ans.*	$6\frac{2}{5}$ *Ans.*

Exercises for the Slate.

From 12 take $\frac{5}{8}$	*Ans.* $11\frac{3}{8}$	From 3 take $\frac{3}{16}$	*Ans.* $2\frac{13}{16}$
From 6 take $\frac{7}{8}$	*Ans.* $5\frac{1}{8}$	From 9 take $5\frac{1}{2}$	*Ans.* $3\frac{1}{2}$
From 10 take $\frac{1}{10}$	*Ans.* $9\frac{9}{10}$	From 100 take $99\frac{99}{100}$	*Ans.* $\frac{1}{100}$

3. To subtract a fraction from a mixed number; or a mixed number from the same, when the fractional part to be subtracted is greater than that from which it is to be subtracted.

RULE :—Change $\frac{1}{2}$ to $\frac{2}{2}$, make the less fraction equal to unity; add to that (in the numerator) the original numerator; then subtract from that sum the other fraction, adding 1 to the integral subtrahend before taking it from the integers.

EXAMPLES.

From $6\frac{5}{7}$ take $\frac{6}{7}$ OPERATION.
$$\begin{array}{ccccc} \frac{7}{7} & 5 & 12 & 6 & 5\frac{6}{7}\ \textit{Ans.} \\ & \underline{7} & \underline{6} & \underline{1} & \\ & 12 & 6 & 5 & \end{array}$$

From $16\frac{2}{4}$ take $7\frac{3}{4}$ OPERATION.
$$\begin{array}{ccccc} \frac{4}{4} & 4 & 6 & 16 & 8\frac{3}{4}\ \textit{Ans.} \\ & \underline{2} & \underline{3} & \underline{8} & \\ & 6 & 3 & 8 & \end{array}$$

From $8\frac{1}{2}$ take $5\frac{3}{4}$ OPERATION.
$$\begin{array}{cccccc} \frac{2}{4} & \frac{4}{4} & 4 & 6 & 8 & 2\frac{3}{4}\ \textit{Ans.} \\ & & \underline{2} & \underline{3} & \underline{6} & \\ & & 6 & 3 & 2 & \end{array}$$

Exercises for the Slate.

From $105\frac{1}{2}$ take $99\frac{3}{4}$	*Ans.* $5\frac{3}{4}$	From $19\frac{2}{5}$ take $11\frac{3}{5}$	*Ans.* $7\frac{4}{5}$
From $70\frac{3}{4}$ take $69\frac{1}{2}$	*Ans.* $1\frac{1}{4}$	From $69\frac{1}{8}$ take $48\frac{5}{8}$	*Ans.* $20\frac{4}{8}$
From $24\frac{1}{3}$ take $12\frac{2}{3}$	*Ans.* $11\frac{2}{3}$	From $16\frac{1}{4}$ take $7\frac{3}{4}$	*Ans.* $8\frac{2}{4}$

Examples for Practice.

1. From 24 dollars take $\frac{1}{5}$ of a dollar. *Ans.* \$$23\frac{4}{5}$

2. Bought $7\frac{3}{4}$ yards of cloth, sold $5\frac{1}{2}$ yards; how many yards remained unsold? *Ans.* $2\frac{1}{4}$ *yds.*

3. Borrowed $16\frac{2}{4}$ dollars, paid $7\frac{3}{4}$ dollars; how much remains unpaid? *Ans.* \$$8\frac{3}{4}$

4. A. owned 85 acres of land, sold B. $37\frac{7}{8}$ acres; how many acres had A. remaining? *Ans.* $47\frac{1}{8}$ *acres.*

MULTIPLICATION OF FRACTIONS.

This is repeating a whole or broken number; by a part or the parts of an integer.

1. To multiply a fraction by a fraction.

RULE:—Multiply the numerators together, for the numerator of the product; and the denominators together, for the denominator of the product, which will give the answer.

Ex. Mult. $\frac{3}{8}$ by $\frac{4}{5}$ OPERATION. $\left.\begin{matrix}\text{3 times 4 are 12}\\ \text{8 times 5 are 40}\end{matrix}\right.$ *written* $\left\{\frac{12}{40}\right.$ *A.*

Exercises for the Slate.

Multiply $\frac{5}{7}$ by $\frac{8}{11}$	*Ans.* $\frac{40}{77}$
$\frac{4}{5}$ by $\frac{15}{16}$	*Ans.* $\frac{60}{80}$
$\frac{11}{13}$ by $\frac{18}{49}$	*Ans.* $\frac{198}{637}$
$\frac{21}{22}$ by $\frac{27}{29}$	*Ans.* $\frac{567}{638}$

Examples for Practice.

1. At $\frac{6}{25}$ of a dollar a yard; what will $\frac{7}{8}$ of a yard of cloth cost? *Ans.* \$$\frac{42}{200}$

2. If $\frac{1}{4}$ of a yard of cloth cost $\frac{2}{3}$ of a dollar; what will $\frac{7}{8}$ of a yard come to? (*invert* $\frac{1}{4}$) *Ans.* \$$\frac{56}{24}$

3. A man owned $\frac{4}{25}$ of a factory, and sold $\frac{6}{11}$ of his share; what part of the whole capital did he sell? *Ans.* $\frac{24}{275}$

4. During a storm, the master of a vessel was obliged to throw overboard $\frac{7}{15}$ of the cargo. A. owned $\frac{3}{8}$ of the cargo; what part of the whole cargo was his share of the loss? *Ans.* $\frac{21}{120}$

5. If $\frac{3}{4}$* of a yard of cloth cost $\frac{4}{5}$ of a dollar; what will $\frac{5}{6}$ of a yard cost? *Ans.* \$$\frac{80}{90}$

6. If $\frac{3}{5}$ of a pound of butter cost $\frac{2}{9}$ of a dollar; what will $\frac{1}{5}$ of 1 pound cost? *Ans.* \$$\frac{10}{135}$

7. If $\frac{3}{4}$ of a pound of raisins cost $\frac{2}{15}$ of a dollar; what will $\frac{1}{4}$ of 1 pound cost? *Ans.* \$$\frac{8}{180}$

8. If $\frac{1}{2}$ of a yard of cloth cost $\frac{3}{4}$ of a dollar; what cost $\frac{7}{8}$ of a yard? *Ans.* \$$\frac{42}{32}$

2. To multiply a whole number by a fraction; when the numerator is 1.

RULE:—Divide the integer by the denominator of the fraction, the quotient will be the answer.

EXAMPLE.	OPERATION.
Multiply 43784 by $\frac{1}{2}$	2)43784
	21892 *Ans.*

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* Invert the fractions $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{4}$ and $\frac{1}{2}$ in Examples 5, 6, 7 and 8th, before multiplying.

Exercises for the Slate.

Multiply 8736 by $\frac{1}{3}$ *Ans.* 2912
8948 by $\frac{1}{4}$ *Ans.* 2237
9735 by $\frac{1}{5}$ *Ans.* 1947

Examples for Practice.

1. If 1 acre of land will produce 126 bushels of potatoes; how many bushels will $\frac{1}{2}$ of an acre produce? *Ans.* 63 *bush.*

2. The whole of a factory is valued at $2587; G. owns $\frac{1}{20}$ of it; how much is his share? *Ans.* 129\frac{7}{20}$

3. A ship worth $8400 being lost at sea, of which $\frac{1}{4}$ belonged to A., $\frac{1}{3}$ to B., and the remainder to C.; what loss will each sustain, supposing they have $6000 insured?
Ans. A's. loss $600; B's. $800; and C's. $1000

4. A farm of 48 acres is owned, viz.—A. $\frac{1}{3}$, B. $\frac{1}{6}$, C. $\frac{1}{9}$, and D. the remainder; what is he to receive for his part, if the whole was sold for $918? *Ans.* $357

5. A man gave $2568 for a house, and then paid $\frac{1}{24}$ part as much for having it repaired. For how much must he sell the house, in order to lose nothing? *Ans.* $2675

Obs. When the multiplier is a proper fraction, such as $\frac{3}{4}$, $\frac{3}{8}$, &c.

Rule:—Multiply the whole number by the numerator of the fraction, and divide *that* product by the denominator; the quotient will be the answer.

EXAMPLE. OPERATION.

Multiply 8743 by $\frac{3}{4}$

$$\begin{array}{r} 8743 \\ 3 \\ \hline 4)26229 \\ \hline 6557\frac{1}{4} \end{array}$$ *Ans.*

Exercises for the Slate.

Multiply 9734 by $\frac{3}{8}$ *Ans.* 3650$\frac{2}{8}$
4745 by $\frac{7}{8}$ *Ans.* 4151$\frac{7}{8}$
3472 by $\frac{6}{8}$ *Ans.* 2604

Examples for Practice.

1. What is $\frac{3}{8}$ of a yard of calico worth, at $2 per yard? *Ans.* 75 *cts.*

2. What is $\frac{3}{5}$ of a barrel of flour worth, at $5 : 25 per barrel? *Ans.* $3 : 15

3. What is $\frac{1}{3}$ of a bushel of corn worth at 75 cts. per bushel? *Ans.* 25 *cts.*

4. Suppose an acre of land to be worth $48 : 16; what is the value of $\frac{3}{4}$ of an acre? *Ans.* $36 : 12

5. A man purchased a farm for $5642, and paid $\frac{4}{7}$ of the price in cash, and gave his note for the remainder; how many dollars did he pay down? *Ans.* $3224

6. Two men A. and B. traded in company and gained $456 of which A. is to have $\frac{5}{8}$ and B. $\frac{3}{8}$; what was the share of each? *Ans.* A's. $285 B's. $171

3. To multiply a whole by a mixed number, whose numerator is 1.

Rule:—Multiply first by the whole number; then divide the multiplicand by the denominator of the fraction, and add the quotient to the product of the integral multiplier; their sum will be the answer.

EXAMPLE.	OPERATION.	
Multiply 3842 by $13\frac{1}{3}$	3842	3)3842
	13	$1280\frac{2}{3}$
	11526	
	3842	
	49946	
	$1280\frac{2}{3}$	
	$51226\frac{2}{3}$ *Ans.*	

Exercises for the Slate.

Multiply	3684 by $81\frac{1}{8}$	*Ans.* $298864\frac{4}{8}$
	4739 by $47\frac{1}{9}$	*Ans.* $223259\frac{5}{9}$
	4382 by $41\frac{1}{12}$	*Ans.* $180027\frac{2}{12}$

Examples for Practice.

1. What cost 1518 pounds of coffee, at $33\frac{1}{2}$ cts. per pound? *Ans.* $508 : 53

2. Bought a firkin of butter weighing 56 pounds, at $15\frac{1}{3}$ cts. per pound, to what did it amount? *Ans.* $8 : $58\frac{2}{3}$

3. Bought 37 bushels of corn at $51\frac{1}{4}$ cts. per bushel; what did the whole cost? *Ans.* $18 : $96\frac{1}{4}$

4. Bought 35 yards of cloth at $35\frac{1}{4}$ cts. per yard, 46 pounds of silk at $85\frac{1}{2}$ cts. per pound, 36 dozen of buttons at $21\frac{1}{2}$ cts. per dozen, and 3 pairs of worsted stockings at $35\frac{1}{2}$ cts. per pair; what is the whole amount. *Ans.* $60 : $47\frac{1}{4}$

Obs. When the numerator is more than 1.

Rule:—First multiply the multiplicand by the whole number, then multiply it by the numerator of the fraction and divide the product by the denominator; add the quotient to the first product obtained, and the result will be the answer.

EXAMPLE.		OPERATION.
Multiply 8438 by 51$\frac{3}{4}$	$\begin{array}{r} 8438 \\ 51 \\ \hline 8438 \\ 42190 \\ \hline 430338 \\ 6328\frac{2}{4} \\ \hline 436666\frac{2}{4} \end{array}$ *Ans.*	$\begin{array}{r} 8438 \\ 3 \\ \hline 4)25314 \\ \hline 6328\frac{2}{4} \end{array}$

Exercises for the Slate.

Multiply 6742 by 6$\frac{3}{8}$	*Ans.* 42980$\frac{2}{8}$
9734 by 121$\frac{8}{16}$	*Ans.* 1182681
7465 by 39$\frac{7}{8}$	*Ans.* 297666$\frac{7}{8}$

Examples for Practice.

1. Bought 304 pounds of sugar at 10$\frac{3}{4}$ cts. per pound; to what did the whole amount? *Ans.* $32 : 68

2. 36 pounds of tea at 47$\frac{5}{9}$ cts. per pound will amount to what sum? *Ans.* $17 : 12

3. 325 reams of foolscap paper at 1\frac{5}{8}$ per ream; to what will it amount? *Ans.* $528 : 12$\frac{4}{8}$

4. 50 barrels of mackerel at 4\frac{3}{4}$ per barrel; will amount to what sum? *Ans.* $237 : 50

4. To multiply a mixed number by a whole number, the numerator being 1.

RULE:—Multiply the multiplicand by the whole number. Then divide the multiplier by the denominator of the fraction, and add the quotient to the product.

If there be a remainder in dividing the multiplier, write it fraction-wise, and annex it to the product.

EXAMPLE.	OPERATION.	
Multiply 68$\frac{1}{4}$ by 39	$\begin{array}{r} 68 \\ 39 \\ \hline 612 \\ 204 \\ \hline 2652 \\ 9\frac{3}{4} \\ \hline 2661\frac{3}{4} \end{array}$ *Ans.*	$\begin{array}{r} 4)39 \\ \hline 9\frac{3}{4} \end{array}$

Exercises for the Slate.

Multiply 375$\frac{1}{8}$ by 13	*Ans.* 4877$\frac{3}{8}$
467$\frac{1}{6}$ by 37	*Ans.* 17285$\frac{1}{6}$
8634$\frac{1}{7}$ by 81	*Ans.* 699365$\frac{4}{7}$

Examples for Practice.

1. What cost 19½ yards of silk at 219 cts. per yard? *Ans.* $42 : 70½

2. What will 8¼ pounds of pepper amount to at 106 cts. per pound? *Ans.* $8 : 74$\frac{2}{4}$

3. How much will 17⅛ pounds of coffee cost at 21 cts. per pound? *Ans.* $3 : 59⅝

4. At $2 : 50 per day, to what sum would your wages amount in 31½ days? *Ans.* $78 : 75

Obs. To multiply a mixed number by a mixed number, when the numerators are 1.

Rule :—Multiply the integers together; then multiply them by their respective fractions alternately, add these products together, their sum will be the answer.

Multiply 54¼ by 24½

OPERATION.
54¼
24½
216
108
27
6⅛
Ans. 1329⅛

Obs. When the multiplicand has ¼ annexed to the integer, multiply the integers together; then take half of the multiplicand and a fourth of the multiplier, and add to the product.

Exercises for the Slate.

Multiply 7¼ by 8½	*Ans.* 61+
7½ by 9¼	*Ans.* 69+
12⅓ by 16$\frac{1}{7}$	*Ans.* 198+
7½ by 2$\frac{1}{18}$	*Ans.* 15+
120¼ by 48½	*Ans.* 5832
54¼ by 24½	*Ans.* 1329$\frac{2}{4}$

1. If one hundred weight of wool is bought at $40⅓; what will 17¾ hundreds cost? *Ans.* 715\frac{11}{12}$

2. What cost 236¾ yards of carpeting, at $1 : 31¼ per yard? *Ans.* $310 : 73¼

It is necessary that the pupil should now be taught the

REDUCTION OF VULGAR FRACTIONS.

1. To change a whole number to an improper fraction.

Rule :—Place the whole number for the numerator, and a unit for the denominator; or assign a denominator to the integer and multiply it by the assigned denominator, and the product will be the numerator to the denominator.

Ex. 1st. Change 8 to an improper fraction. OPERATION. $\frac{8}{1}$

Ex. 2d. Reduce 9 to an improper fraction whose denominator will be 5.

OPERATION.	EXPLANATION.
$\begin{array}{r} 9 \\ 5 \\ \hline 45 \end{array}$	9 is the proposed numerator, which, multiplied by 5, the denominator, gives 45 for the *new* numerator, thus $\frac{45}{5}$, which is equal to 9 integers

written $\frac{45}{5}$ *Ans.*

2. To reduce an improper fraction to a whole number.

Rule :—Divide the numerator by the denominator, the quotient will be the whole number.

Ex. Reduce $\frac{192}{8}$ to a whole number. OPERATION. $8\overline{)192}$

$\underline{\quad}$ 24 *Ans.*

3. To reduce a mixed number to an improper fraction.

Rule :—Multiply the whole number by the denominator of the fraction, and take in the numerator; write the denominator under the product.

Ex. Reduce $12\frac{3}{4}$ to an improper fraction.

OPERATION.

$$\begin{array}{r} 12 \\ 4 \\ \hline 48 \\ 3 \\ \hline 51 \end{array} \text{ written } \frac{51}{4} \text{ Ans.}$$

Note. To change an improper fraction to a mixed number, see CASE 2d.

4. To reduce fractions to their lowest terms.

Rule :—Divide the numerator and denominator of the given fraction by any number that will divide them without a remainder, and the quotient again in the same manner, and so continue to do, till it appears that there is no number greater than one, that will divide them; and the last quotients will express the given fraction in its lowest terms.

Ex. Reduce $\frac{84}{168}$ to its lowest terms.

OPERATION.

$$\begin{array}{l|l|l|l|l} \text{by} & \text{by} & \text{by} & \text{by} & \\ 2)\ 84 & 2)42 & 7)21 & 3)3 & 1 \\ 2)168 & 2)84 & 7)42 & 3)6 & 2 \end{array} \text{ written } \frac{1}{2} \text{ Ans.}$$

5. To reduce fractions of different denominators to one common denominator.

Rule :—Multiply each numerator by all the denominators, except its own, take the respective products for new numerators: then multiply all the denominators together, their product will be the common denominator, which written under each numerator will show the common denominator to each fraction.

Ex. Reduce $\frac{2}{5}$ $\frac{3}{7}$ and $\frac{8}{9}$ to a common denominator.

OPERATIONS.

2 × 7 14 × 9 126 *first numerator.*
3 × 5 15 × 9 135 *second numerator.*
8 × 7 56 × 5 280 *third numerator.*
5 × 7 35 × 9 315 *the common denominator to all the* numerators, which is written thus—$\frac{126}{315}$ $\frac{135}{315}$ $\frac{280}{315}$ *Ans.*

6. To reduce a vulgar fraction to a decimal.

RULE:—Annex one or more ciphers to the numerator of the fraction; then divide it by the denominator, the quotient will be the decimal with a comma prefixed.

Ex. Reduce $\frac{3}{4}$ to a decimal.

OPERATION.

4)300
,75 *Ans.*

7. To change a decimal to a vulgar fraction.

RULE:—Set to the decimal its proper denominator; then divide the numerator and the denominator by any number which will divide both without a remainder.

Ex. Change ,680 to its equivalent vulgar fraction.

OPERATION.

8)$\frac{680}{1000}$($\frac{85}{125}$ 5)$\frac{85}{125}$($\frac{17}{25}$ *Ans.*

Examples for Practice in the 7 previous Cases of Reduction.

CASE 1. How many thirds are there in 14? *Ans.* $\frac{42}{3}$
How many fifths of a dollar are there in $16? *Ans.* $\frac{80}{5}$
In 31 pounds, how many sixths of a pound? *Ans.* $\frac{186}{6}$
In 73 yards, how many eighths of a yard? *Ans.* $\frac{584}{8}$

CASE 2. In $\frac{245}{5}$ of a dollar, how many dollars?
Ans. $49

How many pounds are there in $\frac{132}{6}$ of a pound?
Ans. 22 *lbs.*

If 8 pounds of sugar can be bought for $1; how many pounds will $\frac{2650}{10}$ of a dollar pay for? *Ans.* 2120 *lbs.*

If $\frac{1}{5}$ of a dollar will pay for 1 pound of coffee; how many dollars will 312 pounds cost? *Ans.* 62\frac{2}{5}$

If 1 pound of butter cost $\frac{1}{6}$ of a dollar; what would 491 pounds cost? *Ans.* 81\frac{5}{6}$

A man, spending $\frac{1}{6}$ of a dollar a day, in 83 days would spend $\frac{83}{6}$ of a dollar; how many dollars would that be?
Ans. 13\frac{5}{6}$

CASE 3. How many fourths are there in 15$\frac{3}{4}$? *Ans.* $\frac{63}{4}$

Hr[illegible] Bush' Book [illegible]

How many eighths of a mile in 57$\frac{1}{8}$ miles? *Ans.* $\frac{457}{8}$

In 156$\frac{17}{24}$ days; how many 24ths of a day? *Ans.* $\frac{3761}{24}$

If $\frac{1}{5}$ of a dollar will pay for one gallon of beer; how much can be bought for \$6$\frac{4}{5}$? *Ans.* 34 *gals.*

If $\frac{1}{8}$ of a dollar will pay for 3 yards of ribin; how many yards can be bought for \$6$\frac{5}{8}$? *Ans.* 159 *yds.*

Case 4. Reduce $\frac{1}{180}$ of a dollar to the fraction of a cent. *Ans.* $\frac{5}{9}$ *ct.*

Express the value of $\frac{14}{42}$ in the smallest numbers possible. *Ans.* $\frac{2}{6}$

Employ the smallest numbers to express the value of $\frac{21}{35}$. *Ans.* $\frac{3}{5}$

Reduce $\frac{384}{1152}$ to its lowest terms. *Ans.* $\frac{1}{3}$

Is $\frac{28}{35}$ of a dollar in its lowest terms? No. Why? Because it can be divided by 7 without a remainder, and then retain the same value, thus $\frac{4}{5}$.

Why is $\frac{15}{32}$ of a dollar in its lowest terms? Because there is no number greater than 1 that will divide both numerator and denominator without a remainder.

Case 5. Reduce $\frac{3}{5}$ $\frac{5}{6}$ to a common denominator. *Ans.* $\frac{18}{30}$ $\frac{25}{30}$

Bought $\frac{3}{5}$ of a tun of iron at one time, and $\frac{4}{7}$ of a tun at another; how much in the whole? *Ans.* 1$\frac{6}{35}$ *tuns.*

Bought 8$\frac{3}{5}$ cwt. of sugar at one time, and 5$\frac{2}{3}$ cwt. at another; how much in the whole? *Ans.* 14$\frac{4}{15}$ *cwt.*

What is the amount of 16$\frac{3}{4}$ yards, 17$\frac{1}{5}$ yards and 3$\frac{1}{3}$ yards? *Ans.* 37$\frac{17}{60}$ *yds.*

From a piece of cloth containing 47$\frac{3}{8}$ yards, a merchant sold 23$\frac{9}{25}$; how much remained unsold? *Ans.* 24$\frac{3}{200}$ *yds.*

A man bought four loads of hay, the first containing 17$\frac{3}{5}$ cwt.; the second, 19$\frac{1}{4}$ cwt.; the third, 24$\frac{5}{8}$ cwt.; and the fourth, 14$\frac{1}{2}$ cwt.; how many cwt. in the whole? *Ans.* 75$\frac{312}{320}$ *cwt.*

Case 6. Reduce $\frac{5}{8}$ of a dollar to a decimal? *Ans.* ,625

How many whole dollars and cts. in \$$\frac{56}{24}$? *Ans.* \$2 : 33'3

Express $\frac{7}{16}$ decimally. *Ans.* ,4375 Change $\frac{3}{32}$ to a decimal. *Ans.* .09375.

What is the sum of 79$\frac{1}{2}$ 6$\frac{1}{4}$ and of $\frac{3}{4}$ when added together? *Ans.* 86, 50

From a piece of cloth, containing 36$\frac{5}{8}$ yards, a merchant sold, at one time, 7$\frac{3}{10}$ yards, and, at another time, 12$\frac{5}{8}$ yards; how much of the cloth had he left? *Ans.* 16,7 *yds.*

From 24 dollars take $\frac{4}{5}$ of a dollar. *Ans.* \$23 : 20

A man raised wheat in five fields, in the first, 47$\frac{3}{10}$ bushels; in the second, 94$\frac{8}{25}$; in the third, 87$\frac{15}{32}$; in the fourth 143$\frac{1}{8}$; and in the fifth 387 bushels; how many bushels in the whole? *Ans.* 759,77625 *bush.*

A man bought 3 sheep; for the first he gave 6¾ dollars; for the second, 8⅝; and for the third, 9½; how many dollars did he give for the whole? *Ans.* $24 : 87'5

How many yards of cloth are there in 7 pieces, each piece 19⅞ yards? *Ans.* 139,125

How much cotton is there in 3 7/10 bales, each bale 4¾ cwt.? *Ans.* 17,575 *cwt.*

What cost 5⅜ tuns of hay, at $27 19/50 per tun? *Ans.* $147 : 16'750

Case 7. Change the decimal ,375 to a vulgar fraction. *Ans.* ⅜

Prove that ,875 is equivalent to ⅞; that ,428 is to 107/250; ,8312 is to 1039/1250.

Add ,6 ,27 ,685 together; give the *Ans.* in a vulgar fraction. *Ans.* 1555/1000

Subtract 52,6088 from 406,91 *Ans.* in vulgar fractions. *Ans.* 354 1506/5000

Multiply ,75 by ,25, express the *Ans.* in a vulgar fraction. *Ans.* 375/2000

Divide 26,25 by 2,5 give the *Ans.* in a mixed number. *Ans.* 10½

DIVISION OF FRACTIONS.

This is finding how often a part or the parts of an integer is contained in a given sum.

1. To divide a fraction by a fraction.

Rule:—Invert the divisor, and then multiply the numerators and denominators together: reduce the Answer to the lowest terms or a whole number.

EXAMPLES. OPERATION.

Divide 3/5 by 2/4 $\frac{3\times4=12}{2\times5=10}$ *Ans. reduced.* 10)12(1 2/10 2) 2 (1 *Ans.* 1⅕
10 2)10(5
2

Divide 9/12 by 3/12 $\frac{12\times9=108}{3\times12=36}$ *Ans. reduced.* 36)108(3 *Ans.*
108

Exercises for the Slate.

Divide ¼ by 2/4 *Ans.* ½ Divide 4/7 by ⅔ *Ans.* 6/7
[illegible] by [illegible] *Ans.* [illegible] 2/17 by ¾ *Ans.* 8/51
⅘ by ⅖ *Ans.* 2 3/7 by 3/14 *Ans.* 2
6/11 by 3/11 *Ans.* 2 4/7 by 1/38 *Ans.* 21 5/7

Examples for Practice.

1. At ⅜ of a dollar a yard; how many yards of calico can be bought with ¾ of a dollar? *Ans.* 2 *yds.*

2. At $\frac{1}{7}$ of a dollar a bushel, how many bushels of oats can be bought for $\frac{4}{7}$ of a dollar? *Ans.* 4 *bush.*

3. At $\frac{1}{7}$ of a dollar per bushel; how many bushels of potatoes may be bought for $\frac{4}{5}$ of a dollar? *Ans.* $5\frac{3}{5}$ *bush.*

4. At $\frac{5}{6}$ of a dollar per bushel; how much rye may be bought for $\frac{1}{3}$ of a dollar? *Ans.* $\frac{2}{5}$ *bush.*

5. If $\frac{1}{3}$ of a chaldron of coal will supply a fire 1 week; how many weeks will $\frac{3}{4}$ of a chaldron supply it? *Ans.* $2\frac{1}{4}$ *ws.*

6. If $\frac{3}{10}$ of a tun of hay will keep a horse 1 month; how many horses will $\frac{9}{10}$ of a tun keep the same time? *Ans.* 3 *hs.*

7. If $\frac{1}{5}$ of a yard of cloth cost $\frac{4}{7}$ of a dollar; what will a yard cost? *Ans.* $\$2\frac{6}{7}$

8. If $\frac{3}{7}$ of a yard of cloth cost $\frac{5}{8}$ of a dollar; what is that a yard? *Ans.* $\$1\frac{11}{24}$

2. To divide a fraction by a whole number.

RULE:—Divide the numerator of the fraction by the whole number, which is the divisor; or (if this would leave a remainder) multiply the denominator by the whole number; write the numerator over the product.

EXAMPLES.	OPERATIONS.
Divide $\frac{25}{80}$ by 5	5)25(5 *written* $\frac{5}{80}$ *Ans.*
	25
Divide $\frac{7}{8}$ by 4	8
	4
	32 *written* $\frac{7}{32}$ *Ans.*

Exercises for the Slate.

Divide $\frac{8}{16}$ by 4	*Ans.* $\frac{2}{16}$	Divide $\frac{18}{25}$ by 8	*Ans.* $\frac{18}{200}$
$\frac{4}{9}$ by 2	*Ans.* $\frac{2}{9}$	$\frac{2}{6}$ by 4	*Ans.* $\frac{2}{24}$
$\frac{144}{250}$ by 12	*Ans.* $\frac{12}{250}$	$\frac{3}{4}$ by 7	*Ans.* $\frac{3}{28}$

Examples for Practice.

1. At 3 dollars a barrel; what part of a barrel of cider may be bought for $\frac{1}{8}$ of a dollar? *Ans.* $\frac{1}{24}$ *bar.*

2. At 7 dollars a barrel; what part of a barrel of flour may be bought for $\frac{1}{3}$ of a dollar? *Ans.* $\frac{1}{21}$ *bar.*

3. If 2 yards of cloth cost $\frac{2}{3}$ of a dollar; what does 1 yard cost? *Ans.* $\$\frac{1}{3}$

4. If $\frac{25}{28}$ of a barrel of flour be divided equally among 5 families; how much will each family receive? *Ans.* $\frac{5}{28}$ *bar.*

5. A man divided $\frac{1}{2}$ of a dollar equally among 2 persons; what part of a dollar did he give to each? *Ans.* $\$\frac{1}{4}$

6. If 7 pounds of coffee cost $\frac{21}{25}$ of a dollar; what is that per pound? *Ans.* $\$\frac{3}{25}$

7. $\frac{18}{20}$ of an acre produce 24 bushels; what part of an acre will produce 1 bushel? *Ans.* $\frac{18}{480}$ *acre.*

8. If a horse will eat $\frac{1}{3}$ of a bushel of oats in a day; how long will 12 bushels last him? *Ans.* 36 *days.*

3. To divide a whole number by a fraction.

RULE:—Multiply the whole number by the fraction inverted.

EXAMPLE.	OPERATION.
Divide 72 by $\frac{1}{4}$	$\frac{1}{4}$ *inverted* 72
	4
	288 *Ans.*

Exercises for the Slate.

Divide 43183 by $\frac{1}{2}$	*Ans.*	86366
37047 by $\frac{1}{3}$	*Ans.*	111141
418096 by $\frac{1}{4}$	*Ans.*	1672384

Examples for Practice.

1. A man lost $1272, which was $\frac{1}{4}$ of his whole estate; what was the value of his estate? *Ans.* $5088

2. If $\frac{1}{5}$ of a yard of cloth be worth $3; what is one yard worth? *Ans.* $15

3. If $\frac{1}{7}$ of a farm be worth $565; what is the value of the whole farm? *Ans.* 3955

4. If $\frac{1}{6}$ of a bank be worth $3912; what is the whole worth? *Ans.* 23472

OBS. When the divisor is a proper fraction, as $\frac{3}{4}$, $\frac{2}{7}$, $\frac{3}{5}$, &c.

RULE:—*Multiply* the dividend by the *denominator* of the fraction, and *divide* the product by the *numerator.*

EXAMPLE.	OPERATION.
Divide 28 by $\frac{4}{5}$	28
	5
	4)140
	35 *Ans.*

Exercises for the Slate.

Divide 9 by $\frac{3}{4}$	*Ans.* 12		Divide 13 by $\frac{5}{7}$	*Ans.* $18\frac{1}{5}$
18 by $\frac{6}{5}$	*Ans.* 15		68 by $\frac{5}{7}$	*Ans.* $95\frac{1}{5}$
24 by $\frac{18}{21}$	*Ans.* 28		5 by $\frac{7}{10}$	*Ans.* $7\frac{1}{7}$
125 by $\frac{5}{9}$	*Ans.* 225		9 by $\frac{2}{5}$	*Ans.* $22\frac{1}{2}$

Examples for Practice.

1. At $\frac{5}{16}$ of a dollar a bushel; how many bushels of rye can be bought for $80? *Ans.* 256 *bush.*

2. A man bought $\frac{5}{12}$ of a tun of iron for $40; what was the iron a tun? *Ans.* $96

3. Bought $\frac{3}{5}$ of a tun of hay for $17; what was it a tun? *Ans.* $28\frac{1}{3}$

4. A merchant sold a quantity of goods for $252, which was $\frac{6}{5}$ of what it cost him. How much did it cost him, and how much did he gain? *Cost* $210 *gained* $42

5. A merchant sold a quantity of goods so as to gain $43, which was $\frac{2}{7}$ of what the goods cost him. How much did they cost? *Ans. Cost* $150 : 50

6. If 192 men will perform $\frac{8}{9}$ of a certain piece of work in a week, what number of men will it take to perform the whole of the work in a week? *Ans.* 216 *men.*

7. Suppose a ship to sail 105 miles in $\frac{5}{12}$ of a day; what distance will the ship sail in $\frac{1}{12}$ of a day; what distance will she sail in the whole day? *Ans.* 252 *miles in* 1 *day.*
21 *miles in* $\frac{1}{12}$ *do.*

4. To divide a whole number by a mixed number.

Rule:—Reduce both the divisor and dividend to like terms; then divide as in whole numbers, the quotient will be the answer.

EXAMPLE.	OPERATION.
Divide 92 by $4\frac{1}{2}$	$4\frac{1}{2}$ 92
	2 2
	9)184(20$\frac{4}{9}$ *Ans.*
	18
	4

Exercises for the Slate.

Divide 3724 by $83\frac{1}{5}$	*Ans.* $44\frac{316}{416}$
7280 by $37\frac{1}{8}$	*Ans.* $194\frac{222}{297}$
8870 by $12\frac{1}{9}$	*Ans.* $732\frac{42}{109}$

Obs. When the divisor has $\frac{3}{4}$, $\frac{5}{8}$, &c. annexed to the integer, add it to the product when reducing the whole number.

EXAMPLES.

Divide 129 by $7\frac{6}{7}$	*Ans.* $16\frac{23}{55}$
642 by $21\frac{9}{20}$	*Ans.* $29\frac{399}{429}$
86 by $15\frac{4}{8}$	*Ans.* $5\frac{68}{124}$

Examples for Practice.

1. If $7\frac{3}{8}$ yards of cloth will make a suit of clothes; how many suits will 48 yards make? *Ans.* $6\frac{30}{59}$

2. If it take $1\frac{1}{2}$ bushel of oats to sow an acre; how many acres will 18 bushels sow? *Ans.* 12 *acres.*

3. If it take $1\frac{1}{5}$ bushel of wheat to sow an acre; how many acres will 23 bushels sow? *Ans.* $19\frac{1}{6}$ *acres.*

4. At $$4\frac{2}{5}$ a yard; how many yards of cloth may be bought for $37? *Ans.* $8\frac{9}{22}$ *yards.*

5. At $11\frac{2}{7}$ cents per pound, how much steel can I buy for $50? *Ans.* $443\frac{3}{79}$ *lbs*

6. If a man can perform a journey in 580 hours, how many days will it take him to perform it if he travel $9\frac{3}{10}$ hours in a day? *Ans.* $62\frac{34}{93}$

7. How many coats may be made of 187 yards of cloth; if $3\frac{4}{17}$ yards make 1 coat? *Ans.* $57\frac{4}{5}$ *coats.*

8. If $2\frac{3}{4}$ bushels of oats sow an acre; how many acres will 22 bushels sow? *Ans.* 8 *acres.*

9. If $1\frac{2}{3}$ yard of cloth is worth $11; what is a yard worth? *Ans.* $6 : 60

10. If $8\frac{9}{13}$ bushels of wheat cost $15; what is it a bushel? *Ans.* $\$1\frac{82}{113}$

5. To divide a mixed number by a mixed number.

Rule:—Change the fractions annexed to the integers to decimals; then divide as in decimal division, or

Reduce the mixed numbers to improper fractions, and then to whole numbers or mixed numbers.

Ex. Divide $26\frac{1}{4}$ by $2\frac{1}{2}$ Divide $106\frac{3}{8}$ by $2\frac{7}{8}$ Divide $6\frac{5}{8}$ by $3\frac{1}{4}$

OPERATION.

2,5)26,25(10,5 *Ans.*
25
125
125

OPERATION.

2 106
8 8
23)851(37 *Ans.*
69
161
161

3 6
4 8
$\frac{13}{4}$ $\frac{53}{8}$

OPERATION.

$\frac{4}{13} \cdot \frac{53}{8} = \frac{212}{104}$
104)212($2\frac{1}{26}$ *Ans.*
208
4) 4(1
4)104(26

Exercises for the Slate.

Divide $8\frac{3}{4}$ by $5\frac{1}{2}$ *Ans.* 1,59+
$1\frac{1}{2}$ by $4\frac{8}{10}$ *A.* ,3125=$\frac{5}{16}$
$43\frac{7}{8}$ by $1\frac{3}{4}$ *A.* 25,0714+
$845\frac{1}{5}$ by $26\frac{4}{5}$ *A.* $31\frac{72}{134}$
$236\frac{12}{13}$ by $48\frac{4}{13}$ *A.* $4\frac{142}{157}$
$10\frac{3}{8}$ by $2\frac{1}{8}$ *A.* $4\frac{15}{17}$

Divide $1260\frac{3}{4}$ by $30\frac{1}{2}$ *A.* $41\frac{41}{122}$
$2\frac{1}{4}$ by $1\frac{1}{2}$ *A.* $1\frac{1}{2}$
$5\frac{1}{8}$ by $7\frac{3}{4}$ *A.* $\frac{41}{62}$
$5\frac{8}{15}$ by $2\frac{1}{6}$ *A.* $2\frac{58}{65}$
$2\frac{4}{5}$ by $3\frac{3}{4}$ *A.* $\frac{56}{75}$
$19\frac{1}{20}$ by $2\frac{1}{15}$ *A.* $9\frac{27}{124}$

Examples for Practice.

By Decimals.

1. At $31\frac{1}{2}$ dollars for $10\frac{1}{2}$ barrels of cider, what is that per barrel? *Ans.* $3

2. At $\$1\frac{1}{2}$ per pound; how many pounds of indigo will $\$27\frac{857}{1000}$ buy? *Ans.* 18,57 *lbs.*

3. If 1 gallon of wine cost $\$1\frac{3}{8}$; how many gallons will $\$24\frac{3}{4}$ buy? *Ans.* 18 *gals.*

4. If you pay $\$316\frac{11}{16}$ for $92\frac{1}{8}$ cords of wood; what is that a cord? *Ans.* $3,4375=$\frac{7}{16}$

5. At $\$3\frac{1}{4}$ a yard; how many yards of broadcloth can be bought for $\$64\frac{3}{16}$? *Ans.* 19,75=$\frac{3}{4}$ *yds.*

By Vulgar Fractions.

1. At $$7\frac{3}{5}$ a yard; how many yards of cloth may be bought for $$57\frac{5}{8}$? *Ans.* $7\frac{177}{304}$ *yds.*

2. If $3\frac{5}{8}$ barrels of flour cost $$23\frac{2}{7}$; what is that a barrel? *Ans.* $$6\frac{86}{203}$

3. If $2\frac{1}{2}$ bushels of oats will keep a horse 1 week; how long will $18\frac{3}{5}$ bushels keep him? *Ans.* $7\frac{11}{25}$ *weeks.*

4. If $4\frac{3}{7}$ yards of cloth will make a suit of clothes; how many suits will $87\frac{4}{9}$ yards make? *Ans.* $19\frac{208}{279}$ *suits.*

5. If a man can build $4\frac{5}{17}$ rods of wall in a day; how many days will it take him to build $84\frac{4}{19}$ rods? *Ans.* $19\frac{847}{1387}$ *days.*

CORRESPONDENT EXAMPLES.

1. A person, having $\frac{3}{5}$ of a coal mine, sells $\frac{3}{4}$ of his share for $171; what is the whole mine valued at? *Ans.* $380

2. A man bought $\frac{3}{16}$ of an acre of land for $84; what was that an acre? *Ans.* $448

3. A captain has on board 170 bales, each paying freight $1 : 25; 305 packages, each paying $87\frac{1}{2}$ cts.; 230 tuns of other goods, each tun paying $12 : $62\frac{1}{2}$; and 6 passengers, each paying $78 : 50; how much does his whole freight and passage money amount to? *Ans.* $3854 : $12\frac{1}{2}$

4. From $1153\frac{4}{7}$ tuns of iron, there were sold $684\,{}^{5}{}_{1}$ tuns; how much remained unsold? *Ans.* 468,8312 *tuns.*

5. If a man leave $6509 to his wife and two sons; thus, to his wife $\frac{3}{8}$; to his elder son $\frac{3}{5}$ of the remainder, and his other son the rest; what was the share of each?

Ans. {
Wife $2440 : 87'5
1st. *Son* $2440 : 87'5
2d. *Son* $1627 : 25

6. If $\frac{5}{16}$ of a ship cost $781 : 25; what is the whole ship worth? *Ans.* $2500

7. A farmer has mowed $78\frac{1}{2}$ acres of meadow land, which yielded on an average $2\frac{2}{3}$ tuns per acre, and besides having wintered with the hay 63 head of cattle, he has sold $62\frac{1}{3}$ tuns; at what average per head has his cattle consumed the hay? *Ans.* $2\frac{1}{3}$ *tuns.*

And how much hay did he make in the whole? *Ans.* $209\frac{1}{3}$ *tuns.*

8. A man bought 12 yards of cloth at $$5\frac{3}{15}$ a yard; how much did the whole come to? *Ans.* $$62\frac{2}{5}$

9. A merchant bought 5 pieces of cloth, the first $40\frac{3}{4}$ yards; the second $38\frac{1}{2}$ yards; the third $40\frac{3}{8}$ yards; the fourth $41\frac{1}{16}$ yards; and the fifth $42\frac{1}{4}$ yards; how many yards in all? *Ans.* $202\frac{15}{16}$ *yds.*

10. A man bought $\frac{5}{7}$ of a barrel of flour for $4 : 85 ; what would be the price of a barrel at that rate? *Ans.* $6 : 79

11. A gentleman divided his estate of $4785 among his children, he gave his eldest son $\frac{3}{4}$ of it, the youngest $\frac{1}{4}$; what was their portion?

Ans. { *Eldest.* $3588 : 75
Youngest, $1196 : 25

12. From 37$\frac{5}{8}$ gallons of oil, were sold 28$\frac{4}{9}$ gallons; how much remained? *Ans.* 9,1372 *gals.*

13. If $\frac{3}{5}$ of a yard of broadcloth cost $6; what will $\frac{1}{5}$ cost? What will a yard cost? *Ans.* $\frac{1}{5}$ $2, yard $10

14. If 3 men can do a piece of work in 4$\frac{1}{2}$ hours; in how many hours will 10 men do the same work? *Ans.* 1$\frac{7}{20}$

15. There was a farm of which A. owned $\frac{2}{7}$ and B. $\frac{5}{7}$; the farm was sold for $1764; what was each one's share of the money? *Ans.* A's. $504, B's. 1260

16. A person having $\frac{2}{5}$ of a vessel, sells $\frac{3}{7}$ of his share for $8400; what part of the whole vessel did he sell? What was the whole vessel worth? *Ans. sold* $\frac{6}{35}$ *worth* $49000

17. Four men traded together on a capital of $3000, of which A. put in $\frac{1}{2}$, B. $\frac{1}{4}$, C. $\frac{1}{6}$, and, D. $\frac{1}{12}$; at the end of 3 years, they had gained $2364; what was each one's share of the gain?

Ans. { A's. $1182
B's. $ 591
C's. $ 394
D's. $ 197

18. If 7$\frac{3}{4}$ yards of linen cost $8 : 30; how much will 63$\frac{1}{2}$ yards cost at the same rate? *Ans.* $68 : 00'5

REVIEW.

1. A *unit* is a whole thing of any kind, as 1 dollar, 1 bushel, 1 day, &c. A *fraction* is a part of a thing, as 1-half, 3-fourths, 5-eighths of a dollar, and the like.

To express a part of a thing, we take two whole figures, and place one above the other, thus—

$\frac{1}{2}$ one-half, $\frac{1}{3}$ one-third, $\frac{2}{3}$ two-thirds, $\frac{1}{6}$ one-sixth, $\frac{4}{5}$ four-fifths, $\frac{3}{9}$ three-ninths.

The figure, above the line, shows what part of a unit is taken; the lower, into how many it is divided, thus $\frac{1}{2}$ is one unit divided into 2 parts; $\frac{1}{3}$ into 3 parts; &c. When a unit is divided into 4 parts, 1 part is $\frac{1}{4}$, 2 parts $\frac{2}{4}$, 3 parts $\frac{3}{4}$, 4 parts is $\frac{4}{4}$, or the whole.—When 1 part is taken $\frac{3}{4}$ remains; 2 parts, $\frac{2}{4}$; 3 parts, $\frac{1}{4}$; and 4 parts is the whole. There are $\frac{4}{4}$ in the whole of any thing when divided into 4 equal parts—5 equal parts, $\frac{5}{5}$; 6 equal parts $\frac{6}{6}$, &c.

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1. What is a unit? Give an ex. What a fraction? Give an ex. What fraction is expressed, when there is a 4 with a 1 over it? 7 with a 2 over it? 8 with 5 over it? If a unit is divided into 2 parts, what is one of these parts called? Into 3? Into 5? Into 9? Into 15? How many halves make a whole one? How many thirds? How many fifths? How many eighths? What is meant by 1-half of a thing? 1-third? 1-fifth?

2. The *same thing* that is a unit in one case, may be considered as a *fraction* in another, thus, a dollar is a *unit* or a whole thing, *of itself considered*, but a fraction, when considered as the *tenth part* of an *eagle:* a cent is a *unit*, or a *tenth part* of a dime: a mill is a *unit* or the *tenth* part of a *cent:* a day is a *unit*, or the *seventh* part of a *week:* a week is a *unit* or the *fourth* part of a *month:* a month is a *unit*, or the *twelfth* part of a year, &c. Showing that an integer is a unit or a fraction, if it can be considered as making a part of another thing.

3. The more parts a thing is divided into, the *smaller* these parts must be. If one thing is divided into twice as many parts as another thing, each part is twice as small. Thus, $\frac{1}{2}$ is twice as large as $\frac{1}{4}$, for there are twice as many *fourths* as halves in a thing—therefore $\frac{1}{2}$ is twice as large as $\frac{1}{4}$. An eighth is *twice* as small as $\frac{1}{4}$, for there are *twice* as many pieces in $\frac{1}{8}$ as in $\frac{1}{4}$; $\frac{1}{6}$ is *twice* as large as $\frac{1}{12}$, for 12 is *twice* as many as 6; the parts are more in $\frac{1}{12}$, than in $\frac{1}{6}$, but less in size.

4. Figures are of two kinds; to express whole things and parts of a thing, called unit figures and fractional figures. A *unit* figure expresses any number of *whole* things, as *three*, (thus 3). A fractional figure, thus $\frac{1}{2}$ *one-half;* which shows into how many parts a whole thing is divided, and how many of these parts are expressed; or *what part* is taken from *several whole* things, thus $\frac{3}{4}$; three-fourths shows that one thing is divided into 4 *parts*, and 3 of them are taken; or that 3 *whole* things, have a *fourth* taken from *each* of them. If the fraction is considered as showing how *many parts* are taken from *one* unit, then the lower figure shows *into how many parts* a unit is divided, and the upper figure shows *how many of these parts* are taken. But if the fraction is considered as showing *what part* is taken out of *several units*, then the upper figure shows the *number of units*, and the lower figure shows *what part is taken from each.* Thus the fraction $\frac{2}{6}$ may be considered as expressing, *two sixths* of *one thing*, or as *one sixth* of *two* things. For *two sixths* of one whole thing, is the same quantity as *one sixth of two whole things.*

5. Names are given to the figures above and below the line, viz. *Numerator* and *Denominator*. Fractions are also divided into *proper* and *improper* fractions. A *proper* expresses a quantity less than a unit; thus $\frac{2}{7}$; it shows *how many parts* are taken out of *one thing* —READ 2-sevenths of *one thing;* or it shows *what part* is taken out of *several things*—READ 1-seventh of 2 *things.* An *improper*

1-sixth? Divide any thing into 2 equal parts, take 1 of these parts, how much will be left? Into 3, take 2? Into 4, take 2? Into 8, take 5? Into how many parts must any thing be divided so that 1 part shall be 1-eleventh? 1-thirteenth?

2. Of what order is one dollar *a unit?* Of what order is it *a fraction?* Of what order is one cent *a unit?* Of what order is it *a fraction?* One day is *a unit*, of what order, and *a fraction* of what order?

3. Which is the greatest fraction; 1-half or 1-third? 1-fifth or 1-third? 1-seventh or 1-ninth? 1-fourth or 2-fourths? 6-sevenths or 2-sevenths? What part of a thing is twice as small as 1-half? As 1-third? As 1-fourth? As 1-fifth? What part of a thing is twice as large as 1-fourth? As 1-sixth? As 1-twelfth?

4. If 4-sixths is considered as expressing *how many parts* are taken out of *one unit*, what does the 6 show, and what does the 4 show? If it is considered as expressing *what part* is taken out of *several units*, what does the 4 show, and what does the 6 show?

fraction expresses a quantity, equal to a unit, or greater than a unit: $\frac{7}{2}$; it always expresses *what part* is taken out of *several units.* When we use the expression 7 *halves*, we do not mean 7 halves of *one* thing, (because nothing has more than 2 halves). But halves of 7 things.

Fractional Multiplication is where one or both factors are fractions. RULE FOR MULTIPLYING WHEN ONLY THE MULTIPLICAND IS A FRACTION. *Multiply the numerator, or divide the denominator by the multiplier.*

When we multiply the numerator, the *number of parts* is multiplied, and when we divide the denominator the *size of the parts* is multiplied. **Ex.** Multiply $\frac{4}{12}$ by 2.

OPERATION. $\frac{4}{12} \times 2 = \frac{8}{12}$ or $\frac{4}{12} \div 2 = \frac{4}{6}$

Now $\frac{8}{12}$ and $\frac{4}{6}$ are the *same quantity*.

The only difference is, that in one case the unit is divided into 12 parts and 8 are expressed, and in the other case, the unit is divided into 6 parts, and 4 are expressed. In one case, we make *twice as many* pieces, and in the other we make them *twice as large*.

RULE FOR MULTIPLYING WHEN ONLY THE MULTIPLIER IS A FRACTION. *Multiply by the numerator first, and then divide the product by the denominator; or, divide by the denominator, to obtain one part, and multiply by the numerator, to obtain the required number of parts.*

Ex. Multiply 12 by $\frac{4}{6}$. OPERATION. $12 \times 4 = 48 \div 6 = 8$

In multiplying by a fraction, we take such *a part* of a number as is expressed by the *denominator*, and repeat it as often as there are units in the *numerator*.

Thus in multiplying 12 by $\frac{4}{6}$ we take a sixth part of 12, and repeat it 4 times, and the answer is 8.

The multiplicand is made smaller when multiplied by a fraction. Because we do not repeat the *whole* number, but only a *part* of it.

If a number is to be both multiplied and divided by two figures, it makes no difference which is done *first*, provided the same figures are used as multiplier and divisor.

If we multiply 12 by $\frac{3}{4}$ we divide by 4, to find *one* fourth of 12, and multiply by 3, to obtain *three* fourths, and the answer is 9. But if we should multiply 12 by 3, and then divide the product by 4, the answer would be the same. Thus $12 \times 3 = 36$ and $36 \div 4 = 9$. Thus 9 is the same answer as is obtained by dividing 12 by the denominator, and multiplying the answer by the numerator. In fractions, as in whole numbers, it makes no difference in the product, which factor is used as *multiplier*. When the *whole number* is used as multiplier, the answer is an improper fraction, which can be changed to whole numbers,

Thus $12 \times \frac{3}{4} = 9$
And $\frac{3}{4} \times \frac{12}{1} = \frac{36}{4} = 9$

If we multiply 4-twelveths by 3, in what two ways can it be done? If we divide the denominator, what is it that is multiplied? If we multiply the numerator, what is it that is multiplied? Multiply by 3 in both ways, and tell what each method multiplies.

If you multiply *twelve* by *three*, do you make it larger or smaller? If you multiply it by three-fourths, do you make it larger or smaller? What are the two ways in which 18 can be multiplied by 4-sixths? What will be the answer, if it is divided by 6 first, and the quotient multiplied by 4? What will be the answer, if it is multiplied by 4 first, and then the product divided by 6?

RULE FOR MULTIPLYING WHEN BOTH FACTORS ARE FRACTIONS.—*Multiply the denominators together to obtain one part, and the numerators together to obtain the required number of parts.*

Multiply $\frac{5}{8}$ by $\frac{3}{12}$ $\frac{5}{8} \times \frac{3}{12} = \frac{15}{96}$

Multiplication by a fraction implies the taking some part or parts of the multiplicand, and therefore may be truly expressed by a compound fraction. Thus $\frac{3}{4}$ multiplied by $\frac{5}{8}$ is the same as $\frac{3}{4}$ of $\frac{5}{8}$.

If a unit is divided into 6 parts, and each of these parts into 8, the unit would be divided into 48 parts, and each part is $\frac{1}{48}$ of the whole. After finding what *one* fourth of a fraction is, we can find *three* fourths by multiplying by 3.

Thus $\frac{1}{4}$ of $\frac{2}{6}$ is $\frac{2}{24}$, therefore $\frac{3}{4}$ of $\frac{2}{6}$ is 3 times as much or $\frac{6}{24}$. What is $\frac{4}{6}$ of $\frac{2}{3}$? *One* sixth of *one* third is $\frac{1}{18}$. *One* sixth of *two* thirds $\frac{2}{18}$. *Four* sixths of two thirds is 4 times as much, or $\frac{8}{18}$. What is $\frac{3}{5}$ of $\frac{4}{6}$?

One fifth of $\frac{4}{6}$ is $\frac{4}{30}$, and this is made by multiplying the *denominator* 6, by the *denominator* 5.

Three fifths of $\frac{4}{6}$ is three times as much, or $\frac{12}{30}$, and this is made, by multiplying the *numerator* 4, by the *numerator* 3.

Therefore multiplying the *denominators* together obtained *one* fifth of $\frac{4}{6}$, and multiplying the *numerators* together, obtained *three* fifths.

Division is finding how often one number is contained in another, and thus finding what part of one number is another number. **RULE FOR FRACTIONAL DIVISION WHERE ONLY THE DIVISOR IS A FRACTION.** *Multiply the dividend by the denominator, and divide the product by the numerator.* When we divide by a fraction (unless it be an *improper* fraction) the quotient is *larger* than the dividend. Thus 12 divided by $\frac{1}{4}$ is 48, for there are 48 *one fourths* in 12 *units*. Again, 9 divided by $\frac{1}{3}$ is 27, for there are 27 *one thirds* in 9 *units*. Divide 8 by $\frac{1}{3}$ the answer is 24, for there are 24 one thirds in 8. But if we are to divide 8 by $\frac{2}{3}$ there will be but *half* as many. For there is but half as many *two* thirds as *one* thirds in a number. Therefore if 8 divided by $\frac{1}{3}$ is 24, when divided by $\frac{2}{3}$ it is half as much, or 12. It is seen by the preceding examples, that when a number is to be divide by a fraction, it is *multiplied by its denominator*, and *divided by its numerator.*

Thus if we are to divide 2 by $\frac{3}{4}$ we multiply by the denominator 4 to change 2 into *fourths* and then divided by the 3 to find how many *three* fourths there are.

RULE FOR DIVISION WHERE THE DIVIDEND IS A FRACTION.—*Divide the numerator of the Fraction by the Divisor, or, (if this would leave a remainder,) multiply the denominator by the Divisor.*

When the *dividend* only is a fraction, and we divide it by a *whole number*, we are to find how many *parts of a time*, a certain number is contained in *certain parts of a unit.*

How many 2 sixths in 3? *Ans*. 9. Divide 4 by 2 sixths. If we have $6; how many thirds in the whole? *Ans*. 18. How often is 2 contained in 1 ninth? *Ans*. 1 eighteenth of one time. How often is 3 contained in 4 sixths? *Ans*. *four* eighteenths of one time. Divide 5 tenths by 3. Divide 3 sevenths by 5. What operation have you performed? How is a fraction divided by an integer? How is an integer divided by a fraction?

If $\frac{1}{2}$ is divided by 1 *unit*, we find that it contains it, *not once*, but $\frac{1}{2}$ of once. It can contain *two* units but *half* as many times as *one* unit. Therefore $\frac{1}{2}$ contains 1 *one half* a time, and it contains 2 just half as often, or $\frac{1}{4}$ of a time. $\frac{1}{2}$ divided by 2 then is $\frac{1}{4}$. If $\frac{1}{2}$ is to be divided by 3, we reason in the same way. $\frac{1}{2}$ contains 1, $\frac{1}{2}$ a time. It contains 3 only a *third* as often. $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$, and therefore $\frac{1}{2}$ contains 3, $\frac{1}{6}$ of a time. But there is another method. Let $\frac{8}{16}$ be divided by 4. Now the quotient of 8 *units* divided by 4, is 2 *units*. Of course the quotient of 8 *sixteenths* divided by 4, is 2 *sixteenths*. In this case we have *divided the numerator* by the divisor 4. This can be done in all cases where the numerator can be divided *without remainder*.

But when a remainder would be left, it is best to divide, by *multiplying the denominator*. The answer is of the *same value* either way, though the name is different.

RULE FOR DIVIDING ONE FRACTION BY ANOTHER.—*Invert the Divisor and then multiply the numerators and denominators together*. When one fraction is to be divided by another, the same principle is employed, as when whole numbers are divided by a fraction.

We *invert* a fraction when we *exchange the places of the numerator and the denominator*. When the divisor is thus inverted we can *multiply the numerators together* for a new numerator, and the *denominators for a new denominator* and the process is the same. Divide $\frac{4}{6}$ by $\frac{2}{3}$. Inverting the divisor $\frac{3}{2}$ the two fractions would stand together thus $\frac{4}{6}$ $\frac{3}{2}$. We now multiply the numerators and denominators together and the answer is $\frac{12}{12}$ and it is the same process, as if we had not inverted the divisor.

In like manner, if we wish to find how many times, or *parts* of a time, $\frac{3}{4}$ is contained in $\frac{2}{12}$, we first find how often *one* fourth is contained in it, by reasoning thus: *One* unit would be contained in $\frac{2}{12}$, two twelfths of one time. *One fourth* would be contained *four* times as often, or $\frac{8}{12}$ of one time.

But *three* fourths would be contained only *one third* as often, and we find a third of $\frac{8}{12}$ by *multiplying its denominator* by 3, and thus *make the parts* represented by the denominator, *three times smaller*, hence the *twelfths* are changed to *thirty-sixths*; and a thirty-sixth is a *third* of one twelfth. *Ans.* $\frac{8}{36}$.

INTEREST.

Interest is an allowance made by the borrower to the lender for the use of money or other property. It has reference to four particulars; *viz.* The principal, time, rate per cent per annum, and amount.

The principal is the sum on which interest is computed. The time, is the period for which it is computed. The rate per cent per annum, is the sum allowed for the use of $100 for ONE year. The amount is the principal and interest added together.

1. When the principal and rate per cent, are given to find the interest,

Rule:—Make the principal, the multiplicand; the rate per cent, the multiplier; and the product will be the interest sought.

Remove the decimal point between dollars and cents, (if there be any) two figures towards the left hand, this will give the interest at 1 per cent, and will show the pointing necessary in the product.

Obs. 1. To compute interest two rules are used; multiplication and division. Decimal computation requires only multiplication.

Obs. 2. To find the interest at $1\frac{1}{2}$, $2\frac{3}{4}$, $3\frac{5}{8}$, &c. per cent, find the interest at 1 per cent; then multiply the interest so found by the rate mentioned in the question. For the $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, &c. per cent, apply the rule for multiplying a whole number by a fraction and add the result to the interest first found, &c.

Ex. 1. What is the interest of $250 at 6 per cent for one year? *Ans.* $15

OPERATION.

Principal 250 dollars. Interest at 1 per cent $2 : 50.
$2 : 50 Multiplicand.
6 Multiplier.
$15 : 00 *Ans.* 15 *dollars.*

Ex. 2. What is the interest of $89 : 12'6 at 6 per cent?
Interest at 1 per cent $0 : 89'1,26
: 89'1,26
6
$5 : 34'7,56 *Ans.* 5 *dols.* 34 *cts.* 7 *m.*

Examples for Practice.

Principal.	*Rate.*	*Interest.*	*Principal.*	*Rate.*	*Interest.*
$3500	1	$35*	$1500	½	$7:50
5780	2	115:60	7846	1¼	98:07‘5
6755	3	202:65	6500	¾	48:75
437	4	17:48	1627:18	2½	40:67‘9
650	5	32:50	2317:46	3¼	75:31‘7
537:24‘6	6	32:23‘4	37800	4½	1701
5868:85	7	410:81‘9	4723	5½	259:76‘5
111:99	8	8:95‘9	7234	6¾	488:29‘5
7236	9	651:24	1000	7⅝	78:33‘3
1237	10	123:70†	25156:88	⅝	157:23
3723	12	446:76	2423:60	⅜	9:08‘8

* Remove the decimal point 2 figures towards the left hand, for 1 per cent.

† Remove the decimal point 1 figure towards the left hand, for 10 per cent.

APPLICATION.

☞ The following examples are for 1 year at 6 and 7 per ct.

1. What is the interest of $500? *Ans.* $30—35
2. What is the interest of $325? *Ans.* $19 : 50—22 : 75
3. What is the interest of $842? *Ans.* $50 : 52—58 : 94
4. What is the interest of $789? *Ans.* $47 : 34—55 : 23
5. What is the interest of $846? *Ans.* $50 : 76—59 : 22
6. What is the int. of $476 : 75? *Ans.* $28 : 60‘5—33 : 37‘2
7. What is the int. of $537 : 24? *Ans.* $32 : 23‘4—37 : 60‘6
8. What is the int. of $856 : 89‘5? *Ans.* $51 : 41‘3—59 : 98‘2
9. B's. note of $853 : 34 was on interest at 7 per cent one year; what is the interest thereon? *Ans.* 59 : 73‘3
10. D's. note of $254 payable in 12 months, at 6 per cent has expired, and is now due; what is the interest thereon? *Ans.* $15 : 24
11. T's. note of $419 on interest for one year, has become due; what is the interest thereon? *Ans.* $25 : 14
12. G's. promissory note of $158 has run a year on interest: what is the interest thereon? *Ans.* $9 : 48

2. When the principal, time and rate per cent are given to find the amount,

RULE:—Find the interest by multiplying the principal by the rate per cent. Then the principal added thereto will be the amount.

Ex. What is the amount of $476, at 6 per cent per annum? *Ans.* 504 : 56

OPERATION.

$4 : 76
6
28 : 56 *int.*
prin. 476
$504 : 56 *Amt.*

Examples for Practice.

Amount of $236 one year; 6 per cent?	*Ans.* $250 : 16
375 one year; 6 per cent?	*Ans.* 397 : 50
485 one year; 6 per cent?	*Ans.* 514 : 10
763 one year; 6 per cent?	*Ans.* 808 : 78
850 one year; 6 per cent?	*Ans.* 901

APPLICATION.

1. What is the amount of $409, at 6 per cent per annum? *Ans.* $433 : 54

2. What will $173 : 50 amount to, at 6 and 7 per cent for one year? *Ans.* $183 : 91—185 : 64'5

3. Find the amount of $327 : 82'5 for one year, at 6 and 7 per cent. *Ans.* $347 : 49'4—350 : 77'2

4. What is the amount of $278 : 75 at 6 and 7 per cent per annum? *Ans.* $295 : 47'5—298 : 26'2

3. To compute interest for years.

Rule :—First find the interest for one year, then multiply the interest so found by the number of years required.

Or, multiply the rate into the number of years, by which multiply the principal.

Ex. What is the interest of $320 for 2 years at 6 per cent? *Ans.* $38 : 40

OPERATION.

$3 : 20
6
$19 : 20 *for* 1 *year.*
2
Ans. $38 : 40 *for* 2 *years.*

Or thus : 6 *rate.*
2 *years the time.*
12
$3 : 20
12
640
320
Ans. $38 : 40

Examples for Practice.

Interest of $562 for 2 years?	*Ans.* $67 : 44
247 : 19 for 3 years?	*Ans.* 44 : 49'4
781 for 4 years?	*Ans.* 187 : 44
615 : 75 for 5 years?	*Ans.* 184 : 72'5

APPLICATION.

1. What is the interest of $560 for 2 years at 6 and 7 per cent? *Ans.* $67 : 20—78 : 40

2. What is the interest of $846 for 3 years, at 6 and 7 per cent? *Ans.* $152 : 28—177 : 66

3. What is the interest of $936 for 4 years, at 6 and 7 per cent? *Ans.* $224 : 64—262 : 08

4. What is the interest of $1500 for 5 years, at 6 and 7 per cent? *Ans.* $450—525

4. To compute interest for months.

RULE:—Multiply the principal by the number of months, divide the product by 12; then multiply the quotient by the rate per cent, the product will be the interest.

OBS. When the months are an even number, (rate 6 per cent,) multiply the principal by half the number of months, and the product will be the interest required. Or, multiply by the whole number of months and divide by 2.

EX. What is the interest of $1600 for 4 months at 6 and 7 per cent? *Ans.* $31 : 99‘8—37 : 33‘1

OPERATION.

$1600
4
12)6400
5 : 33‘3
6
$31 : 99‘8 *int.* 6 *per cent.*

$5 : 33‘3 *int.* 1 *per ct.*
7
$37 : 33‘1 *int.* 7 *per ct.*

Examples for Practice.

	Ans. at 6 pr. ct.	at 7 pr. ct.
Interest of $100 for 1 mo.	$0 : 50	$0 : 58‘3
105 for 2 mo.	1 : 05	1 : 22‘5
108 for 3 mo.	1 : 62	1 : 89
110 for 4 mo.	2 : 20	2 : 56‘6
120 for 5 mo.	3 :	3 : 50
125 for 6 mo.	3 : 75	4 : 37‘5
130 for 7 mo.	4 : 55	5 : 30‘8
135 for 8 mo.	5 : 40	6 : 30
140 for 9 mo.	6 : 30	7 : 35
145 for 10 mo.	7 : 25	8 : 45‘8
150 for 11 mo.	8 : 25	9 : 62‘5

APPLICATION.

1. What is the interest of $765 : 23 for 7 months? *Ans.* $26 : 78‘3

2. What is the interest of $284 : 85 for 3 months? *Ans.* $4 : 27‘2

3. What is the interest of $123 for 2 months? *Ans.* $1 : 23

5. To find the interest for years and months.

Rule:—Reduce the years to months by multiplying them by 12: add in the given months: by which multiply the principal, then divide the product by 2, the quotient will be the answer.

Ex. What is the interest of $123 : 65 for 1 year and 1 month, at 6 per cent? *Ans.* $8 : 03‘7

OPERATION.

12 *mo.* 1 *year.*
1 *mo. added.*
13 *Reduced to mo.*

$123 : 65
13
37095
12365
2)160745
$8 : 03·7,25 *int.* 1 *year* 1 *mo.*

Examples for Practice.

	Ans. at 6 per ct.
Interest of $638 : 20 for 1 year, 3 mo.?	$47 : 86‘5
284 : 60 for 2 years, 4 mo.?	39 : 84‘4
83 for 4 years, 9 mo.?	23 : 65‘5
64 : 80 for 2 years, 6 mo.?	9 : 72
274 : 60 for 4 years, 3 mo.?	70 : 02‘3

APPLICATION.

1. What is the interest of $225 : 50 for one year, 6 months? *Ans.* $20 : 29‘5

2. What is the interest of $326 for 2 years, 1 month? *Ans.* $40 : 75

3. What is the interest of $476 for 1 year, 2 months? *Ans.* $33 : 32

6. When interest is for days only.

Rule:—Divide the principal by 6, the quotient will give the interest for one day at 6 per cent; this multiplied by the number of days specified in the question, will give the interest sought.

Obs. To find the interest for any other than 6 per cent divide the interest of 6 per cent by 6, which will give 1 per cent; this multiplied by (7 or) the rate mentioned will give the interest required.

Ex. What is the interest of $60 for 15 days at 6 and 7 per cent? *Ans.* 15 *cts.*—17 *cts.* 5 *m.*

OPERATIONS.

6)60
10 1 *day's int.* 6 *per ct.*
15
15 *cts. int. for* 15 *days.* 6 *per ct.*

6)15 6 *per. ct. int.*
2‘5 1 *per ct. int.*
7
17‘5 7 *per ct. int.*

Examples for Practice.

	At 6 per ct. $ cts.	m.	At 7 per ct. $ cts.	m.
Interest of $24 for 1 day? *Ans.*		4	*Ans.*	4,66
360 for 2 days?	12		14	
150 for 3 days?	7	5	8	7
225 for 4 days?	15		17	5
312 for 5 days?	26		30	3
450 for 6 days?	45		52	5
250 for 8 days?	33	2	38	8
320 for 9 days?	47	9	55	9
112 : 25 for 10 days?	18	7	21	8
237 : 75 for 11 days?	43	5	50	8
384 : 85 for 12 days?	76	9	89	7

APPLICATION.

What is the interest of $600 for 55 days, at 6 per ct.? Ans $5 : 50. Of $24 for 10 days? Ans. 04 cts. Of $360 : 60 for 19 days? Ans. $1 : 14'1. Of $180 for 29 days? Ans. 87 cts. Of $1321 for 21 days? Ans. $4 : 62'2. Of $4111 for 60 days? Ans. $41 : 10'6. Of $413 : 83 for 35 days? Ans. $2 : 41'3. Of $374 : 13 for 20 days? Ans. $1 : 24'7. Of $2 for 6 days? Ans. :01'9. Of $95 : 36 for 17 days? Ans. 27 cts. Of $8673 for 10 days? Ans. $14 : 45'5. Of $15000 for 1 day? Ans. $2 : 50 Of $180 : 75 for 5 days? Ans. 15 cts.

7. To compute interest for years, months, and days.

Rule :—Find the whole number of months in the given years and months, by which multiply the principal. Divide the product by 2, the quotient will be the interest for the years and months. For days, and 7 per cent, divide and multiply as in the 6th case.

Ex. What is the interest of $95 : 36 for one year, 5 months, 17 days, at 7 per cent? *Ans.* $9 : 77'1

OPERATION.

```
   $95 : 36   one year is 12 mo. and 5 mo. more are 17
        17    months.
   --------
   $667  52                    6)95 : 36
     953  6                    ---------
   --------                    : 15'8,93 one day's int.
 2)1621  12                          17
   --------                    ---------
   $8 : 10'5,6                   111251
        27'0,181                 15893
   -------------              -----------
   $8 : 37'5,781               : 27'0,181 17 day's int.
            6)8 : 37'5,781 6 per ct. int.
              1 : 39'5,963 1 per ct. int.
         ------------------
      Ans. $9 : 77'1,741 7 per ct. int.
```

Examples for Practice.

Interest of	yrs.	mo.	days.	6 per cent.	7 per cent.
$240:30	3	4		$48:06	$56:07
75	2	6		11:25	13:12'5
1600	1	3		120	140
17:68	0	11	28	1:05'4	1:22'9
200	0	8	4	8:13'2	9:48'5
10:91	1	9	10	1:16'3	1:35'7
112:12'5	1	2	3	7:90'4	9:22'1
600:50	2	8	1	96:18	112:21
180:60	2	4	20	25:88'6	30: 20

APPLICATION.

1. Interest of $647 : 65'4, years 4, months 10, days 20, 5 per cent? *Ans.* $158 : 31'5

2. Interest of $276 : 45'5, years 4, months 8, days 5, 6 per cent? *Ans.* $77 : 63'7

3. Interest of $84 : 09'4, years 2, months 9, days 20, 6 per cent? *Ans.* $14 : 15'5

Another Method.

Rule :—Find the interest for one year at the rate mentioned; then multiply it by the number of years demanded. For months take the aliquot parts of a year; For days take the aliquot parts of a month, allowing 30 days to the month agreeable to the following

TABLE.

1 month is $\frac{1}{12}$ of a year.	1 day is $\frac{1}{30}$ of a month.
2 $\frac{1}{6}$ of a year.	2 days $\frac{1}{15}$ of a month.
3 $\frac{1}{4}$ of a year.	3 days $\frac{1}{10}$ of a month.
4 $\frac{1}{3}$ of a year.	5 days $\frac{1}{6}$ of a month.
5 $\frac{1}{3}$ & $\frac{1}{12}$ of a year.	6 days $\frac{1}{5}$ of a month.
6 $\frac{1}{2}$ of a year.	10 days $\frac{1}{3}$ of a month.
7 $\frac{1}{2}$ & $\frac{1}{12}$ of a year.	15 days $\frac{1}{2}$ of a month.
8 $\frac{1}{2}$ & $\frac{1}{6}$ of a year.	Note. 4, 7, 8, 9, 11, 12, 13, 14—and 16 to 29 days, cannot be divided in 30 without a remainder :—Therefore find the interest for one day and multiply the quotient by the number of days.
9 $\frac{1}{2}$ & $\frac{1}{4}$ of a year.	
10 $\frac{1}{2}$ & $\frac{1}{3}$ of a year.	
11 $\frac{1}{2}$, $\frac{1}{3}$ & $\frac{1}{12}$ of a year.	

Ex. What is the interest of $365 : 14'6 for 3 years, 7 months and 6 days, at 6 per cent? *Ans.* $78 : 87'1

16on

8. To compute interest for Days.

Rule :—Multiply the principal by ⅓ of the days; divide the product by 2, the quotient will be the interest at 6 per cent.

Examples for Practice.

What is the interest of $563, 18 days? *Ans.* $1 : 68'9. Of $454, 15 days? *Ans.* $1 : 13'5. Of $150, 12 days? *Ans.* 30cts. Of $481, 25 days? *Ans.* 1 : 99'6. Of $425, 20 days? *Ans.* 1 : 42'3. Of $806, 10 days? *Ans.* $1 : 32'9.

For Months and Days.

Rule :—Multiply the principal by the number of months and ⅓ of the days placed at the right hand of the months; divide the product by 2, the quotient will be the interest at 6 per cent.*

Examples for Practice.

What is the interest of $'00, 2 months, 6 days? *Ans.* 1 : 10. Of $600, 4 months, 18 days? *Ans.* $13 : 80. Of $800, 6 months, 21 days? *Ans.* $26 : 80. Of $500, 1 month, 12 days? *Ans.* $3 : 50. Of $450, 10 months, 18 days? *Ans.* $23 : 80. Of $25 : 50, 9 months, 12 days? *Ans.* $1: 19'8.

For Years, Months, and Days.

Rule :—Multiply the principal by the number of months and ⅓ of the days; then divide the product by 2.*

Examples for Practice.

What is the interest of $91 : 83, 3 years, 2 months, 13 days? *Ans.* $17 : 52. Of $8 : 50, 1 year, 9 months, 12 days? *Ans.* 90'9. Of $281 : 72, 1 year, 10 months, 18 days? *Ans.* $31 : 83'4. Of $457 : 93, 2 years, 4 months, 24 days? *Ans.* $65 : 94'1.

TABLE.

⅓ of 1 day is	,03	⅓ of 7 days is	,23	⅓ of 13 days is	,43
2 days	,07	8 days	,27	14 days	,47
4 days	,13	10 days	,33	16 days	,53
5 days	,17	11 days	,37	17 days	,57

9. For various rates per cent.

For One Year.

Rule :—Multiply the principal by the rate per cent.

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* The reason of this rule is obvious. If the given principal be multiplied by the whole number of months, the interest will be 12 per cent. Example, 100 × 12 = 12. But 5 × 12 = 6; or 10 × 6 = 6, or 6 per cent. It is hence immaterial whether the principal or the time be halved to obtain 6 per cent. 30 days are usually called a month, and are composed of 3 tens. One third, therefore, brings the days into a proper ratio with months: so that, if months be halved, the third of the days must be halved.

Obs. Where there is one odd day, ,3 is the decimal; 2 ,7 is the decimal,

For Months.

Rule :—Multiply the principal by the number of months; divide the product by 12, which will give 1 *per cent interest*—this multiplied by the rate per cent, will give the interest sought.

For Days.

Rule :—Divide the principal by 6; multiply the quotient by the number of days; then divide the answer by 6 and multiply by the rate mentioned.

For Years and Months.

Rule :—Reduce the years to months; add in the odd month, multiply the principal by the No. of months; divide the product by 12: then multiply the quotient by the rate mentioned.

For Years, Months, and Days.

Rule :—Multiply the principal by the number of months, divide the product by 2—then divide the principal by 6, multiply the quotient by the number of days; add the same to the interest of the years and months :—then divide by 6, and multiply by the rate mentioned.

Examples for Practice.

1. What is the interest of $784 for one year at 4 per cent? *Ans.* $31 : 36. Of $235 at 7 per cent. *Ans.* $16 : 45. Of $375 at 8 per cent. *Ans.* $30.

2. What is the interest of $150 for 2 years, at 4 per cent? *Ans.* $12. Of $180 for 3 years, at 7 per cent? *Ans.* $37 : 80. Of $250 for 4 years, at 5 per cent? *Ans.* $50.

3. What is the interest of $225 for 3 months, at 4 per cent? *Ans.* $2 : 25. Of $175 for 7 months, at 5 per cent? *Ans.* $5 : 10'4. Of $350 for 9 months, at 4 per cent? *Ans.* $10 : 50.

4. What is the interest of $950 for 5 days, at 7 per cent? *Ans.* : 92'3. Of $85 for 20 days, at 5 per cent? *Ans.* : 23'6. Of $1700 for 28 days, at 8 per cent? *Ans.* $10 : 57'7. Of $360 for 27 days, at 10 per cent? *Ans.* $2 : 70. Of $450 : 78 for 13 days, at 4 per cent? *Ans.* : 65'1.

5. What is the interest of $750 for 1 year 3 months, at 5 per cent? *Ans.* $46 : 87'5. Of $875 for 1 year 6 months, at 3 per cent? *Ans.* $39 : 37'5. Of $225 for 5 years 6 months, at 7 per cent? *Ans.* $86 : 62'5. Of $243 : 23 for 1 year 2 months, at 2 per cent? *Ans.* $5 : 67'5.

6. What is the interest of $34 : 25 for 3 years, 8 months and 15 days, at 5 per cent? *Ans.* $6 : 35'1. Of $372 for 4 years, 3 months and 17 days, at 8 per cent? *Ans.* $127 : 88'4.

10. To find the intermediate time from one date to another.

Rule :—First set down the years, then the months, then the days last mentioned ; then place the first mentioned time under the same. Over the days place 30. Over each month place its own order ; and if necessary, subtract from the whole number of days, or months contained in the given days or months, the remainder will give the time sought.

Ex. 1. What is the intermediate time from May 15, 1821, to Sept. 20th, 1822? *Ans.* 1 *yr.* 4 *mo.* 5 *da.*

Ex. 2. Find the time from Oct. 17th, 1820, to July 14th, 1821? *Ans.* 8 *mo.* 27 *da.*

OPERATIONS.

		(12)	(30)			(12)	(30)
	yrs.	*mo.*	*da.*		*yrs.*	*mo.*	*da.*
1st Ex.	1822	9	20	2d Ex.	1821	7	14
	1821	5	15		1820	10	17
	1	4	5 *Ans.*			8	27 *Ans.*

Examples for Practice.

1. What is the intermediate time from Sept. 21st, 1817, to August 16th, 1819? *Ans.* 1 *yr.* 10 *mo.* 25 *da.*

2. What is the time from Sept. 29th, 1816, to April 2d, 1819? *Ans.* 2 *yrs.* 6 *mo.* 3 *da.*

3. Find the time from May 20th, 1819, to April 4th, 1821? *Ans.* 1 *yr.* 10 *mo.* 14 *da.*

4. What is the intermediate time from Nov. 2d, 1818 to Nov. 1st, 1821? *Ans* 2 *yrs.* 11 *mo.* 29 *da.*

5. A note bearing date Dec. 28th, 1826, was paid Jan. 2d, 1827 ; how long was it at interest? *Ans.* 4 *da.*

6. A note given the 25th of Dec. 1828, was paid May 20th, 1829, how long was it at interest? *Ans.* 4 *mo.* 25 *da.*

APPLICATION.

1. What is the interest of $75 : 33 from Oct. 17th, 1830, to July 14th, 1831, at 6 per cent? *Ans.* $3 : 35'2

2. What is the interest of $796 : 67 from Nov. 2d, 1828, to Nov. 1st, 1831, at 7 per cent? *Ans.* $167 : 14'5

3. What is the interest of $59 : 50 from August 12th, 1826, to March 10th, 1830, at 7 per cent? *Ans.* $14 : 90'1

4. What is the interest of $492 : 48 from June 1st, to Dec. 20th, at 6 per cent? $16 : 33'4

5. What is the interest of $170 from July 15th, 1827, to Oct. 9th, 1830, at 6 per cent? *Ans.* $32 : 98

6. What is the interest of nine hundred twelve dollars, from Nov. 24th, 1830, to Oct. 15th, 1831, at 6 per cent? *Ans.* $48 : 79'2

7. What is the amount of $49 : 25 from March 16th, 1821, to Oct. 25th, 1827, at 7 per cent? *Ans.* $72 : 03'2

8. What is the interest of $1000 from Feb. 1st, 1825, to April 1st, 1826, at 6 per cent? *Ans.* $70

11. To find the PRINCIPAL, when the time, rate, and amount are given.

Rule:—Find the amount of $1 for the given time; then divide the sum given by this amount.

Ex. If in 1 yr. 4 mo. the interest and principal of a sum at 6 per cent, amount to $61 : 02 what is the principal?

Ans. 56 : 50

OPERATION. EXPLANATION.

Amt. of $1, 1 yr. 4 mo. is $1 : 08

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We first find what will be the amount of *a dollar with its interest*, for the given time, which is $1 : 08. Now as every dollar in the original sum gained 8 cents interest, there were as many dollars as there are $1 : 08 in $61 : 02.

APPLICATION.

1. A sum of money has been on interest 2 years, at 6 per cent, and it amounts to $112; what is the sum?

Ans. $100

2. What principal, at 8 per cent, will amount to $85 : 12 in 1 year, 6 months? *Ans.* $76

3. A note in 2 years amounted to $275, at 5 per cent; what was the principal of the note? *Ans.* $250

12. To find the PRINCIPAL, when the time, rate, and interest are given.

Rule:—Find the interest of $1 for the given rate and time; then divide the interest given by this interest, and the quotient will be the principal.

Ex. What sum put to interest 1 year 4 months, at 6 per cent, will gain $10 : 50? *Ans.* $131 : 25

OPERATION. EXPLANATION.

Interest of $1, 1 yr. 4 mo. is : 08

8)1050(13125

One dollar put to interest 1 yr. 4 mo. would gain 8 cts.; therefore it requires as many 1 dollars as there are 8 cts. in $10 : 50.

APPLICATION.

1. A man paid $4 : 52 interest at the rate of 6 per cent, at the end of 1 year 4 months; what was the principal?

Ans. $56 : 50

2. What principal will gain $60 in 2 years? *Ans.* $500

3. What principal will amount to $642, at 6 per cent, in 1 year, 2 months? *Ans.* $600

13. To find the RATE, when the principal, interest, and time are given.

RULE:—Find the interest on the given sum, at 1 per cent, for the given time, by which divide the given interest; the quotient will be the rate per cent.

Ex. If $3 : 78 is paid for $36, 1 year, 6 months; what is the rate per cent? *Ans.* 7 *per cent.*

OPERATION.

The interest on $36 1 yr. 6 mo. is : 54
54)378(7
378

EXPLANATION.

If $36 were at interest 1 yr. 6 mo. at 1 per cent, it would produce 54 cts. As many times, therefore, as 54 cts. is contained in $3 : 78 *so much more* than 1 per cent is the rate.

APPLICATION.

1. If $6795 gain $407 : 70 interest in a year; what is the rate per cent? *Ans.* 6

2. A. lent $75 for 8 months, and received $4; what was the rate per cent? *Ans.* 8 *per cent.*

3. A note for $930 : 25, was given on the 11th of March, 1823, and on the 29th of May, 1824, its interest was $111 : 63; what was the rate? *Ans.* 9'3 *per cent.*

14. To find the TIME when principal, rate and interest are given.

RULE:—Find the interest on the principal, at the rate mentioned for 1 year; divide the given interest by this, that is found, the quotient will be the time.

Ex. What is the time required to gain $3 : 78 on $36 at 7 per cent? *Ans.* 1 *year* 6 *mo.*

OPERATION.

Int. on $36, at 7 per ct. 1 yr. is $2 : 52
252)378(1. 6 *mo.*
252
126 *which is* ½ *of* 252

EXPLANATION.

We first find what would be the interest of $36 for *one year*, at 7 per cent. This would be $2 : 52. As many times as this sum is contained in the interest mentioned in the sum, *so much more time* than one year is required.

APPLICATION.

1. In what time will $85 amount to $108 : 80, at 7 per cent per annum? *Ans.* 4 *years.*

2. In what time will $75 amount to $102 at 6 per cent per annum? *Ans.* 6 *years.*

3. Paid $20 for the use of $600 at 8 per cent; what was the time? *Ans.* 5 *mo.*

4. Paid $28 : 24'2 for the use of $217 : 25 at 4 per cent; what was the time? *Ans.* 3 *yrs.* 3 *mo.*

INTEREST BY DECIMALS.

1. When the rate per cent is less than unity or more than 1 per cent, and a fractional part of one or more per cent,

Rule:—Reduce the rate per cent or time to a decimal; multiply the rate by the principal; the years by the interest of one year; the product will give the interest for years.

Note. The method of finding a decimal is shown at page 120, Case 6.

TABLE OF RATIOS.

Rate per cent.	Ratio.	Rate per cent.	Ratio.	Years.	Ratio.
¼	,0025	3¼	,0325	½	,5
½	,005	4½	,045	¾	,75
¾	,0075	5½	,055	¼	,25
⅝	,00625	6½	,065	3½	,35
1¾	,0175	6¼	,0625	5½	,55
2½	,025	5¾	,0575	6¼	,625
2¾	,0275	12½	,125	6½	,65
		15½	,155	7¾	,775

Ex. 1. What is the interest of $4000, at ¾ per cent?
Ans. $30

Ex. 2. What is the interest of $1627 : 18, at 2½ per cent?
Ans. $40 : 67'9

Ex. 3. What is the interest of $100, ¾ of a year at 6½ per cent?
Ans. $4 : 87'5

OPERATIONS.

1st Example.
4000
From table ,0075 *rate pr. ct.*
20000
28000
$30:0000 *Ans.*

2d Example.
1627 : 18
From table ,025 *rate pr. ct*
813590
325436
$40 : 67'950 *Ans.*

3d Example.
100
,065 *rate per cent.*
500
600
6500 *int. one year.*
,75 *ratio.*
32500
45500
$4:87'500 *Ans.*

APPLICATION.

What is the interest of $3000 at ½ per cent? *Ans.* $15. Of $5643 at ¾ per ct.? *Ans.* $42 : 32‘2. Of $1600 at ⅝ per cent? *Ans.* $10. Of $1286 at 2½ per cent? *Ans.* $32 : 15. Of $9458 at 2¾ per cent? *Ans.* $260 : 83‘7. Of $2317 : 46 at 3¼ per cent? *Ans.* $75 : 31‘7. Of $32 : 15 at 4½ per cent? *Ans.* $1 : 44‘6. Of $3000 at 5½ per cent? *Ans.* $165. Of $2491 : 78 at 5¾ per cent? *Ans.* $143 : 27‘7. Of $1200 : 30 at 12½ per cent? *Ans.* $150 : 03‘7.

What is the interest of $155 : 49 for ¼ of a year at 6¼ per cent? *Ans.* $2 : 42‘9. Of $970 : 99 for ½ of a year at 5½ per cent? *Ans.* $26 : 70‘2. Of $4000 for 5 years at 3½ per cent? *Ans.* $700. Of $2000 for 5½ years at 7 per cent? *Ans.* $770. Of $7200 for 6½ years at 5 per cent? *Ans.* 2340. Of $7981 : 25 for 3½ years at 7¾ per cent? *Ans.* $2164 : 91‘4. Of $156 : 90‘5 for 4 years at 6¼ per cent? *Ans.* $39 : 22‘6.

The RULES to find the interest on the following sums are given Page 136, 140.

Find the interest, at 6 per cent.		*Ans.*
Of $150	for 2 months,	$1 : 50
200	8	8 : 00
700	6	21 : 00
800	4	16 : 00
600	1	3 : 00
750	6	22 : 50
100	5	2 : 50
300	9	13 : 50
400	3	6 : 00
500	4	10 : 00

Find the interest, at 6 per cent.			*Ans.*
Of $250	for 1 yrs.	6 m.	$22 : 50
125	1	2	8 : 75
225	1	6	20 : 25
275	1	3	20 : 62‘5
300	1	4	24
350	1	6	31 : 50
400	1	8	40
475	1	6	42 : 75
425	2	2	55 : 25
550	2	4	77

Find the interest, at 6 per cent.		*Ans.*
Of $120	for 12 days,	$00 : 24
200	9	30
250	10	41‘2
340	11	62‘9
460	12	92
580	13	1 : 24‘7
600	14	1 : 41
400	15	1 : 25
540	16	1 : 16‘6
620	17	1 : 76‘7

Find the interest, at 6 per cent.			*Ans.*
Of $700	for 6mo.	1 days,	$21 : 10‘5
140	6	2	4 : 24‘9
260	9	3	11 : 83
380	3	4	5 : 94‘7
900	4	5	18 : 76‘5
860	6	6	26 : 66
180	6	7	5 : 60‘7
240	7	21	9 : 24
560	3	22	10 : 44‘4
640	9	23	31 : 26‘4

1. What is the interest of $450, at 6 per cent, for 10 months and 18 days? *Ans.* $23 : 85.

2. What is the interest of $137 : 45, at 6 per cent, for 8 months and 27 days? *Ans.* $6 : 11‘6.

NOTES AND OBSERVATIONS.

In the New England States, the law allows the lender to take 6 per cent from the borrower. In New-York, the per cent established by law is 7 cents for the use of one dollar for the term of one year.* Five per cent is what is allowed in England.

Legal or lawful interest is that which is allowed by the laws of the State, and is different under different governments, and in different States. The lender is not permitted to receive a greater per cent than is allowed by the laws of his State.

When no mention is made of a particular rate, but the note or agreement says simply *with interest*, the legal interest is always understood, and may be collected by law.

QUESTIONS ON INTEREST.

What is Interest? What is meant by the Principal? By rate per cent? per day? per month? per annum? What is the amount? *V. p.* 133 *and Dic. A. Terms.*

How is Interest computed? Of how many kinds is it? What is Simple Interest? What Compound? How do they differ? What is the rate per cent in New-England? What in New-York? What is legal interest? What usury? *V. p.* 147 *and Dic. A. Terms.*

1. How is the interest on any sum found from 1 to 12 per cent? Give an example. Which is the principal, which the interest, and which the rate, in the example you gave? When the principal is dollars, multiplied by the rate, what will the answer be in? When dollars and cents? When dollars, cents and mills? Divide dollars by the decimal ,6 what will the quotient be? If dollars and cents what? If dollars, cents and mills what? How can the interest be found at 1 per cent? Give an example. *V. p.* 133, 134.

2. How is the amount found, when the principal, time and rate are given? Give an example. Which is the interest, and which the amount in the example you gave? How is the interest found when the amount is given? *Ans.* Subtract the principal from the amount. *V. p.* 134.

3. What two methods are there to find the interest for 2, 3, 4 or more years? Why multiply the interest of one year by the number of years? Why multiply the rate into the number of years? What will the product be? *V. p.* 135.

4. How is the interest found for months, at 6 or 7 per cent? Why multiply by the number of months? *Ans.* To find the interest at 12 per cent? Why divide by 12? *Ans.* To find 1 per cent interest why multiply by 6? *Ans.* To find 6 per cent interest? Why by 7? By 8? By 9? &c. *Ans.* To find the interest at the required rate. Give an example. *V. p.* 136.

5. How is the interest found for years and months? Why multiply the years by 12? Why add in the given months? Why divide their sum by 2? Why divide that sum by 6? Why multiply the quotient by 7? By 8? Or why is the amount of the months divided by 12 and multiplied by 6? By 7? By 8? &c. *V. p.* 137.

6. How is the interest found for days? Why divide the principal by 6? Why multiply the quotient? How is the interest found at any other rate than at 6 per cent? Give an example. *V. p.* 137.

7. How is the interest found for years, months and days? What is the principal multiplied by? Why divide the product by 2? How is the interest for the days found, at 6 per cent? At 7? At 8? At 9? What other method is there? *V. p.* 138.

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* Till 1833, then established at 6 per cent.

8. How is the interest found for months and days? How many days in a year? What are the years multiplied by? For what? How many days in a month? What are the months multiplied by? For what? *V. p.* **140.**

9. How is the interest found, at **any rate per** cent, for 1 year? For months? For days? For **years and months?** For years, months and days? Give an example of each case. *V. p.* **140.**

10. How is the intermediate time from **one date to another** found? Why place **30** over the days? *Ans.* **Because 30 days make a month.** Why 12 over the months? *Ans.* **Because 12 months make one year.** Are these numbers always placed over the months and days? *Ans.* When we can subtract without carrying they are not. *V. p.* **142.**

11. If the time, rate and amount of a sum were given, how is the principal found? Give an example. *V. p.* **143.**

12. If the time, rate and interest were given, how is the principal found? Give an example. *V. p.* 143.

13. How is the rate found, when the principal, interest and time are given? Give an example. *V. p.* 144.

14. When the principal, rate and interest are given, how is the time found? Give an example. *V. p.* 144.

Decimals. 1. How is the interest found for ½, ¼ or 1½, 2½, and 3¾ per cent? *V. p.* 145.

2. How is the interest found for months by decimals? *V. p.* 146.

3. How is the interest found for years and months? *V. p.* 146.

INSURANCE.

Insurance is security given in consideration of a premium paid down, to restore to a certain value, for which the premium is advanced, the loss or damage on ships, houses, goods, &c. by storms, fire, &c.

⁂ The method of working questions in this rule is the same as in simple interest.

APPLICATION.

1. What is the insurance of property valued at $2500, at 6 per cent? *Ans.* $150

2. What insurance must be paid on $37500, at 5 per cent? *Ans.* $1875

3. What is the insurance of a house worth $1065, at 3 per cent? ***Ans.*** **$31 : 95**

TABLE.

¼=,025	4½=,045
1¼=,0125	5½=,055
1¾=,0175	7¾=,0775
3½=,035	8½=,085

4. What must he paid for the insurance of a house worth $2540, at 1¾ per cent? ***Ans.*** **$44 : 45**

5. What is the insurance of property worth $3000 at 5½ per cent? ***Ans.*** **$165**

6. What is the insurance on $5630, at 7¾ per cent? ***Ans.*** **$436 : 32'5**

7. What is the insurance of a vessel and cargo valued at $90146 : 46, at 8½ per cent? *Ans.* $7662 : 449

COMMISSION.

Commission is an allowance from a merchant to his factor or correspondent abroad, for buying and selling goods, and is at a certain rate per cent according to the custom of the country where the factor resides.

⁂ The work is performed the same as in simple interest.

APPLICATION.

1. What sum must be demanded on $3444 : 29 at 7 per cent? *Ans.* $241 : 10

2. What must be allowed on $750, at 5 per cent commission? *Ans.* $37 : 50

3. B. sold goods for A. to the amount of $3450, and charged 4½ per cent commission, what did A. pay? *Ans.* $155 : 25

4. A. sold goods to the amount of $2186 : 15, and charged the owner 3½ per cent commission, and 1¼ per cent storage; what was the amount of his bill? *Ans.* $103 : 84

ANNUITIES.

An annuity is a sum payable periodically, for a certain length of time, or for ever.

An annuity, in the proper sense of the word, is a sum paid *annually*, yet payments made at different periods, are called annuities. Pensions, rents, salaries, &c. belong to annuities.

When annuities are not paid at the time they become due, they are said to be in *arrears*.

The sum of all the annuities in arrears, with the interest on each for the time they have remained due, is called the *amount*.

The *Present worth* of an annuity, is the sum which should be paid for an annuity *yet to come*.

When an annuity is to continue for ever, its present worth is a sum, whose yearly interest equals the annuity.

Now as the principal, multiplied by the rate, will give the interest, the interest, divided by the rate, will give the principal.

Hence to find the *present worth* of an annuity, continuing for ever,

Divide the annuity by the rate per cent.

1. What is the worth of $100 annuity, to continue for ever, allowing to the purchaser 20 per cent? *Ans.* $500

2. What is an estate worth, which brings in $7500 a year, allowing 6 per cent? *Ans.* $125000

BROKERAGE.

Brokerage is an allowance of so much per cent to a person called a broker, for assisting merchants or factors in procuring or disposing of goods.

⁂ The principles of simple interest control the solution of questions in this rule.

APPLICATION.

1. What brokerage has B. on $2150, at 2 per cent?
Ans. $43

2. A broker demanded 3 per cent on $1000; to what did it amount? *Ans.* $30

3. D. sold A's. note of $1356, and charged 1¼ per cent; to what did his brokerage amount? *Ans.* $16:95

4. Suppose I employ a broker who sells goods to the value of $8970; what is the brokerage at 15 cts. on each dollar?
Ans. $1345:50

5. If I allow a broker 1¾ per cent; what is his demand for disposing of goods to the value of $1000? *Ans.* $17:50

STOCK.

Stock is a general name for the capitals of our trading companies, and the buying and selling certain sums of money in these funds has become a general practice.

⁂ The operation of the work is the same as in simple interest; multiply the stock by the rate per cent, the product is the worth of the stock.

APPLICATION.

1. What is the value of $6000 of stock at 115 per cent?
Ans. $6900

2. What is the value of $1500 of bank stock worth 75 per cent? *Ans.* $1125

3. What is the value of $1000 of insurance stock at 95 per cent? *Ans.* $950

4. Required the sum that will purchase $4039 stock at 94 per cent? *Ans.* $3796:66

DISCOUNT.

Discount is an allowance made for the payment of any sum of money, before it becomes due, according to a certain rate per cent agreed on between the parties concerned.

The present worth of any sum, or debt, due sometime hence, is such a sum as, if put to interest, for that time at a certain rate per cent would amount to the sum or debt then due.

Rule :—Find the amount of $1 for the given rate and time for a divisor, by which divide the given sum, and the quotient will be the answer.

Ex. What is the present worth of $983 : 84, discount at 4 per cent? *Ans.* $946

OPERATION.

1:00
‘4
4‘00
100
104)983:84(946 *dols. Ans.*
936
478
416
624
624

Proof. Reverse the question—will $946 amount to $983: 84 in 1 year, at 4 per cent?

$946 *pr. worth.*
4
$37 : 84 *int.*
$946 *prin.*
$983 : 84 *amt.*

APPLICATION.

1. What is the present worth of $397 : 50, discount at 6 per cent? *Ans.* $375

2. How much ready money for a note of $173 due one year hence at 5 per cent per annum? *Ans.* $164 : 76‘1

3. What is the present worth of $775 due 4 years hence at 5 per cent per annum? *Ans.* $645 : 83‘3

4. What is the present worth of $312 for 6 months discounting at 6 per cent? *Ans.* $302 : 91‘2

5. What is the present worth of $500 due 2 years hence at 7 per cent per annum? *Ans.* 438 : 59‘6

6. What is the present worth of $648 payable in one year discounting at 8 per cent? *Ans.* $600

*** To find the discount, subtract the present worth from the given sum, the remainder will be the discount.

7. What is the discount of $100 for one year at 6 per cent per annum? *Ans.* 5 : 66‘1

8. What is the discount of $420 for 2 years at 6 per cent per annum? *Ans.* $45

9. What is the discount of $321 : 63 due 4 years hence, discounting at the rate of 6 per cent? *Ans.* $62 : 25‘1

10. What is the present worth, and what is the discount of $100 payable in one year, at 7 per cent a year?

Ans. { $93 : 45‘7 present worth.
$6 : 54‘3 discount.

Obs. When discount is to be made for present payment without time, the interest is the discount.

11. What is the discount of $853, at 2 per cent? *Ans.* $17 : 06

12. What is the discount on $476 : 50, at 7 per cent? *Ans.* $33 : 35'5

N. B.—*To find the bank discount on notes that have* 33, 63, *or any number of days to run, including* 3 *days of grace:* multiply the amount of the note by the number of days it has to run, and divide that product by 6000 : the quotient will be the discount required.

13. What is the discount on a note of $500, for 33 days at 6 per cent per annum? *Ans.* $2 : 75

14. What is the discount of $9760, for 63 days, at 6 per cent per annum? *Ans.* $102 : 48

15. What is the discount of $870 : 75, for 93 days, at 6 per cent per annum? *Ans.* $13 : 49'6

LOSS AND GAIN.

Loss and Gain is a rule by which we discover the gain or loss of any parcel of goods, and it instructs us how to raise or fall the price of any commodity in such proportions, that neither our gain may be so exorbitant as to injure our customers, nor our loss so great as to impoverish ourselves, which is generally at so much per cent.

1. To find the rate per cent of Gain or Loss.

Rule:—Find the difference between the buying and selling price by subtraction, then annex ciphers thereto, divide the same by the first cost, the quotient will be the gain or loss per cent.

Ex. A merchant bought a quantity of goods for $318 : 50 and sold them again for $299 : 39, what was lost per cent? *Ans.* 6

OPERATIONS.

$318 : 50 *first cost.*
299 : 39
318 : 50)191100(6 *pr. ct. loss.*
191100

Proof of Ex. 1*

$1 : 10
1 : 25
5 50
22 0
110
$1 : 37'50 *selling price.*

Proof. Reverse the question, see if the loss per cent will amount to the total loss—or see if the gain per cent will amount to the selling price of the article sold.

$318 : 50
6
$19:1100 *total loss.*

APPLICATION.

1. If I purchase wheat at $1 : 10 per bushel, and sell it at $1 : 37'5 per bushel, what do I gain per cent? *Ans.* 25*

2. If I buy salt for 75 cts. per bushel and sell it again at 90 cts. per bushel, what do I gain per cent? *Ans.* 20

3. Bought tea at : 87‘5 per pound which I sold at $1 per pound; what do I gain per cent? *Ans.* 14‘2

4. Sold merchandise which cost $1856 for $2784; at what rate per cent was the gain? *Ans.* 50

5. If I buy cotton at 15 cts. a pound and sell it for : 16‘5; what should I gain in laying out $100? *Ans.* $10

6. If A. buy cloth at $2 per yard, and sell it for $2 : 50 per yard; what would he gain in laying out $100? *Ans.* $25

7. Bought indigo at $1 : 20 per pound and sold the same at 90 cts. per pound what was lost per cent? *Ans.* 25

8. If I buy tea at one dollar per pound and sell it again at : 87‘5 per pound; what is lost per cent? *Ans.* 12‘5

9. Bought broadcloth at $4 : 50 per yard and sold it for $1 : 12‘5; what was lost per cent? *Ans.* 75

10. A merchant bought a quantity of goods for $94 : 30, and sold them for $83 : 92‘7; what did he lose per cent? *Ans.* 11

11. A merchant bought a quantity of tea for $365, which proving to have been damaged, he sold for $332 : 15; what did he lose per cent? *Ans.* 9

2. To know how a commodity must be sold, to gain so much per cent.

Rule :—Prefix unity to the proposed gain per cent, multiply the same by the purchase price, and the product will be the selling price.

Ex. D. bought 1 cwt. of iron for $343, at what price must it be sold to gain 15 per cent? *Ans.* $394 : 45

OPERATION.

343
115
1715
343
343
$394 : 45 *Ans.*

Proof. Find the total gain—then see if the rate per cent will amount to the same.

Cost $343—*sold for* $394 : 45
difference $51 : 45

$343
15
1715
343
$51 : 45

APPLICATION.

1. If a merchant buy sugar at 9 cts. per pound, how must he sell it to gain 25 per cent? *Ans.* : 11‘2

2. Bought wine at 90 cts. per gallon, at what rate must it be sold to gain 20 per cent? *Ans.* $1 : 08

3. How must tea, which cost 92 cts. per lb. be sold to gain 25 per cent? *Ans.* $1 : 15

4. Bought a hogshead of melasses for $60, for how much must I sell it to gain 20 per cent? *Ans.* $72

5. If B. pays $50 for a piece of broadcloth; how must he sell the same so as to gain 20 per cent? *Ans.* $60

3. To know at what price a commodity must be sold to lose so much per cent.

Rule:—Subtract the proposed rate per cent from $1, and multiply the first cost by the remainder; the product will be the selling price.

Ex. Bought flour at $3 : 15 a barrel, how must it be sold per barrel to lose 20 per cent? *Ans.* $2 : 52

OPERATION.

$1 : 00	$3 : 15
20	80
80	$2 : 52'00 *Ans.*

APPLICATION.

1. Bought calico at 20 cts. per yard; how must I sell it to lose 20 per cent? *Ans.* 16 cts.

2. Bought broadcloth at $2 : 50 per yard, but it being damaged I must lose 12 per cent; how much will that be per yard? *Ans.* $2 : 20

3. If I give $15 for a cow, how must I sell her to lose 10 per cent? *Ans.* $13 : 50

4. Bought oil at $1 : 25 per gallon which not proving as good as expected, I am obliged to lose 18 per cent; how must it be sold per gallon? *Ans.* $1 : 02'5

4. To find the first cost of a commodity when there is a gain of so much per cent.

Rule:—Prefix unity to the rate per cent of gain for a divisor, by which divide the price of the article or commodity per yard or bushel; the quotient will be the first cost.

Ex. By selling wheat at $1 : 07 per bushel I gain at the rate of 30 per cent; what was the first cost of the wheat per bushel? *Ans.* : 82'3

OPERATION.

130)107000(: 82'3 *Ans.*
1040
300
260
400
390
10

Proof. If the first cost multiplied by the amount per dollar equals the selling price the answer is correct.

: 82'3 *first cost.*
1 30 *amount.*
246 90
823
$106 : 990 *nearly* 107.

APPLICATION.

1. A. sold B. 375 yards of cloth for $490 and gained 20 per cent, what did it cost? *Ans.* $408 : 33'3

2. B. sold cloth at $1 : 50 per yard and gained 20 per cent; what did it cost? *Ans.* $1 : 25

5. To find the first cost of a commodity when there is a loss of so much per cent.

Rule :—Subtract the loss per cent from $1, and divide the price of one yard, pound, &c. by the remainder; the quotient will be the first cost.

Ex. When 20 per cent loss is made on coffee sold at 20 cts. per pound; what was the first cost? *Ans.* 25 *cts.*

OPERATION.

```
$1 : 00      80)2000(25 cts. Ans.
  20            160
  ——            ———
  80             400
                 400
```

APPLICATION.

1. If 12 per cent be lost by selling 100 yards of broadcloth for $500; what was the prime cost per yard? *Ans.* $5 : 68'1

2. By selling broadcloth at $3 : 25 per yard, I lose at the rate of 20 per cent; what is the prime cost of said cloth per yard? *Ans.* $4 : 06'2½

3. If a merchant, by selling wine at $2 : 25 per gallon, lose 18 per cent; what was the prime cost? *Ans.* $2 : 74'3

4. If by selling tea at : 87'5 per pound there is lost 12½ per cent; what did it cost per pound? *Ans.* $1

BARTER.

Barter is exchanging one commodity for another, according to the price or value agreed upon by the parties concerned, so that neither shall sustain loss.

⁂ The operation is performed by multiplication and division.

Rule :—Find the amount of the commodity whose quantity and price are given, then divide that sum by the price of the article to be exchanged, the quotient will be the required quantity.

Ex. How much indigo at $2 per pound, must be given for 19 yards of broadcloth at $7 per yard? *Ans.* 66½

OPERATION. Proof. Multiply the Ans. by its respective price —if the amount be the same as the known quantity, the solution is correct.

```
    19
     7                        66½
  ————                         2 price, &c.
2)133                       ————
———————                     $133 price of 19 yds. $7 per yd.
Ans. 66½ lbs.
```

APPLICATION.

1. How many pounds of butter, at 23 cts. per pound must be given for 5 quintals of fish worth $2: 25 per quintal?

Ans. 49 *nearly.*

2. How much corn at 40 cts. per bushel, must be given for 60 bushels of wheat at 80 cts. per bushel? *Ans.* 120 *bush.*

3. How much corn at 45 cts. per bushel, equals in value 357 bushels of wheat at 93 cts. per bushel? *Ans.* $737\frac{4}{5}$ *bush.*

4. How much sugar at 8 cts. per pound, must be given for 525 pounds of tobacco at 10 cts. per pound? *Ans.* $656\frac{1}{4}$ *lbs.*

5. How many bushels of salt at $1 : 25 per bushel, must be given for 50 bushels of corn at 70 cts. per bushel? *Ans.* 28

6. A merchant bartered 41 cwt. of rice at $4 : 50 per cwt. for 195 bushels of salt at 80 cts. per bushel and received the balance in cash; how much money did he receive?

Ans. $28 : 50

⁂ Find the separate value of the two given commodities; subtract the less from the greater, and the difference will be the money required.

7. A merchant bartered 40 yards of cloth at $1 : 16 per yard, for 28 pounds of tea at $1 : 80 per pound; how much money did he pay? *Ans.* $4

8. A. lets B. have 50 bushels of rye at 45 cts. per bushel, for which he receives 60 bushels of oats at 25 cts. per bushel; how much money must A. receive? *Ans.* $7 : 50

9. A. and B. barters; A. has 75 bushels of wheat at $1 : 45 per bushel, and 64 bushels of rye at 95 cts. per bushel, for which B. gives 2 quintals of fish at $2 : 30 per quintal, and 8 hogsheads of salt at $4 : 30 per hogshead; how much money must A. receive of B.? *Ans.* $130 : 55

10. A farmer sold a grocer 20 bushels of rye at 75 cts. per bushel, 200 pounds of cheese, at 10 cts. per pound, in exchange for which he received 20 gallons of melasses at 22 cts. per gallon and the balance in money; how much money did he receive? *Ans.* $30 : 60

11. A farmer sold 6 loads of wood at $3 a load; and 336 pounds of butter at 9 cts. per pound, and 70 bushels of wheat at $2 : 12'5 per bushel, and 93 bushels of rye at 75 cts. per bushel; in return he received 3 plows at $12 a-piece, one yoke of oxen $60, one span or horses at $75 a-piece and the rest in money; how much money did he receive? *Ans.* $20: 74

EQUATION OF PAYMENTS.

Equation of Payments is the finding a time to pay at once, several debts due at different times, so that no loss may be sustained by either party.

Rule:—Take the debt, or sum total of the several debts for a divisor, and the sum of the products made by multiplying each payment with its respective time for a dividend, and the quotient will be the equated time.

Ex. A. owes B. $400, of which $200 is to be paid in 2 months and $200 in 4 months, but they agree that the whole shall be paid at one time; at what time must it be paid?

OPERATION.

```
     200×2=400
     200×4=800
     ———   ————
     400)  1200(3 mo. Ans.
           1200
```

Proof. If the interest of the sum payable at the equated time, equal the interest of the several payments for their respective times, the work is right.

APPLICATION.

1. Find the equated time for the following payments, viz. $100 due in 6 months, $120 in 7 months, and $160 in 10 months? *Ans.* 8 *mo.*

2. D. owes H. $200 whereof $40 is to be paid in 3 months, $60 in 5 months, and the remainder in 10 months; at what time may the whole be paid? *Ans.* 7$\frac{20}{200}$

3. D's. bond note for $636 falls due thus:—200 in 33 days, 150 in 93 days, and the balance in 123 days, but he will pay the whole at the equated time; what is it?

Ans. 88 *days nearly.*

4. A. owes C. $500, payable as follows: $250 in 6 months, and $250 in 8 months; if paid at one time, what would that be? *Ans.* 7 *mo.*

5. E. holds F's. bond for $500, payable, $125 in 5 months, $150 in 8 months, and the balance in 13; what is the equated time for the payment of the whole? *Ans* 9½

FELLOWSHIP.

Fellowship means copartnership. It is the agreement of two or more persons to trade on a joint stock; of which each owns a part, and participates proportionally in the gain or loss.

Rule:—Add the several stocks together for a divisor, then make the whole gain or loss the dividend, the quotient will be the gain or loss on a dollar; this multiplied by each partner's sum invested in trade, will give the gain or loss of each, in proportion to his stock.

Ex. D., E. and F. traded in company: D. put in $140, E. $300, and F. $160; they gained $120: what is each partner's share? *Ans. A's share* $28. *E's.* $60. *F's.* $32.

OPERATION.

```
D's. stock 140            140       300       160
E's.       300             20        20        20
F's.       160         ------    ------    ------
          ----         $28:00    $60.00    $32:00
          600)120:00(20 cts. on a dol.
              120:00
```

Proof. Find whether the sum of the *shares* of the gain is equal to the *whole* gain.

APPLICATION.

1. A. and B. contracted to do a piece of work for $150; to complete which, A. worked 70 days, and B. 50 days; what share of the money did each receive?

Ans. A. $37 : 50—B. $62 : 50

2. A. B. C. and D. enter into partnership; A. puts in stock $1000, B. $2000, C. $3000 and D. $4000 with which they gained $1000; what is the share of each?

Ans. A's. $100—B's. $200—C's. $300—D's. $400.

3. A bankrupt is indebted to A. $780, to B. $460, and to C. $760, his estate is worth only $600; how must it be divided?

Ans. A. $234—B. $138—C. $228

4. A. B. and C. companied; A. put in $400, B. $600, and C. $800, they gained $540; what must each have of the gain?

Ans. A's. $120—B's. $180—C's. $240.

5. A. and B. trade in company with a joint capital of $600; A. put in $350 : 50, and B. $249 : 50, and by trading they gained $120; what was the gain on $1 and what is each person's share of the gain?

Ans. { *gain on* 1 *dollar is* 20 *cts.*
A's. $70 : 10—B's. $49 : 90 }

6. A bankrupt is indebted to A. $277 : 33, to B. $305 : 17, to C. $152, and to D. $105, his estate is worth only $677 : 50; how must it be divided?

Ans. A. $223:80'5 B. $246:27'2 C. $122:66'4 D. $84:73'5

7. Three merchants formed a company for the purpose of trade; the first advanced $25000, the second $18000, and the third $42000; after some time they separated and wished to divide the joint profit, which amounted to $57225; how much ought each to have?

Ans. 1*st.* 16830\frac{75}{85}$—2*d.* 12118\frac{20}{85}$—3*d.* 28275\frac{75}{85}$

8. Two merchants traded in company; A. put in stock $250, and B. $150, they lost by trading $100; what was the loss of each?

Ans. A's. $62 : 50—B's. $37 : 50

COMPOUND FELLOWSHIP.

Compound Fellowship is concerned in cases in which the stock of partners is employed for different times.

Rule:—Multiply each man's stock by the time it was continued in trade, then divide the whole gain or loss by the sum of the products, and it gives the gain or loss on a dollar; this multiplied separately by each man's respective capital gives the gain or loss of each.

Ex. A. B. and C. trade in company, and gain $480; A's. capital is $800 for 4 months, B's. $600 for 2 months; and C's. $720 for 5 months; what is each man's share of the gain?

Ans. A's. $192 B's. $72 C's. $216

OPERATION.

800×4=3200	3200	1200	3600
600×2=1200	6	6	6
720×5=3600	$192:00	$72:00	$216:00

8000)480 : 00(6 *cts. gain on a dollar.*

N. B. Proved the same as Single Fellowship.

APPLICATION.

2. Three merchants, A. B. and C. enter into partnership: A. puts in $80 for 4 months: B. $60 for 2 months, C. $72 for 5 months: but by misfortune they lose $48; how much loss must each man sustain? *Ans.* A. $19 : 20 B. $7 : 20 C. $21 : 60

3. Three butchers, A. B. C. hire a pasture for $24: A. puts in 40 sheep for 4 months, B. 30 sheep for 2 months, and C. 36 sheep for 5 months; what share of the rent must each pay?

Ans. A's. $9 : 60 B's. $3 : 60 C's. $10 : 80

4. Two partners gained by trading $800: A's. stock was $1200 for 8 months, and B's. $800 for 10 months; what is each partner's share of the gain?

Ans. A's. $436 : 32 B's. $363 : 68

5. Two men, A. and B. traded in company; A. put in $700 for 8 months, and B. $1280 for 10 months; they gained $500; what was the share of each?

Ans. A's. $152 : 17'3 B's. $347 : 82'7

QUESTIONS ON INSURANCE, &c.

1. What is Insurance? What is meant by premium? By policy? How is the work performed? *V. p.* 148, *and Dic. A. Terms.*

2. What is Commission? Who is a factor? How is the commission at a certain per cent, found? *V. p.* 149, *and Dic. A. Terms.*

3. What is brokerage? Who is a broker? What does he receive for his services? What rule controls the solutions of questions in brokerage? *V. p.* 150, *and Dic. A. Terms.*

4. What is a general name for the capitals of our trading companies? What is understood by stock being at par? Above par? Below par? The operation is the same as in what? *V. p.* 150, *and Dic. A. Terms.*

5. What is Discount? What is the present worth? Is the discount on a given sum the same as the interest on the same sum for the given, time and rate? How do you find the present worth of any sum? Repeat *Rule p.* 151. How do you find the discount on any sum at a given rate per cent and for a given time? *V. p.* 151.

6. What is Loss and Gain? How do you find what is gained or lost per cent? *V. Rule, p.* 152. How do you find *how* the commodity must be sold to gain a certain per cent? Repeat *Rule, p.* 153. How can you find the price at which a commodity must be sold to lose a certain rate per cent? Repeat *Rule, p.* 154. How do you find the first cost of a commodity when there is a gain of so much per cent? *V. Rule, p.* 154. When there is a loss of so much per cent? *V. Rule, p.* 155.

7. What is Barter? How is the operation performed? When you have the price of one commodity, how do you find the quantity of the other, which is to be exchanged for it? *V. Rule, p.* 155. When the price and quantity of both are given, how do you find the balance due in money? *V. p.* 156.

8. What is Equation of Payments? What is the method of operation? *V. p.* 157. What does Fellowship mean? *V. p.* 157. Compound Fellowship? What is the Rule? *V. p.* 159.

COMPOUND ADDITION.

Compound Addition teaches to collect several numbers of different denominations into one sum.

The denominations must be of the same kind; otherwise we could add eagles, dimes, and dollars, to years, months and days. Dimes and dollars are different denominations from eagles, but they are of the same kind, viz: money.

In addition of several denominations it must be observed, how many of the smaller name make one of the next greater; viz: how many mills make a cent; and how many cents make a dollar.

The currency of the United States, is called Federal Money, and the terms by which it is known, are eagles, dollars, dimes, cents and mills. The coins of Federal Money are two of gold, four of silver, and two of copper. The gold coins are an eagle, and half eagle; the silver, a dollar, half-dollar, dime, and half-dime; and the copper a cent and a half-cent.

FEDERAL MONEY.

A Table of the Parts.

10 mills (m.)	make	one cent	Marked,	*ct.*
10 cents	"	one dime	"	*d.*
10 dimes	"	one dollar	"	$.
10 dollars	"	one eagle	"	*E.*

Addition of money, weight and measure.

Rule:—Place the given numbers of the same name under each other, separate the columns by dashes, &c.

2. Begin with the right hand column, and work as in addition of whole numbers.

3. Divide the amount by as many of that name as will make one in the next greater.

4. Set the remainder, if any, under the column added; if not place a cipher.

5. When the column does not amount to as much as the dividing denomination, set it down under the column added.

6. Carry the quotient produced by division, to the next higher name; and in this way add all the given columns.

Find the value of 69 E. $7. and 8d. 3 E. $5. and 4d. 91 E. $2. and 2d.

OPERATION.

	(10)	(10)
E.	$.	d.
69	7	8
3	5	4
91	2	2
164	5	4 *Ans.*

DEM. Divide the amount of the first column by 10, because 10d. make one Dollar. 2d, by 10 because $10 make one E. Add the last column as in whole numbers.

AVOIRDUPOIS WEIGHT.

16 drams (dr.)	make	1 ounce, marked	*oz.*
16 ounces	"	1 pound, "	*lb.*
100 pounds†	"	1 hundred, "	*cwt.*
25 pounds	"	1 quarter, "	*qr.*
10 hundred	"	1 thous. lbs.	*mwt.*

† In our principal seaports, merchants at the present time buy and sell by the hundred pounds; and the terms *gross* and *net* weight have become obsolete in business. It appears perfectly needless, childish and nonsensical, to continue the theory of that which is laid aside in practice.

It is farther evident that if the several denominations of money, weight and measure proceeded in a decimal ratio, the fundamental operations might be performed upon these as upon abstract numbers.

Add 15 lbs. 6 oz. 11 drs., 24 lbs. 9 oz. 10 drs., 11 lbs. 15 oz. 11 drs., and find their amount?

OPERATION.

	(16)	(16)
lbs.	*oz.*	*drs.*
15	6	11
24	9	10
11	15	11
52	00	00 *Ans.*

DEM. Divide the amt. of drs. by 16, because 16 drs. make 1 oz. Divide the amt. of oz. by 16, because 16 oz. make 1 lb.

Find the amt. of 16 lbs. 10 oz. 10 drs., 17 lbs. 15 oz. 14 drs., 18 lbs. 12 oz. 11 drs., and 19 lbs.
Ans. 72 *lbs.* 7 *oz.* 3 *dr.*

APOTHECARIES' WEIGHT.

What is the amt. of 19℔. 10 oz. 4 drs. 2 sc. 16 grs., 9 oz. 7 drs. 17 grs., and 3℔. 6 oz. 5 drs. 1 sc. 18 grs.

OPERATION.

	(12)	(8)	(3)	(20)
℔	*oz.*	*dr.*	*sc.*	*gr.*
19	10	4	2	16
00	9	7	0	17
3	6	5	1	18
24	3	1	2	11 *A.*

DEM. Divide the amt. of grs. by 20, because 20 grs. make 1 sc. Divide sc. by 3, because 3 sc. make 1 dr. Divide drs. by 8, because 8 drs. make 1 oz. Divide oz. by 12, because 12 oz. make 1 ℔.

Find the amt. of 5 drs. 1 sc. 15 grs., 6 drs. 2 sc. 18 grs., 4 drs. 17 grs., 6 drs. 2 sc. 11 grs. *Ans.* 2 *oz.* 7 *drs.* 2 *sc.* 1 *gr.*

CLOTH MEASURE.

Find the amount of 36 yds. 1 qr. 2 na., 41 yds. 2 qrs. 3 na. 65 yds. 3 qrs. and 1 na.

OPERATION.

	(4)	(4)	
yds.	*qr.*	*na.*	
36	1	2	
41	2	3	
65	3	1	
143	3	2	*A.*

Dem. Divide the amt. of na. by 4, because 4 na. make 1 qr. Divide qrs. by 4, because 4 qrs. make 1 yd.

A. sold four pieces of cloth; the first contained 57 yds. 2 qrs.; the second 29 yds. 3 qrs. 2 na.; the third, 46 yds. 1 qr.; the fourth 32 yds. 3 qrs.; find the amount of all the pieces? *Ans.* 166 *yds.* 1 *qr.* 2 *na.*

LONG MEASURE.

Add 63 deg. 44 mi. 6 fur. 30 po. 14 ft. 10 in. 2 bc., 56 deg. 54 mi. 7 fur. 35 po. 11 ft. 6 in. 2 bc., and 10 deg. 57 mi. 6 fur. 16 po. 7 ft. 8 in. 1 bc. together.

OPERATION.

*(69½)	(8)	(40)	(16½)	(12)	(3)	
deg.	*mi.*	*fur.*	*po.*	*ft.*	*in.*	*bc.*	
63	44	6	30	14	10	2	
56	54	7	35	11	6	2	
10	57	6	16	7	8	1	
131	18	5	3	1	1	2	*Ans.*

Dem. Divide the amt. of bc. by 3, because 3 bc. make 1 in. Divide in. by 12, because 12 in. make 1 ft. Divide ft. by 16½, because 16½ ft. make 1 po. Divide po. by 40, because 40 po. make one fur. Divide fur. by 8, because 8 fur. make 1 mi. Divide mi. by 69½, because 69½ mi. make one deg.

B. went in one day 27 mi. 2 fur.; the next, 32 mi. 7 fur. the third 19 mi. 7 fur. 16 po.; and in the fourth, 15 mi. 5 fur. 32 po.; how far did he travel? *Ans.* 95 *mi.* 6 *fur.* 8 *po.*

LAND MEASURE.

Add 123 A. 11 po., 164 A. 2r. 21 po., 464 A. 3r. 32 po., 602 A. 1r. 6 po. into one sum. *Ans.* 1354 *A.* 3*r.* 30 *po.*

OPERATION.

	(4)	(40)	
A.	*r.*	*po.*	
123	0	11	
164	2	21	
464	3	32	
602	1	6	
1354	3	30	*Ans.*

Dem. Divide the amt. of po. by 40, because 40 po. make 1r. Divide r. by 4, because 4r. make 1 acre.

A gentleman has 3 farms which measure as follows, viz: 150 A. 3r. 14 po. 175 A. 2r. 18 po. 100 A. 1r. 39 po.; what do they measure in all? *Ans.* 426 *A.* 3*r.* 31 *po.*

---

* To divide by 16½ double it: the amount of ft. also; and half of what remains, after dividing, will be the remainder. Also, to divide by 69½, &c. pursue the same principle.

SOLID MEASURE.

What are the solid contents of 45 T. 6 ft. 1718 in., 31 T. 16 ft. 416 in., 46 T. 19 ft. 500 in., 75 T. 24 ft. 900 in?

Ans. 216 *T.* 27 *ft.* 78 *in.*

OPERATION.

	(40)	(1728)
T.	*ft.*	*in.*
45	6	1718
31	16	416
46	19	500
75	24	900
198	27	78 *Ans.*

Dem. Divide the amt. of in. by 1728 because 1728 solid in. make 1 solid foot. Divide ft. by 40, because 40 solid feet, make 1 tun.

Harry disposed of wood at sundry times, viz: 79 cords, 120 ft., 91 cords, 8 ft., 56 cords, 127 ft., 247 cords, 64 cords 34 ft., 110 cords, and 9 ft.; of how much wood did Harry dispose. *Ans.* 649 *cords*, 42 *ft.*

A man brings to market 4 loads of wood, containing the first, 1 cord 69 ft. 860 in.; the second, 1 cord 67 ft. 68 in.; the third, 1 cord 30 ft. 300 in.; the fourth, 1 cord 30 ft. 631 in.; how much in all? *Ans.* 5 *cords*, 69 *ft.*, 131 *in.*

LIQUID MEASURE.

Add 81 hhds. 38 gals. 1 qt. 1 pt.; 75 hhds. 18 gals. 3 qts.; 18 hhds. 55 gals. and 1 pt.

OPERATION.

	(63)	(4)	(2)
hhds.	*gals.*	*qts.*	*pts.*
81	38	1	1
75	18	3	0
18	55	0	1
175	49	1	0 *Ans.*

Dem. Divide the amt. of pts. by 2, because 2 pts. make 1 qt. Divide qts. by 4, because 4 qts. make 1 gal. Divide gals. by 63, because 63 gals. make 1 hhd.

A grocer purchased at several times, 1 hhd. 59 gals. 2 qts.; 62 gals.; 1 hhd. 9 gals. 2 qts.; and 46 gals. of melasses; how much melasses did he purchase? *Ans.* 4 *hhds.* 51 *gals.*

DRY MEASURE.

Add 45 bu. 3 pks. 5 qts. 1 pt.; 84 bu. 2 pks. 7 qts.; 95 bu. 1 pk. 6 qts. 1 pt.

OPERATION.

	(4)	(8)	(2)
bu.	*pk.*	*qt.*	*pt.*
45	3	5	1
84	2	7	0
95	1	6	1
226	0	3	0 *Ans.*

Dem. Divide the amt. of pts. by 2, because 2 pts. make 1 qt. Divide qts. by 8, because 8 qts. make 1 pk. Divide pks. by 4, because 4 pks. make 1 bu.

F. has 5 granaries; in 3 he has 756 bu. 2 pks. 6 qts. 1 pt.; in the other two, 854 bu. 5 qts.: what is the amount of all?

Ans. 1610 *bu.* 3 *pks.* 3 *qts.* 1 *pt.*

TIME.

Add the following sums together; 49 yrs. 320 da. 14h. 49 min. 37 sec.; 360 da. 19 h. 8 min. 45 sec.; 76 yrs. 200 da.; and 16 yrs. 150 da. 20h. 54 min. 45 sec.

OPERATION.

	(365)	(24)	(60)	(60)	
yrs.	*da.*	*h.*	*min.*	*sec.*	
49	320	14	49	37	
	360	19	8	45	
76	200	00	00	00	
16	150	20	54	45	
143	302	6	53	7	*Ans.*

Dem. Divide the amt. of sec. by 60, because 60 sec. make 1 min. Divide min. by 60, because 60 min. make 1 h. Divide h. by 24, because 24 hours make 1 day. Divide days by 365, because 365 days make 1 year.

D. was born in Boston, lived there 18y. 0 mo. 3w.; went to Hartford in 2 days, and lived there 3y. 6 mo.; went in 1 day to New-Haven, lived there 1y. 2 mo. 3w.; he then moved to Albany in 5 days, lived there 12y. 0 mo. 2w. 6d. What was his age, 3 weeks and 3 days after?

Ans. 34 *yrs.* 11 *mo.* 1*w.* 3 *da.*

MOTION.

Add together the following sums, viz: 9 S. 20 deg. 34 min. 37 sec.: 17 deg. 36 min. 44 sec.; 7 S. 28 deg. 39 min. 14 sec.; and 8 S. 24 deg. 38 min. 55 sec.

OPERATION.

	(30)	(60)	(60)	
S.	*deg.*	*min.*	*sec.*	
9	20	34	37	
	17	36	44	
7	28	39	14	
8	24	38	55	
27	1	29	30	*Ans.*

Note—In adding signs, reject all the twelves, and set down only the excess; because 12 signs complete the great circle of the Zodiac.

Add 11 S. 24 deg. 55 min. 25 sec.; 8 S. 19 deg. 59 min. 17 sec.; 8 S. 29 deg. 11 min. 18 sec.

*Ans.*29*S.* 14*deg.* 6*min.*

REVIEW.

There are four different processes of addition.

The *first* is *Simple Addition*, in which *ten* units of one order make *one* unit of the next higher order. Thus, 10 and 20 are 30.

The *second* is *Decimal Addition*, in which *decimal fractions* are added to each other. Thus, ,5 ,50 505 are added together.

The *third* is the Addition of *Vulgar Fractions*, in which *parts of units* are added to each other. Thus, $\frac{1}{5}$, $\frac{4}{5}$ and $\frac{2}{5}$ are added to each other.

The *fourth* is *Compound Addition*, in which other numbers besides ten, make units of higher orders. Thus, 13 units of the order of drams. make one unit of the order of ounces. Sixteen units of the order of ounces, make one of the pound order. 1 hundred of the pound order, make 1 hundred-weight.

1. How many kinds of Addition are there? How many units of one order make one of a higher order in S. Addition? Name the rule for Decimals, For Vulgar Fractions? For Compound Addition? Wherein do each of these operations differ? Give an example in each rule. Tell the method of performing Simple Addition—Decimal—Fractional—Compound. Why do you carry by different numbers in Compound Addition? How are the orders placed in each operation in Addition? In Decimals? In Vulgar Fractions? In Compound Addition?

COMPOUND SUBTRACTION.

Compound Subtraction is taking a less sum or quantity of different denominations from another. The denominations must be of the same kind as in Compound Addition. The borrowing depends upon the same principle, as in Simple Subtraction, only it is of different denominations.

Rule:—Place the less sum or quantity under the greater, so that those of the same denomination, may stand directly under each other.

2. Begin with the least denomination and if it exceed that directly above it, subtract it from as many of that denomination as shall equal a unit in the next higher, and to the difference add the upper denomination; when you have thus subtracted, add one to the next lower denomination.

3. But when the lower denomination does not exceed that directly above, you merely set down the difference, and proceed to the next without carrying.

MONEY.

	(10)	(10)	(10)	(10)
E.	$	*d.*	*cts.*	*m.*
470	1	8	4	1
387	3	2	8	4
82	8	5	5	7

What is the difference between 583 E, and 16 E. $8, 37 cts. 8 mills? *Ans.* 566 *E.* $1:62 *cts.* 2 *m.*

AVOIRDUPOIS WEIGHT.

	(16)	(16)
lbs.	*oz.*	*ds.*
46	13	10
27	15	14
18	13	12

If from a box of butter, containing 20 lbs. there be sold 10 lbs. 8 oz., how much will there be left? *Ans.* 9 *lbs.* 8 *oz.*

TROY WEIGHT.

	(12)	(20)	(24)
lbs.	*oz.*	*pwt.*	*grs.*
24	9	14	12
19	7	15	20
5	1	18	16

D. had 34 lbs. 9 oz. 10 pwt. of gold, and gave to his sons, 19 lbs. 0 oz. 15 pwt. 10 grs.; what had he left for his daughters? *Ans.* 15 *lbs.* 8 *oz.* 14 *pwt.* 14 *grs.*

APOTHECARIES' WEIGHT.

℔.	*oz.*	*drs.*	*sc.*	*grs.*
	(12)	(8)	(3)	(20)
40	7	3	2	7
24	4	5	2	8
16	2	5	2	19

Subtract 34℔. 9 oz. 4 drs. 2 sc. 16 gr., from 93℔. 10 oz. 5 drs. 1 sc. and 19 grs. *Ans.* 59℔. 1 *oz.* 2 *sc.* 3 *grs.*

CLOTH MEASURE.

yds.	*qrs.*	*na.*
	(4)	(4)
54	2	2
27	2	3
26	3	3

E. E.	*qrs.*	*na.*
	(5)	(4)
420	3	2
347	0	3
73	2	3

E. Fl.	*qrs.*	*na.*
	(3)	(4)
89	2	1
20	2	2
68	2	3

E. Fr.	*qrs.*	*na.*
	(6)	(4)
79	3	2
60	5	3
18	3	3

From a piece of cloth containing 47 yards, 2 quarters, a merchant sold 4 suits, each 6 yards 2 quarters; how much of the piece remained unsold? *Ans.* 21 *yds.* 2 *qrs.*

LIQUID MEASURE.

T.	*hhds.*	*gals.*	*qts.*	*pts.*	*gi.*
	(4)	(63)	(4)	(2)	(4)
88	3	55	3	0	0
48	0	62	2	0	2
40	2	56	0	1	2

C. bought wine, gallons-154, qts. 2, of A.; gals. 161, qt. 1, pt. 1, of B.; and sold gals. 39, qts. 2, pt. 1, to C.; and gals, 100, qts. 3, pt. 0, to D.; what had he left?

Ans. 175 *gals.* 2 *qts.*

DRY MEASURE.

bu.	*pks.*	*qts.*	*pts.*
	(4)	(8)	(2)
34	3	6	0
31	0	7	1
3	2	6	1

A. bought 368 bu. 3 pks. 5 qts. of wheat, and sold 188 bu. 2 pks. 6 qts.; what has he left? *Ans.* 180 *bushels*, 7 *quarts.*

A farmer has on hand 27 loads and 34 bushels of wheat: should he sell 19 loads, 10 bushels and 3 pecks; how much would remain unsold? *Ans.* 8 *loads*, 23 *bushels*, 1 *peck.*

LONG MEASURE.

deg.	*min.*	*fur.*	*po.*	*yds.*	*ft.*	*in.*	*bc.*
	(69½)	(8)	(40)	(5½)	(3)	(12)	(3)
58	48	7	28	4	2	2	2
36	25	6	39	0	2	8	0
22	23	0	29	3	2	6	2

Subtract 30 miles, 7 furlongs, 20 poles, from 45 miles; what then remains? *Ans.* 14 *mi.* 20 *po.*

LAND MEASURE.

A.	*r.*	*po.*
	(4)	(40)
36	3	28
28	2	39
8	0	29

E. had 500 A. 1 rood of land, and gave his son 150 acres, 3 roods, 25 poles; what had he left? *Ans.* 349 *A.* 1 *rood*, 15 *po.*

SOLID MEASURE.

	(50)	(1728)
T.	*ft.*	*in.*
44	44	884
39	39	982
5	4	1630

If from a pile of wood, containing 40 cords and 64 ft. there be sold 39 cords and 32 ft.; how much will there be left?

Ans. 1 *cord*, 32 *feet.*

TIME.

	(12	(4)	(7)
yrs.	*mo.*	*w.*	*da.*
176	8	3	4
91	9	2	6
84	11	0	5

W. went as an apprentice for 7 years, and has served 3 years 5 mo.; how long has he to stay? *Ans.* 3 *years* 7 *months.*

MOTION.

	(30)	(60)	(60)
S.	*deg.*	*min.*	*sec.*
6	25	45	38
4	29	38	49
1	26	6	49

Boston is 71 degrees 3 minutes, and the city of Washington is 77 degrees 43 minutes West longitude; what is the difference of longitude between the two places? *Ans.* 6 *deg.* 40 *min.*

REVIEW.

There are four kinds of Subtraction.

The *first is Simple Subtraction*, in which the minuend and subtrahend are whole numbers, and ten units of one order, make one unit of the next higher order.

The *second* is *Decimal Subtraction*, in which the minuend and subtrahend are Decimals.

The *third* is *Subtraction of Vulgar Fractions*, in which the minuend and subtrahend are vulgar fractions.

The *fourth* is *Compound Subtraction*, in which other numbers beside ten, make units of a higher order.

How many kinds of subtraction are there? What is S. Subtraction? Decimal? Vulgar Fractions? Compound? Rule for each—The minuend and subtrahend in S. Subtraction are what? In Decimals? In Vulgar Fractions? In Compound? How do you borrow and carry in each rule? Which is the larger sum *taken as a whole*, the minuend or subtrahend? Which if each order is taken *separately?* Name a case! Name a case in which the numbers in the orders of the subtrahend are larger than those of the minuend. What must be done in such a case?

COMPOUND MULTIPLICATION.

Is repeating a number of different denominations a certain proposed number of times; or it is the shortest method of performing compound addition where the same number is to be repeated a definite number of times.

RULE :—Make the compound number the multiplicand, and the simple number the multiplier; multiply the lowest denomina-

tion of the multiplicand, by the multiplier—divide the product by the number, which it takes to make one of the next superior denomination; set down the remainder—add the quotient to the product of the next denomination: reduce the sum—set down the remainder, and reserve the quotient as before. Proceed in this manner through all the denominations to the last, which is to be multiplied like a simple number.

MONEY.

Multiply 621 E. $3 and 5 dimes by 3.

OPERATION.

	(10)	(10	
E.	$	*d.*	
621	3	5	
		3	
1864	0	5	*Ans.*

Explanation.

3
3
10)10
E. 1. $0.

5
3
10)15
$1. 5d.

TROY WEIGHT.

Multiply 16 lbs. 9 oz. 19 pwt. 23 grs. by 11.

OPERATION.

	(12)	20)	(24)	
lbs.	*oz.*	*pwt.*	*grs.*	
16	9	19	23	
			11	
185	1	19	13	*Ans.*

Explanation.

23
11
24)253
pwt. 10 13 *g.*

19
11
20)219
oz. 10 19 *pwt.*

9
11
12)109
lbs. 9 1 *oz.*

A gilder bought 42 packs of gold leaf, each containing 1 lb. 2 oz. 10 pwts. 22 grs.; how much did they all contain?

Ans. 50 *lbs.* 10 *oz.* 18 *pwt.* 12 *grs.*

APOTHECARIES' WEIGHT.

Multiply 7℔ 10 oz. 7 drs. 2 sc. 19 grs. by 9.

OPERATION.

	(12)	(8)	(3)	(20)	
℔.	*oz.*	*drs.*	*sc.*	*grs.*	
7	10	7	2	19	
				9	
71	2	7	2	11	*Ans.*

What is the weight of 3 parcels of medicine, each weighing 1℔ 10 oz. 5 drs. 0 sc. 15 grs.

Ans. 5℔. 7 *oz.* 7 *drs.* 2 *sc.* 5 *gr.*

CLOTH MEASURE.

Multiply 19 yards 3 quarters 1 nail by 27

OPERATION.

	(4)	(4)	
yds.	*qrs.*	*na.*	
19	3	1	
		27	
534	3	3	*Ans.*

How many yards in 12 pieces of cloth, containing 18 yards, 2 quarters, 1 nail?

Ans. 222 *yards.* 3 *quarters.*

LIQUID MEASURE.

Multiply 1 hhd. 34 gals. 3 qts. by 7

OPERATION.

	(63)	(4)
hhds.	*gals.*	*qts.*
1	34	3
		7
10	54	1 *Ans.*

F. sold 45 barrels of cider, each 31 gallons, 2 quarts; find the gallons in the whole? *Ans.* 1417 *gallons*, 2 *quarts.*

DRY MEASURE.

Multiply 2 bushels 3 pecks 1 quart by 8

OPERATION.

	(4)	(8)
bu.	*pks.*	*qt.*
2	3	1
		8
22	1	0 *Ans.*

In 18 loads of wheat, each containing 23 bushels, 3 pecks, 6 quarts, how many bushels.

Ans. 430 *bushels*, 3 *pecks*, 4 *quarts.*

LONG MEASURE.

Multiply 36 leagues 2 miles 3 furlongs by 6.

OPERATION.

	(3)	(8)
L.	*mi.*	*fur.*
36	2	3
		6
220	2	2 *Ans.*

How far will a man travel in 4 days, at the rate of 24 miles, 4 furlongs, 4 poles per day? *Ans.* 98 *miles*, 16 *poles.*

LAND MEASURE.

Multiply 24 acres 2 roods 39 poles by 8.

OPERATION.

	(4)	(40)
A.	*r.*	*po.*
24	2	39
		8
197	3	32 *Ans.*

How much land is contained in 32 lots, each measuring 2 acres, 3 roods, 24 poles?

Ans. 92 *acres*, 3 *roods*, 8 *poles.*

SOLID MEASURE.

Multiply 175 cords 126 feet 1721 inches by 54.

OPERATION.

	(128)	(1728)
cords.	*ft.*	*in.*
175	126	1721
		54
9503	73	1350 *Ans.*

How many cords of wood in 9 piles, each containing 26 cords 98 feet? *Ans.* 240 *cords.* 114 *ft.*

TIME.

Multiply 7 months 3 weeks 5 days by 26.

OPERATION.

(12)	(4)	(7).
mo.	*w.*	*da.*
7	3	5
		26
17. 2	0	4 *Ans.*

If a man work 12 hours 45 minutes per day, for 9 days in succession, for how much time must he be paid? *Ans.* 114*h.* 45 *min.*

MOTION.

Multiply 1 S. 2 deg. 32 min. 42 sec. by 11.

OPERATION.

	(30)	(60)	(60)
S.	*deg.*	*min.*	*sec.*
1	2	32	42
			11
11	27	59	42 *Ans.*

A ship sailed at the rate of 1 deg. 59 min. 44 sec. each day, for 20 days; how far did she sail in that time? *Ans.* 39 *deg.* 54 *min.* 40 *sec.*

REVIEW.

There are four processes of multiplication.

The first is *Simple Multiplication*, where the factors are whole numbers, and ten units of one order make one unit of the next higher order.

The second is *Decimal Multiplication*, where one, or both the factors are decimals.

The third is the multiplication of *vulgar fractions* where one, or both the factors, are vulgar fractions.

The fourth is *Compound Multiplication*, where the multiplicand consists of orders, in which other numbers besides *ten*, make units of a higher order.

How many kinds of multiplication are there? What is S. Multiplication? Decimal? Vulgar Fractions? Compound? Give the Rule for each? The multiplicand is what in whole numbers? The multiplier? In Decimals? In Vulgar Fractions? In Compound what? What is the product divided by in compound numbers? What is set down? What is carried to the next order? Why? What is the object of Compound Multiplication? How do the orders increase in this rule?

REDUCTION.

Reduction is wholly performed by multiplication and division. It teaches to bring and change numbers of one denomination into another, without the least alteration of value, though it be in different terms.

Reduction is of two kinds, viz. Descending and Ascending; the former of which, is performed by multiplication, and the latter by division.

A General Rule.

1. All great names are brought into smaller of equal value, by multiplication; that is, by multiplying the given number by as many of the next less as make one of the greater; as pounds into ounces, days into hours, &c.

2. All small names are brought into greater of equal value by division; that is, by dividing the given number by as many of the lesser, as make one of the next greater; as drams into ounces, seconds into minutes, &c.

⁂ Reduction Descending and Ascending reciprocally prove each other. This is shown by the last example in each case.

MONEY.

In 46 E. $7: 29 cts. 4 m. how many mills?

OPERATION.

46×10=460+7=467×100=46700+29=46729×10=467290+4 =467294 mills *Ans.*

Proof.
1|0)46729|4 m.
1|00)467|29 cts.
1|0)46|7 dols.
E. 46-7-29-4

[Descending.]

In 64 E. $5, 9d. 7 cts. and 6m.; how many mills? *Ans.* 645976

In 9 E. $3, 73 cts.; how many cts. *Ans.* 9373

In 24 E.; how many cts.?

[Ascending.]

How many dimes in 900 mills? *Ans.* 9

In 150 cts. how many dimes? *Ans.* 15

In 24000 cts. how many E.?

AVOIRDUPOIS WEIGHT.

Pounds mult. by 16 are oz.	Drs. div. by 16 are oz.
Oz. mult. by 16 are drs.	Oz. div. by 16 are lbs.*

In 24 lbs. 11 oz. 9 drs. how many drams?

OPERATION.

24×16=384+11=395×16=6320+9=6329 *drs. Ans.*

Proof.
16)6329
16)395-9 drs.
lbs. 24-11-9

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* Let the instructor question the pupil, thus—years multiplied by 365 are what, &c.

[Descending.]

Bring 17 lbs. into ounces. *Ans.* 272

How many ounces in 2 lbs., and in 104000 lbs. *Ans.* 1664032

Bring 20 oz. into drams.

[Ascending.]

In 11468800 drs. how many pounds? *Ans.* 44800

In 48 oz. how many lbs.? *Ans* 3

In 320 drs. how many ounces?

TROY WEIGHT.

Pounds mult. by 12 are oz.
Oz. mult. by 20 are pwts.
Pwts. mult. by 24 are grs.

Grs. div. by 24 are pwts.
Pwts. div. by 20 are oz.
Oz. div. by 12 are lbs.

In 47 lbs. 10 oz. how many grains? *Ans.* 275520

How many ounces in 2 lbs. and in 360 lbs.? *Ans.* 4344

In 4 lbs. 4 oz. 6 pwts. 4 grs. how many grains?

Bring 12960 grs. into ounces. *Ans.* 27

In 1200 pwts. how many lbs.? *Ans.* 5

Reduce 25108 grains to pounds.

APOTHECARIES' WEIGHT.

Pounds mult. by 12 are oz.
Oz. mult. by 8 are drs.
Drs. mult. by 3 are sc.
Sc. mult. by 20 are grs.

Grs. div. by 20 are sc.
Sc. div. by 3 are drs.
Drs. div. by 8 are oz.
Oz. div. by 12 are lbs.

Bring 10℔ to grains. *Ans.* 57600

In 24℔ how many ounces? *Ans.* 288

How many drams in 2 oz. and in 4360 oz.? *Ans.* 34896

In 231℔ 3 oz. 5 grs. how many grains?

How many ℔ in 4896 scruples? *Ans.* 17

Reduce 67109 grs. to ℔. *Ans.* 11℔ 7 oz. 6 drs. 1 sc. 9 grs.

Reduce 1332005 grs. to pounds.

CLOTH MEASURE.

E. Fr. mult. by 6 are qrs.
E. E. mult. by 5 are qrs.
E. Fl. mult. by 3 are qrs.
Yds. mult. by 4 are qrs.
Qrs. mult. by 4 are na.
Na. mult. by 2¼ are in.

In. div. by 2¼ are na.
Na. div. by 4 are qrs.
Qrs. div. by 4 are yds:
Qrs. div. by 3 are E. Fl.
Qrs. div. by 5 are E. E.
Qrs. div. by 6 are E. Fr.

In 24 yds. how many qrs. and na.? *Ans.* 96 qrs. 384 na.

How many qrs. in 2, and in 26872 yds.? *Ans.* 107496

In 340 yds. 2 qrs. 3 na.; how many nails?

In 1520 na. how many yards? *Ans.* 95

Bring 3783 na. into yards. *Ans.* 236 yds. 1 qr. 3 na.

Reduce 5451 na. to yds.

DRY MEASURE.

[Descending.]
Bu. mult. by 4 are pks.
Pks. mult. by 8 are qts.
Qts. mult. by 2 are pts.

[Ascending.]
Pts. div. by 2 are qts.
Qts. div. by 8 are pks.
Pks. div. by 4 are bu.

Bring 7 bu. to pecks. *Ans.* 28

How many pecks in 2 bu. and in 32 bu.? *Ans.* 136

How many pecks in 4 bins, each 25 bu. *Ans.* 400

Bring 24 bu. 1 pk. 2 qts. 1 pt. to pints.

In 1024 pts. how many bushels. *Ans.* 16

In 900 bu. 3 pks. of corn, how many barrels, each containing 3 bu. 3 pks.?

Ans. 240 bar. 3 pks. over.

In 1557 pts. how many bu. pks. qts. and pts.

LIQUID MEASURE.

Tuns mult. by 2 are Pipes.
P. mult. by 2 are hhds.
Hhds. mult. by 63 are gals.
Gals. mult. by 4 are qts.
Qts. mult. by 2 are pts.
Pts. mult. by 4 are gills.

Gills div. by 4 are pints.
Pts. div. by 2 are qts.
Qts. div. by 4 are gals.
Gals. div. by 63 are hhds.
Hhds. div. by 2 are P.
P. div. by 2 are tuns.

Bring 5 hhds. into gallons. *Ans.* 315

How many pints in 2 qts. and in 480 qts.? *Ans.* 964

In 5 T. 1 P. 1 hhd. 61 gals. 3 qts. 1 pt. of port wine, how many pts.?

Reduce 6043 pts. of Madeira wine to hhds.?

Ans. 11 hhds. 62 gals. 1 qt. 1 pt.

Bring 10080 pints into tuns. *Ans.* 5

Reduce 12087 pts. of wine to T. P. hhds. and gals.

LONG MEASURE.

Deg. mult. by 69½ are mi.
L. mult. by 3 are mi.
Mi. mult. by 8 are fur.
Fur. mult. by 40 are po.
Po. mult. by 5½ are yds.
Yds. mult. by 3 are ft.
Ft. mult. by 12 are in.
In. mult. by 3 are b. c.

B. c. div. by 3 are in.
In. div. by 12 are ft.
Ft. div. by 3 are yds.
Yds. div. by 5½ are po.
Po. div. by 40 are fur.
Fur. div. by 8 are mi.
Mi. div. by 3 are L.
Mi. div. by 69½ are deg.

Reduce 27 ft. to inches. *Ans.* 324

How many fur. in 2 mi. and in 26784 mi.? *Ans.* 214288

Reduce 2 mi. 1 fur. 8 po. 3 yds. 2 in. into inches.

Bring 4352 inches into yards. *Ans.* 120 yds. 2 ft. 8 in.

How many mi. in 9187200 inches? *Ans.* 145

How many mi. fur. po. yds. and in. in 136334 in.?

LAND MEASURE.

Mi. mult. by 640 are A.
A. mult. by 4 are r.
R. mult. by 40 are rods.
Rds. mult. by 30¼ are yds.
Yds. mult. by 9 are ft.
Ft. mult. by 144 are in.

In. div. by 144 are ft.
Ft. div. by 9 are yds.
Yds. div. by 30¼ are rds.
Rds. div. by 40 are r.
R. div. by 4 are A.
A. div. by 640 are mi.

[Descending.]

In 50 A. how many roods and po. *Ans.* 200r. 8000 po.

In 34 A. 2r. and 17 po. how many po. *Ans.* 5537

Reduce 17 A. to r. and po. *Ans.* 68r. 2720 po.

In 15 A. how many poles? *Ans.* 2400

How many square acres in 2 square miles, and in 2342 mi.? *Ans.* 1500160

Reduce 11 A. 3r. and 17 po. to poles.

[Ascending.]

Bring 2720 po. into acres. *Ans.* 17

Bring 4392 po. into acres. *Ans.* 27 A. 1r. 32 po.

How many acres in 2400 po.? *Ans.* 15

In 25600 po. how many acres? *Ans.* 160

A field is 60 rods long and 48 rods wide; how many acres therein? *Ans.* 18

In 1897 po. how many A r. and po.

SOLID MEASURE.

Cords mult. by 128 are sol. ft.
Solid ft. mult. by 1728 are in.

Solid feet div. by 128 are cords
Inches div. by 1728 are ft.

In 10 cords of wood how many sol. ft.? *Ans.* 1280

In 25 cords of wood, how many in.? *Ans.* 5529600

Reduce 65 cords of wood to ft. *Ans.* 8320

In 16 cords of wood how many sol. ft.

In 2688 sol. ft. how many cords? *Ans.* 21

In 221184 sol. in. how many cords. *Ans.* 1

Bring 4608 cubic feet to cords? *Ans.* 36

How many solid feet in 345600 sol. in. and in 691200 sol. in.? *Ans.* 600

In 2048 sol. ft. how many cords of wood?

TIME.

Years mult. by 365 are days.
Days mult. by 24 are hours.
Hours mult. by 60 are min.
Min. mult. by 60 are sec.

Sec. div. by 60 are min.
Min. div. by 60 are hours.
Hours div. by 24 are days.
Days div. by 365 are years.

Bring 30 min. to sec. *Ans.* 1800

Bring 12 hours to min. *Ans.* 720

Bring 3 da. 5h. 29 min. into minutes. *Ans.* 4649

Reduce 2w. to days; and 318w. to days. *Ans.* 2240

Reduce 41 weeks to da.

In 144 months how many years? *Ans.* 12

In 105 days how many weeks? *Ans.* 15

In 203 days how many weeks? *Ans.* 29

In 668315 days how many years? *Ans.* 1831

In 287 days how many weeks?

REVIEW.

Reduction is changing units of one order, to those of another.

A unit of the order of eagles is how many units of the order of dollars? Of dimes? Thirty units of the order of dimes, is how many units of the order of dollars?

From the preceding exercises, you learn that *a unit of one order may contain several units of another order.*

How many units of the order of cents, are there in one unit of the order of dimes? How many units of the order of weeks, in one unit of the order of months?

Reduction Ascending, is changing units of a *lower* to a *higher* order. *Reduction Descending*, is changing units of a *higher* to a *lower* order.

In 4 gallons how many quarts? *One* gallon contains *four* quarts, and *four* gallons four times as much. 4 times 4 is 16.

In 4 gallons how many pints? Two bushels how many pecks?

Are the above sums in Reduction Ascending or Descending?

In 32 quarts how many gallons? *One* gallon contains 4 quarts. In 32 quarts therefore, there are as many gallons as there are 4's in 32. In 42 pints how many gallons?

Are the above sums in Reduction Ascending or Descending?

If twenty cents are changed to dimes, which kind of reduction is used? If eight feet are changed to inches, which kind of reduction is used? If four gallons are changed to pints, which reduction is used?

Reduce two yards to quarters, and which kind of reduction is it? Reduce twenty-four inches to feet, and which kind of reduction is it?

How many kinds of reduction are there of fractions? *Ans.* Reduction of whole numbers to fractions: of fractions to whole numbers: of mixed numbers to improper fractions: of fractions to a common denominator: to their lowest terms: a vulgar fraction to a decimal: and a decimal to a vulgar fraction.

CORRESPONDENT EXAMPLES.

1. At 9 cts. a pound, what cost 3 cwt. 2 qrs. 16℔ of sugar? *Ans.* $32 : 94

2. If it take 1 ounce of salts for a dose; what will 75 pounds amount to, at 4 cts. a dose? *Ans.* $36

3. A merchant sold 84 yds. 3 qrs. 2 na. of broadcloth at 50 cts. a nail; to what did it amount? *Ans.* $679

4. What will 1 acre 960 rods of land amount to, at $25 per acre? *Ans.* $175

5. What will 4 cords of wood amount to, at 25 cts. a cord foot? *Ans.* $16

6. At 6 cts. a quart, what will a hhd. of melasses come to? *Ans.* $15 : 12

7. At 320 cts. a yard, what will 64 nails of cloth cost? *Ans.* 12 : 80

8. A certain toper drank 1 gill of rum every forenoon, and 1 in the afternoon, for 6 years; in consequence of which, he died; how many hogsheads did he drink? *Ans. more than* 2 *hhd.*

9. At 6 cents a pint, what will 20 bu. 0 pks. 3 qts. 1 pt. of flax-seed cost? *Ans.* $77 : 22

10. What is the value of a silver cup, weighing 9 oz. 4 pwt. 16 gr. at 3 mills per grain? *Ans.* $13 : 29‘6

11. What would 2 pipes of Madeira wine amount to at 67 cts. per quart? *Ans.* $675 : 36

12. What would 93 reams and 1 quire of paper cost at ½ a cent per sheet? *Ans.* $223 : 32

13. C. sold 53 ells 1 qr. English, at 97½ cts. a yard; to what did the sale amount? *Ans.* $64 : 84 *nearly.*

14. B. sold wrought silver, lb 7 oz. 14 pwts. at 79 cts. an ounce; what did he receive? *Ans.* $15 : 56

15. If a pint of rum a day will kill a man in a year and a half, how many men would a cargo of 600 hogsheads kill in the same time? *Ans.* 552 +

16. If the digging of a mile of canal cost $6500; what will the digging of 30 miles, 7 furlongs and 21 poles cost? *Ans.* $201114 : 06'2½

17. What is the rent of 21 acres, 3 roods, 20 poles of ground, if the rent of 36 acres, 3 roods be $42? *Ans.* $25

18. What is the cost of a farm, containing 256 acres, 1 rood, 30 poles, at $6 : 80 per acre? *Ans.* $1743 : 77'5

19. What cost 135 acres, 3 roods, 20 poles, at $5 : 62½ per acre? *Ans.* $764 : 29$\frac{11}{16}$

COMPOUND DIVISION

Is finding how many times a simple number is contained in a compound number; and is the converse of compound multiplication.

Obs. A compound number may be divided by a simple number by regarding each of the terms of the former, as forming a distinct division, as 14 pounds, 11 ounces, divided by 9. But as each term of the dividend is not exactly divisible by the divisor, the remainders must be reduced to the next lower name and added to it in order to include the fractional parts.

Rule:—Divide each denomination in the dividend by the divisor; whenever there is a remainder in either of the denominations, reduce it to the next less, and add in those of the same name.

2. When the divisor is not contained in the first denomination, reduce it to the next lower, adding in those of a like name; then divide the same by the divisor, &c.

TROY WEIGHT.

Divide 23 pounds 7 ounces 6 pennyweights 12 grains by 7. *Ans.* 3 *lbs.* 4 *oz.* 9 *pwt.* 12 *grs.*

OPERATION.

7)23(3 lbs.	2×12+7=31	3×20+6=66	3×24+12=84
21	7)31(4 oz.	7)66(9 pwts.	7)84(12 grs.
2	28	63	
	3	3	

Twenty-four persons receive equally alike, 31 lbs. 11 oz. 16 pwt.; how much does each person receive?

Ans. 1 *lb.* 3 *oz.* 19 *pwts.* 20 *grs.*

APOTHECARIES' WEIGHT.

Divide 13℔ 1 oz. 2 drs. by 12. *Ans.* 1℔ 1 *oz.* 2 *sc.* 12 *grs.*

OPERATION.

12)13(1℔	1×12+1=13	1×8+2=10	10×3=30
12	12)13(1 oz.	12)10(0 drs.	12)30(2 sc.
1	12	6×24=144	24
	1	12)144(12 grs.	6

If the weight of 17 packages of medicine be 40℔. 10 oz. 7 drs. 2 sc. 11 grs.; what is the weight of each package?

Ans. 2℔ 4 *oz.* 7 *drs.* 3 *grs.*

CLOTH MEASURE.

1. Divide 572 yards 3 quarters 1 nail by 47.

Ans. 12 *yds.* 3 *na.*

2. If 174 yds. 1 qr. 2 na. be divided equally among 5 persons; what will be the share of each?

Ans. 34 *yds.* 3 *qrs.* 2 *na.*

LAND MEASURE.

1. Divide 51 acres 2 roods 22 poles by 51. *Ans.* 1 *A.* 2 *po.*

2. A man divided a piece of land, measuring 664 A. 1r. 22 po. equally among his seven sons; how much did each receive?

Ans. 94 *A.* 3*r.* 26 *po.*

LIQUID MEASURE.

1. Divide 2 hhds. 16 gals. 1 qt. 1 pt. by 67.

Ans. 2 *gals.* 1 *pt.*

2. F. divided 132 gals. among 88 men; what had each?

Ans. 1 *gal.* 2 *qts.*

DRY MEASURE.

1. Divide 420 bushels 3 pecks 2 quarts by 16.

Ans. 26 *bu.* 1 *pk.* 1 *qt.*

2. A gentleman wishes to put 130 bushels of apples into barrels containing 3 bushels 1 peck each; how many barrels does he need? *Ans.* 40

TIME.

1. Divide 120 months 2 weeks 3 days 4 hours 12 minutes by 111. *Ans.* 1 *mo.* 2 *da.* 10*h.* 12 *min.*

2. Divide 12 years 11 months 3 weeks 6 days 23 hours by 11. *Ans.* 1 *yr.* 2 *mo.* 5 *da.* 2*h.* 5 *min.* 27 *sec.*

MOTION.

1. Divide 17 deg. 23 min. 12 sec. by 8. *Ans.* 2 *deg.* 10 *min.* 24 *sec.*

2. A ship sailed 39 deg. 54 min. 57 sec. in 19 days; how far did she sail each day on an average? *Ans.* 2 *deg.* 6 *min.* 3 *sec.*

REVIEW.

There are four kinds of division.

The *first* is *Simple Division,* in which both the dividend and divisor are whole numbers, and ten units of one order, make one unit of the next higher order.

The *second* is *Decimal Division,* in which the dividend, or divisor, (or both) are decimal fractions.

The *third* is *Division of Vulgar Fractions,* in which the dividend, or divisor (or both) are Vulgar Fractions.

The *fourth* is *Compound Division,* in which other numbers besides ten, make units of higher orders. *When the dividend has several orders,* we divide each *separately,* beginning with the highest orders. *When one order of the dividend will not contain the divisor once, it is reduced and added to the next lower order, and then divided.* The *quotient and the remainder are always of the same order as the dividend.*

How many kinds of division are there? What is the divisor and dividend in whole numbers? In decimals? In Vulgar Fractions? In compound numbers? What is the Rule in each of these operations? Wherein do they differ? Give an example in each rule.

PROPORTION
OR
RULE OF THREE.

This rule is nothing more than the application of the two grand operative principles in Arithmetic, multiplication and division. It is called proportion, because there is an actual relative proportion existing between the given terms; and it is called the Rule of Three, because three terms are always given or implied in each question by which a fourth term or answer is found. This rule consists of two parts: *Single* proportion, and *Compound* proportion.

To denote that there is a proportion between numbers, they are written thus:— 6 : 4 : : 12 : 8,

Read, as 6 : is to 4 : : so is 12 : to 8.

Obs. That, of the three given numbers in any question you have two of them always of one kind; that is, if one be *money*, so is the other; or if one be *yards*, the other is also; and one of which must be the first, and the other the third term in stating; and *that* must always be the third term which moves the question, and the other of the same kind, must be the first; and the other term, which is of another denomination, always possesses the middle place, and is of the same kind with the answer.

If 18 yards of calico cost 5 dollars; what will 90 yards cost?

Here, in stating the question for solution, 90 must be the third term; because that is the number which asks the question, for it is required to know what 90 yards will cost; and the other number of the same kind, is to be the first, which is 18; and the last number, which is of the same kind with the number sought, possesses the middle place, and when stated for solution stands thus: yds. $ yds.
18 : 5 : : 90

Then multiply the second and third terms together; and divide the product by the first term; the quotient will be the answer.

Recapitulating the remarks already made, we have the following

Rule:—That must be the third term, which asks the question. The first and third terms must be of one name. The second term of divers denominations. Multiply the second and third terms together, and divide the product by the first term; the quotient thence arising will be the answer.

Obs. This rule is founded on the obvious principle, that the magnitude or result of any effect, varies constantly in proportion to the varying part of the cause; thus, the quantity of articles purchased, is in proportion to the money laid out; and the space gone over by a uniform motion, is in proportion to the time.

ILLUSTRATION.

Ex. If 6 yards of cloth cost $24; what cost 12 yards?

Ans. 48

STATEMENT AND OPERATION.	Proof by Analysis.
6 : 24 : : 12	6)24
12	4 *price of* 1 *yd.*
48	12
24	$48 *price of* 12 *yds.*
6)288	
Ans. $48	

It is plain that Direct Proportion or the Rule of Three Direct, is nothing more than an application of multiplication and division.

Dem. It is evident, that when we multiply 24 dollars, the price of 6 yards, by 12, the product is 6 times too much; then when we divide the product by 6, the first term, the quotient must be the price of 12 yards.

2. When we divide $24, the price of 6 yards, by 6, the quotient must be the price of one yard; and the price of one yard multiplied by 12, evidently gives the price of 12 yards.

QUESTIONS FOR SOLUTION.

1. If 8 yards of cloth cost $24; what will 96 yards cost? *Ans.* $288

2. If 6 pounds of tea cost $3 : 75; what cost 18 pounds? *Ans.* $11 : 25

3. If $50 will buy 40 sheep; how many sheep will $112 : 50 buy? *Ans.* 90

4. If a man can earn $64 in 4 months; how long must he work to earn $320? *Ans.* 20

5. If 4 men build a wall in 20 days, how many men would it require to build the same in 40 days? *Ans.* 2

6. How many men must be employed to finish a piece of work in 15 days, which 5 men can do in 24 days? *Ans.* 8

7. If 5 tuns of hay will beep 25 sheep over the winter; how many sheep can be kept on 12 tuns, at the same rate? *Ans.* 60

Obs. Writers on Arithmetic have distinguished the *Rule of Three* into two kinds, viz.: Direct and Inverse; but this is unnecessary, when the nature of proportion and the enunciation of the questions are well understood. Furthermore, these useless distinctions may be avoided, and the obscurity and labor greatly abridged by adopting one general rule, and making all propositions concur with Direct Proportion.

As the difficulty in this rule consists in the manner of stating the proportion, the following rules and illustrations, will enable the pupil to solve any question that may occur.

Rule:—Put that term which is of the same name and kind with that of the answer, in the third place, for a multiplier.

2. Then, if from the nature of the question, the answer must be more than the third term; place the larger of the two remaining terms in the second place for a multiplicand, and the other in the first place for a divisor.

3. Multiply the second and third terms together, and divide the product by the first, and the quotient will be the answer.

Ex. If 6 apples cost 9 cents, what will 12 apples cost?

STATEMENT AND OPERATION.	Proof by Analysis.
6 : 12 : : 9	6) 9
9	15
6)108	12
Ans. 18 cts.	*Ans.* 18 cts.

Dem. In this question, 9 cents, is of the same name and kind with the answer, and stands in the third place; the answer must be more than the third term, because 12 apples will, at the same rate, cost more than 6; therefore, 12 occupies the second place, and the remaining term, 6, the first place. Then 12 multiplied by 9, equals 108; which divided by 6, gives 18 cents, the answer in the same name and kind with the third term.

QUESTIONS FOR SOLUTION.

1. How many men must be employed to do a piece of work in 5 days, which 8 men can do in 15 days? *Ans.* 24

2. If 100 workmen can finish a piece of work in 12 days; how many are sufficient to do the same in 3 days? *Ans.* 400

3. How much land, at $2 : 50 per acre, must be given in exchange for 360 acres, at $3 : 75 per acre? *Ans.* 540

4. If 23 head of cattle can pasture in a field for 57 days, how long can 17 head of cattle pasture in it? *Ans.* $77\frac{2}{17}$

5. If 15 men can do a piece of work in 84 days; how many men must be employed to perform it in 30 days? *Ans.* 42

Rule:—2. When from the nature of the question, the answer must be less than the third term, then the smaller of the two remaining terms must have the second place, and the larger, the first place.

Thus, If 18 cents buy 12 apples, how many will 9 cents buy? *Ans.* 6

STATEMENT AND OPERATION.	Proof by Analysis.
As 18 : 9 : : 12	12)180
9	15)90
18)108(6 *Ans.*	*Ans.* 6
108	

Dem. Here the answer is in apples, and the third term is in apples, and because 9 cents, at an equal rate, will buy less than 18 cents, the answer will be less than the third term; therefore 9, the smaller of the remaining terms, takes the second place, and 18, the larger, the first place. Hence it is evident from both examples, that a large multiplier, and a small divisor, produce a large quotient; while a small multiplier, and a large divisor, yield a small quotient. All questions in Single Proportion may be stated and worked in the above mode.

METHOD OF PROOF.—In all cases, if the sum be right, the product of the first and fourth terms will be equal to the product of the second and third; as in the preceding examples, multiply 18 by 6, and it is equal to 108; so 9 multiplied by 12 is equal to 108.

QUESTIONS FOR SOLUTION.

1. If 6 men can do a piece of work in 10 days; in how many days will 12 men do it? *Ans.* 5

2. If 3 teams can plow a field in 12 days; in what time will 9 teams do it? *Ans.* 4

3. If 6 men can mow a field of grass in 12 days; in what time will 24 men do it? *Ans.* 3

4. If 8 men will do a piece of work in 24 days; in what time will 16 men do it? *Ans.* 12

5. If 25 men can do a piece of work in 17 days; in how many days will 38 men do it? *Ans.* 11 7/38

6. D. bought goods to the amount of $560, and gained by the sale, $190 : 40; how much would he have gained had he laid out $150? *Ans.* $51

PARTICULAR CASES.

1. A. can do a job of work in 10 days, B. can do the same in 15 days; how long will it take both together to do the same work? *Ans.* 6 *days.**

2. There are 5 mills, the first grinds 7 bushels of corn in an hour, the 2d 5, the 3d 4, the 4th 3, and the 5th 1; I demand in what time the 5 mills will grind 500 bushels, if they work all together? *Ans.* 25 *hours.*

3. If a man can dig a trench in 15 days, and a boy can dig the same trench in 18 days; in what time can they both dig it? *Ans.* 8 6/33 *days.*

4. If A. can do a piece of work alone in 10 days, and A. and B. in 7 days; in what time can B. do it? *Ans.* 23 1/3 *days.*

5. A. can do a piece of work in 10 days, B. alone in 13. Set them both about it together; in what time will it be finished? *Ans.* 5 15/23 *days.*

OBS. It sometimes happens, that the given terms in proportion, are of compound terms; as yards, quarters, nails, &c. in which case, the first and second terms must both be reduced to their lowest or like name, and the third term to its lowest, or some convenient name. Then the answer will appear in the same name, and of the same kind with the third term.†

1. If 175 pieces of India cotton cost $1924 : 65, and each piece contain 39 yards; what does it cost per yard? *Ans.* $0 : 28'2

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* See Key for statements. Page 230.

† All questions relative to Proportion can be solved by the rule of Analysis.

2. If 4 yards 3 quarters cost \$19; what will 3 yards, 2 quarters cost? *Ans.* \$14

3. If 5 yards of cloth cost \$14 : 02; what must be given for 9 pieces, each piece containing 21 yards 1 quarter? *Ans.* \$536 : 26‘5

4. If the price of 1 acre of land be \$ 18 : 25; what will 50 acres, 2 roods, 20 poles come to? *Ans.* \$923 : 90‘5

5. If 36 bushels of wheat in one year yield 216 bushels; how much will 36 quarters yield in 6 years?
[8 bu. make 1 qr] *Ans.* 10368 *bush.*

6. If 57 lb. 7 oz. of spices be bought for \$17 : 25; what must I pay for 87 lb. 10 oz. 7 pwt.? *Ans.* \$26 : 32‘9

*Method of Assessing Town Taxes.**

RULE:—As the valuation of the whole property in the town, is to the tax, so is the valuation of each man's estate, to the tax.

1. The valuation of the property in a certain town according to the town's inventory, is \$610000, and the tax levied on that town, is \$3050; what is B's tax, whose estate according to the inventory, is valued at \$1200? *Ans.* \$6

2. What is the tax on \$5679, at 10 cents on a dollar? *Ans.* \$567 : 90

3. If a tax of \$650 be laid on a certain school district, for erecting a school house, and the inventory of all the estates, in that district, amount to \$120000; what must F. pay whose estate is \$2100? *Ans.* \$11 : 37‘5

4. Suppose a certain town is to pay a tax of \$6145 : 88, and

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* A TAX is a sum of money required of individuals by a government, for the use of the nation; or by a town, county, society, or corporation, for the purpose of defraying the expenses of the whole, in their collective capacity. A tax is sometimes imposed by government, of a small sum upon every male citizen above a certain age, and this being the same to all, and not varying with the amount of an individual's property, is called, *a tax of so much a head*, or, *of so much a poll;* the word *poll*, meaning *head*. This is commonly abbreviated into the more concise expression POLL-TAX. In some States no poll-tax is allowed. Other taxes are usually rated or *assessed* on property. Property is of two kinds, *real* and *personal*. Real property, or, as it is usually called, *real estate*, consists of possessions, which are *fixed* and *immovable;* as *lands* and *buildings*. Personal property, comprehends *all other possessions*, which, of course, are *movables;* as *furniture, jewels, money, cattle*, &c. Taxes are assessed on property, both real and personal. When, therefore, *no particular kind* of property is mentioned, property *generally*, or, *of all kinds*, is understood. In order to *assess* a tax of this kind, then, or, in other words, *to apportion or rate it justly* among the individuals of a town or society, we must have a list, or *inventory* of all the property, both real and personal on which it is to be assessed. If there is likewise a poll-tax included in the whole amount given, we must also have the whole number of *polls*, or rateable *heads*, or *persons*. We must then find the number of all the poll taxes, or the *whole poll-tax*, which we must subtract from the *whole given tax*, and the remainder is to be assessed on property.

the whole property of the town is valued at $153647; how much must a man pay, whose property is valued at $23475 : 67?

Ans. $939 : 02'7

OBS. The above exs. can be solved by Analysis.

RETROSPECTIVE OBSERVATIONS.

The great property of every arithmetical proportion is this; that the sum of the second and third terms is equal to the first and fourth. This property, which we must particularly consider, is expressed by saying that the sum of the *means* is equal to the sum of the *extremes*. Thus, since 2—7=5. 9—14=5, we have 7+9=16. 2+14=16. and the sum we find is 16 in both.

And conversely, if four numbers, 2, 7, 9, 14, are such, that the sum of the second and third is equal to the sum of the first and fourth, that is, if 7+9=16. 2+14=16, we conclude, without a possibility of mistake, that these numbers are in arithmetical proportion.

When in arithmetical proportion, the second term equals the third, we have only three numbers; the property of which is this, the difference between the first and the second number is equal to the difference between the second and the third.

The three numbers 19, 15, 11, are of this kind, since

19—15=4. 15—11=4.

The foundation of Proportion is this. We suppose three numbers given and seek a fourth, which may be in geometrical proportion; so that the first may be to the second, as the third is to the fourth.

It matters not whether the second term be multiplied by the third, or the third by the second, only let one of them be multiplied by the other, and the product divided by the first; the quotient will be the answer.

Ex. If 4 cords of wood cost 8 dollars, what will 16 cords cost?

4 : 16::8 : 32; for 16×8=128÷4=32 *Ans.*

OBS. 1. It may be observed, that of the four terms employed in the proposition, two are referred to wood and two to money. And that they are proportionate; that is as wood is to wood, so is money to money; or as wood is to money, so is wood to money.

OBS. 2. The principles upon which proportion is founded, may be thus illustrated.

If four numbers are proportional, the product of the *extremes*, is equal to the product of the *means*. Therefore, a division, either of the product of the extremes, or of the product of the means, by the first extreme, will give the other extreme.

Thus: as 4 : 8::16 : 32.

And 32×4=128, the product of the extreme.

And 8×16=128, the product of the means.

Now the last product divided by the first extreme, (128÷4=32,) gives the other extreme, and the first product divided by the first means, (128÷8=16,) gives the other means. Hence, the propriety of multiplying the second and third terms together, and dividing the product by the first term.

COMPOUND PROPORTION.

Compound Proportion, or as it is sometimes called, the Double Rule of Three, teaches to resolve by one statement such questions as require two or more statements by Single Proportion, and hence the rule has derived its name. In this rule there is always an odd number of terms given, as five, seven, nine, &c. In questions, generally under this rule, there are five terms given to find a sixth; the three first terms are a supposition, the two last a demand.

Rule:—1. Put the term of the supposition which implies the principal cause of action, gain or loss, in the first place.

2. That term which relates to time, distance, place, &c. in the second place; and the other suppositive term in the third place.

3. Put the two remaining terms of demand directly under those of the same name and kind with the supposition.

4. If the blank place for the term sought, fall under the third term, then multiply the first and second terms together for a divisor, and the other three for a dividend; the quotient will be the answer.

5. But if the blank place fall under the first or second term, multiply the third and fourth terms together for a divisor, and the other three for a dividend; the quotient will be the answer.

Ex. If 2 men can do 12 rods of ditching in 6 days, how much will 8 men do in 24 days?

OPERATION.

2 : 6 : 12 Then, 2×6=12, divisor.
8 : 24 12×8=96×24=2304, dividend.
And 2304÷12=192 *Ans.*

1. If 7 men can reap 84 acres of wheat in 12 days, how many men can reap 100 acres in 5 days, at the same rate?
Ans. 20

2. When the carriage of 24 cwt. for 45 mi. is $18, how much will it cost to carry 76 cwt. 121 mi.? *Ans.* $153 : 26

Obs. There is another method of stating questions in this rule, which is sometimes preferred.

Rule:—1. Put that term which is of the same name and kind with the answer, in the third place.

2. Take one term from the supposition, and one from the demand, both of the same name and kind and put them in the first and second places, as in single proportion.

3. Take the two remaining terms, and place each under its like term in the first and second places; multiply the terms in the second place together, and their product by the third term for a dividend; multiply the terms in the first place together for a divisor, the quotient will give the answer.

Ex. If 6 men eat 10 pounds of bread in 8 days, how much will 12 men eat in 24 days?

OPERATION.

$$\begin{array}{l}6:12\\8:24\end{array}::10$$

Then, 12×24=288×10=2880, dividend.
6×8=48, divisor. 2880÷48=60, *Ans.*

1. If $100 principal in 12 months gain $6 interest, what principal will gain $18 in 15 months? *Ans.* $240

2. If 4 men mow 48 acres in 12 days, how much will 8 men mow in 16 days. *Ans.* 128 *acres.*

3. If $4 pay 8 men for 3 days' work, how long must 20 men work for $40? *Ans.* 12 *days.*

4. If $100 gain $6 interest in 12 months, what will $400 gain in 8 months? *Ans.* $16

NOTE. There is a method of contracting operations, viz.: Divide the two second terms by the two first, respectively, and multiply the product of the quotient by the third term.

Ex. If 6 men eat 10 pounds of bread in 8 days, what will serve 12 men for 24 days?

OPERATION.

$$\begin{array}{l}6:12\\8:24\end{array}::10$$

12÷6=2.
24÷8=3×2=6×10=60, *Ans.*

Any question in Compound Proportion, may be answered by two statings in Single Proportion.

Ex. If $100 in 12 months, gain $6, what will $600 gain in 9 months?

OPERATION.

Thus:

$$\begin{array}{r}100:6::600\\ 6\\ \hline 100)3600(\$36\\ 300\\ \hline 600\\ 600\end{array}\qquad\begin{array}{r}12:36::9\\ 9\\ \hline 12)324\\ \hline \$27\ \textit{Ans.}\end{array}$$

Thus we see the question solved by Single Proportion, in plain and easy statements: More simple by multiplication and division, or Analysis,

Thus: 100)6:00(6 *cts. the gain on a dollar for one year.*

$$\begin{array}{r}600\\ 6\\ \hline 36{:}00\end{array}$$

6 00
12)36:00 *the gain on* $600 *for a year.*
$3:00 *the gain on* $600 *for a month.*
9
$27 *the gain of* $600 *for* 9 *months.*

1. If 10 persons expend $9 in 12 weeks, how much will serve 20 persons 24 weeks? *Ans.* $36

2. If $100 in one year gain $6, what will $400 gain in 9 months? *Ans.* $18

3. If 7 men can build 84 rods of wall in 6 days, in how many days will 10 men build 140 rods? *Ans. 7 days.*

4. If 50 men consume 15 bushels of grain in 40 days, how much will 30 men consume in 60 days? *Ans.* 13½ *bush.*

QUESTIONS ON PROPORTION.

1. What is Proportion, or the Rule of Three?—*V. p.* 178. Why called Proportion? Why Rule of Three? How is the rule divided? How is the proportion between numbers written?—Of what kind are the given numbers?—Obs., *p.* 179. How placed in stating? Where is the third term placed? which terms multiplied together? which used for a divisor? What is the rule of operation? *V. p.* 179.

2. On what obvious principle is Proportion founded.—Obs., *p.* 179. What two Rules are applied to the performance of an operation? What in the Note relative to Analysis? *V. p.* 182. How is the first Ex. demonstrated? *V. p.* 180.

3. Is there any necessity for making a distinction between Direct and Inverse Proportion? *V. p.* 180. Why not? Repeat the Rule for stating all questions, whether in Direct or Inverse Proportion? *V. p.* 180. The proof of the statement? *V. p.* 182. What is the principal difficulty in this rule? *V. p.* 180. How obviated? How is this performed? Can the Rule of Three be dispensed with? *Ans. Yes.*

4. What is the property of proportion? How is it expressed? How can we find without the possibility of a mistake, that numbers are in Arithmetical proportion? What is the foundation of proportion? Is it of any consequence which terms we multiply together, the second by the third, or the third by the second? What is observed in the 1st Obs., p. 184? What in the 2d? What if the terms be compound? *V. p.* 182, &c.

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1. What does Compound Proportion teach? How many terms are given in this rule? Repeat the Rule of operation?—*p.* 185. There is another method, what? the rule?—*p.* 185. What is said in the note respecting contracting operations p. 186? Can any question in Compound Proportion be solved by two or more statements in Single Proportion?—*V. operation, p.* 186. Can questions in this rule be reduced back to simple multiplication and division without any formal statement? *V. operation, p.* 186.

DUODECIMALS.

Duodecimals are parts of a foot, which decrease in a twelvefold proportion. This measure is applied to the admeasurement of joiner's work, and the solid contents of bodies, &c.

They are called Duodecimals, from the Latin word *duodecim*, which signifies *twelve*. Duodecimals are commonly used only for measuring *length*, *surface* and *solidity*.

A lineal, square, or solid foot, is, therefore, considered the unit, or whole number. The lower denominations, or orders, are of course fractions of a foot.

The terms are,	**12 fourths (⁗) make**	**1 third,**	**marked**	**‴**
	12 thirds	**„**	**1 second,**	**„**	**″**
	12 seconds	**„**	**1 inch,**	**„**	**in.**
	12 inches	**„**	**1 foot,**	**„**	**ft.**

ADDITION.

Rule :—Place the numbers, and proceed as in Compound Addition, observing to carry one for every twelve.

10*ft.*	5*in.*	7″	11‴	6⁗
15	9	5	7	7
16	5	1	2	9
12	6	6	5	10
55	2	9	3	8

37*ft.*	8*in.*	11″	6‴	9⁗
42	10	5	3	11
18	8	1	5	10
19	3	2	9	10
118	6	9	2	4

3. Four boards measure as follows: 17 ft. 5 in. 6″; 18 ft. 10 in. 5″ 8‴: 21 ft. 10 in. 4″ 10‴ 11⁗; and 24 ft. 10 in. 9″: what is the amount? *Ans.* 83 *ft.* 1 *in.* 1″ 6‴ 11⁗

SUBTRACTION.

Rule :—Place the numbers, and proceed in the operation, the same as in compound subtraction, observing to borrow twelve when necessary, and carry one,—thus:

30*ft.*	5*in.*	6″	8‴	10⁗
16	8	7	5	6
13	8	11	3	4

330*ft.*	10*in.*	8″	1‴	6⁗
132	8	10	5	9
198	1	9	7	9

3. D's. stock of boards measures 416 ft. 8 in. 9″, and E's. 341 ft. 2 in. 9″: what is the price of the difference, at 3¾ cents a foot? *Ans.* $2 : 83·1

MULTIPLICATION.

Rule :—Place the terms of the multiplier under the corresponding terms in the multiplicand; multiply each term in the multiplicand by each term in the multiplier, placing the product, when multiplying by feet, directly under the multiplicand,—when multiplying by inches, one place to the right hand,—when multiplying by seconds, two places,—by thirds, three places;—observing to carry one for every twelve: add the several numbers in the order in which they stand, carrying one for every twelve—this amount will be the product.

Thus : Multiply	7*ft.*	3*in.*	2″		
By	2	7	3		
	14	6	4		
	4	2	10	2‴	
		1	9	9	6⁗
	18	10	11	11	6

Note. Here it appears that
feet multiplied by feet produce feet;
feet " by inches " inches;
feet " by seconds " seconds;
inches " by inches " seconds;
inches " by seconds " thirds;
seconds " by seconds " fourths.

Obs. The above operation is the ordinary method of solving questions in Duodecimals: but the result is evidently the same, if we begin by multiplying the lowest denomination of the multiplicand, and divide each product by 12, and place each remainder under its like name, as will be seen by comparing the same example, performed according to the directions just given; which is preferable, as it is more strictly conformable to what takes place in the multiplication of numbers accompanied by decimals. This mode of operation is much easier for the pupil. It was taken from Lacroix.

1st. Example by Lacroix's Method.

Multiply			7*ft.*	3*in.*	2″
By			2	7	3
		1	9	9	6
	4	2	10	2	
	14	6 .	4		
	18	10	11	11	6 *Ans.*

	ft.	*in.*	″		*ft.*	*in.*	″		*ft.*	*in.*	″	‴
Mult.	7	2	8	by	5	2	6	*Ans.*	37	7	4	8
Mult.	12	4	0	by	2	7	0	*Ans.*	31	10	4	0
Mult.	14	6	0	by		5	8	*Ans.*	6	10	2	0
Mult.	28	2	0	by	24	6	0	*Ans.*	690	1	0	0

2. Multiply 9 feet 6 inches, by 4 feet 9 inches. *Ans.* 45 *ft.* 1 *in.* 6″

3. Multiply 8 feet 11 inches, by 7 feet 10 inches? *Ans.* 69 *ft.* 10 *in.* 2″

4. Multiply 9 feet 8 inches, 6″, by 7 feet 9 inches, 3″ *Ans.* 75 *ft.* 5 *in.* 3″ 7‴ 6⁗

5. How many square feet in a board 10 feet 8 inches long, and 1 foot 5 inches broad? *Ans.* 15 *ft.* 1 *in.* 4″

6. How many square feet in a board, 14 feet 9 inches long, and 2 feet 6 inches wide? *Ans.* 36 *ft.* 10 *in.* 6″

7. What are the contents of a door 6 feet 9 inches 3″ long, and 3 feet 5 inches wide? *Ans.* 23 *ft.* 1 *in.* 7″ 3‴

8. How many square feet in a flag stone, 5 feet 10 inches long, and 4 feet 7 inches broad? *Ans.* 26 *ft.* 8 *in.* [illegible]″

9. Find the square feet in a board 17 feet 7 inches long, and 1 foot 5 inches wide? *Ans.* 24 *ft.* 10 *in.* 11″

10. What is the price of a marble slab, whose length is 5 feet 7 inches, and breadth 1 foot 10 inches, at one dollar per foot?
Ans. $10 : 23‘5

11. What will the painting of a floor come to, at 10 cts. per square yard, allowing the floor to be 21 feet 8 inches long, and the breadth 14 feet 10 inches? *Ans.* $3 : 57

Note. Divide the square feet by 9, and the quotient will be square yards.

12. What is the contents of 12 windows, each measuring 3 feet 10 inches long, 2 feet 8 inches broad; what will the glazing come to at 25 cents per foot.
Ans. 122 *ft.* 8 *in.* $30 : 66‘6 *amount.*

Obs. In computing solid measure, the given length must be multiplied by the given breadth, and that product by the given height; the last product will be the answer.

1. How many solid feet in a pile of wood 8 feet long, 3 feet wide, and 2 feet 8 inches high? *Ans.* 64 *ft.*

2. A. bought a load of wood, which was 9 feet 6 inches long, 8 feet 4 inches wide, and 3 feet 7 inches high; what did it want of a cord? *Ans.* 14 *ft.* 6 *in.* 4″

3. How much wood in a pile 3 feet, 8 inches wide, 3 feet 5 inches high, and 6 feet 7 inches long?
Ans. 82 *ft.* 5 *in.* 8″ 4‴

4. What are the contents of a solid stick of timber, 7 feet 6 inches long, 1 foot 3 inches wide, and 1 foot 10 inches thick?
Ans 17 *ft.* 2 *in.* 3″

MENSURATION.

Mensuration has reference to that branch of common arithmetic which treats of the admeasurement of surfaces, and the relative magnitude of bodies.

Rule:—When the length of a board or plank is in feet, and the width in feet, multiply them for the contents in feet.

When the length is in feet and the width in inches, multiply the length by the width, and divide the product by 12 for feet.

When the length of timber or joist is in feet, and the side or sides in inches, multiply the *product* of the sides by the length, and divide by 12 for feet, board measure.

Ex. 1. In a board or plank 19 feet long, and 2 feet wide, how many feet? *Ans.* 38

OPERATION.

19×2=38

2. In a board or plank 23 feet long, and 17 inches wide, how many feet? *Ans.* 32 *ft.* 7 *in.*

OPERATION.

$23\times17=391+12=32$ 7 in.

3. How many feet in a joist, 8 inches square and 30 feet long? *Ans.* 160

OPERATION.

$8\times8=64\times30=1920+12=160$

Require the superficial feet in the following lot of boards, viz:

		Answers.	
length.	width.	feet.	inches.
20 feet.	9 inches.	15	0
40	6	20	0
36	12	36	0
30	15	37	6
15	18	22	6

Require the contents of the following pieces of timber in board measure, the length and side of the square being given.

		Answers.	
length.	square.	feet.	inches.
30 feet.	6 inches.	90	0
30	9	202	6
30	23	1322	6
28	16	597	4

To calculate interest on notes, bonds, &c. on which partial payments have been made.—The following Rule was established by the Superior Court of Connecticut, in 1784.

Rule:—" Compute the interest to the time of the first payment, if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and, in like manner from one payment to another, till all the payments are absorbed; *provided,* the time between one payment and another be one year or more. But if any payment be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation for one year, add it to the principal; and compute the interest on the sum paid, from the time it was paid, up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added as above. If any payment be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum, for any period."—*Kirby's Reports, p.* 49.

1. Suppose a note was given on demand for 1000 dollars, dated February 1, 1825. On which were the following indorsements,

1.	April 1, 1826, Received,	$80
2.	August 1, 1826,	30
3.	Oct. 1, 1827,	10
4.	December 1, 1827,	600
5.	May 1, 1828,	200

What was the balance due on the note, Oct. 1, 1828, at 6 per cent? *Ans.* $263 : 93

$1000	Principal.
70	Interest to the 1st ind't., being 1 year and 2 mo.
1070	Amount.
80	1st Indorsement.
990	Principal remaining due, April 1, 1826.
99	Interest to the 4th ind't., being 1 yr. and 8 mo
1089	Amount to do.
30	2d Indorsement.
2 : 40	Interest to the 4th Indorsement.
32 : 40	
10 :	3d Ind't., which does not bear in't. because 1 y's. in't. has accrued on new principal.
600 :	4th Indorsement.
642 : 40	Deduct from 1089 dollars.
446 : 60	Principal remaining due, Dec. 1, 1827.
22 : 33	Interest to Oct. 1, 1828, being 10 months.
468 : 93	Amount to do.
200	5th Indorsement.
5	Interest to Oct. 1, 1828, being 5 months.
205	Amount—deduct from 468 : 93 cts.

$263 : 93 cts. Balance due, Oct. 1, 1828.

2. Suppose a bond or note dated May 10th, 1824, was given for 2000 dollars, interest at 6 per cent, upon which were the following indorsements, viz:

1.	March 10, 1825, Received,	$800
2.	May 10, 1826,	400
3.	Sept. 10, 1827,	300

What remained due January 10, 1829? *Ans.* $831 : 57‘5

The following Rule was established by the Superior Court of the State of Massachusetts in 1821.

Rule:—"Cast the interest up to the first payment, and if the payment exceed the interest, deduct the excess from the principal, and cast the interest upon the remainder to the time of the second payment. If the payment be less than the interest, place it by itself, and cast on the interest to the time of the next payment, and so on, until the payments exceed the interest, then deduct the excess from the principal, and proceed as before."—*Mass. Reports, Vol.* 17, *page* 418.

EXAMPLES.

1. Suppose a note was given on demand for $1000, dated February 1, 1825, on which were the following indorsements.

1.	April 1, 1826, Received,	$80
2.	August 1, 1826,	30
3.	October 1, 1827,	10
4.	December 1, 1827,	600
5.	May 1, 1828,	200

What remained due on the note, Oct. 1, 1828, at 6 per cent?

Ans. $265 : 87‘2

$1000	Principal.
70	Interest to the first ind't., being 1 yr. 2 months.
1070	Amount to do.
80	1st Indorsement.
990	Principal remaining due, April 1, 1826.
19 : 80	Interest to the 2d Indorsement, being 4 mo.
1009 : 80	Amount to do.
30 :	2d Payment.
979 : 80	Principal remaining due, Aug. 1, 1826.
68 : 58‘6	Interest to the 3d Indorsement.
9 : 79‘8	Interest to the 4th ind't., on same principal; the interest exceeding the payment.
1058 : 18‘4	Amount to the 4th Indorsement.
10	3d Indorsement.
600	4th Indorsement.
610	Sum of the 3d and 4th Ind't., deducted.
448 : 18‘4	Principal remaining due, Dec. 1, 1827.
11 : 20‘4	Interest to the fifth payment.
459 : 38‘8	Amount to do.
200 :	5th Indorsement.
259 : 38‘8	Principal remaining due, May 1, 1828.
6 : 48‘4	Interest to Oct. 1, 1828.
$265 : 87‘2	Balance due on note, Oct. 1, 1828.

2. Suppose a bond or note was given for 1200 dollars, at 6 per cent interest, dated Oct. 15th, 1826, on which were the following payments, viz. Oct. 15th, 1827, Received 1000 dollars, April 15th, 1828, Rec'd. 200 dollars. What remained due Oct. 15th, 1828?

Ans. $82 : 56‘4

The following Rule is established for the practice of the Courts in the State of New York:

RULE :—" The *Rule* for casting Interest, where partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due. If the payment exceeds the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due. If the payment be less than the Int., the surplus of interest must not be taken to augment the principal, but interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied towards discharging the principal; and interest is to be computed on the balance of principal as aforesaid."—*Johnson's Chancery Reports, Vol.* I, *page* 17.

1. Suppose you have a bond against B. for 1000 dollars, dated May 15th, 1821, upon which you find the following Indorsements, viz.

			y.	m.	d.
1.	Sept. 20th, 1822. Rec'd.	$150 : 60 cts.	1	4	5
2.	Oct. 25th, 1824 - -	200 : 90	2	1	5
3.	July 11th, 1826 - -	75 : 20	1	8	16
4.	Sept. 20th, 1827 - -	112 : 11	1	2	9
5.	Dec. 25th, 1828 - -	105 : 00	1	3	5

What remained due upon the bond May 20, 1829? Interest to be cast at 7 per cent? *Ans.* $869 : 19'9

		y.	mo.	da.
$1000	Principal.			
94 : 30'5	Int. to the 1st ind't. being	1	4	5
1094 : 30'5	Amt. due, at the time the 1st pay't. was made.			
150 : 60	1st payment deducted.			
943 : 70'5	Principal remaining due, Sept. 20, 1822.			
138 : 54'1	Interest to Oct. 25, 1824.			
1082 : 24'6	Amt. at the time of the 2d payment.			
200 : 90	2d Payment Oct. 25, 1824.			
881 : 34'6	Principal remaining due, Oct. 25, 1824.			
179 : 08'3	$105 : 56'5 Int. ex'ds. the p't. July 11, 1826. 73 : 51'8 Int. to Sept. 20, 1827, upon same principal. 179 : 08'3 sum of the Int's.			
1060 : 42'9	Amount.			
75 : 20				
112 : 11				
187 : 31	Sum of the 3d and 4th pay'ts. deducted Sept. 20, 1827.			

(Carried up.)

873 : 11‘9 Principal remaining due, Sept. 20, 1827.
77 : 24‘6 Interest to Dec. 25th, 1828.
950 : 36‘5 Amount to do.
105 : 5th payment deducted.
845 : 36‘5 Principal remaining due, Dec. 25, 1828.
23 : 83‘4 Interest to May 20, 1829.
$869 : 19‘9 Balance due May 20, 1829.

A note was given, April 20th, 1825, of $800 : 50 cts.; May 25th, 1826, 250 dollars were indorsed; and Dec. 20th, 1828, 300 dollars were indorsed; what was due on the note June 20th, 1829; Interest at 7 per cent? *Ans.* $436 : 82‘4

COMPOUND INTEREST.

Compound Interest is that which accrues on the amount of the principal and Interest. That is, the interest for the given time is added to the principal, and the amount constitutes a principal for another given time, and so on. The time may be three, six, or twelve months, as the parties may agree.

Rule:—1. Find the amount of the given principal, at the given rate and time, as in simple interest, which will form a new principal for another period of time.

2. Subtract the first principal from the last amount, and the remainder will be the interest. Thus:

1. What is the compound interest of $150 for 5 years, at 4 per cent a year?

150×,04=6 : 00 interest for 1 year, and 150+6 : 00=$156, amount of principal for 2d year.
156×,04=6 : 24 interest 2d year, and 156+6 : 24=$162 : 24 amount of principal for 3d year.
162 : 24×,04=6 : 48‘9 interest 3d year, and 162 : 24+6 : 48‘9= $168 : 72‘9 amount or principal for 4th year.
168 : 72‘9×04=6 : 74‘9 interest 4th year, and 168 : 72‘9+6 : 74‘9 =175 : 47‘8 amount and principal 5th year.
175 : 47‘8×,04=7 : 01‘9 int. 5th year, and 175 : 47‘8+7 : 01‘9= $182 : 49‘7 amount 5th year. Then
$182 : 49‘7—150=32 : 49‘7 com. int. 5 years. *Ans.*

2. What is the compound interest of $210 : 50 for 3 years, at 6 per cent a year? *Ans.* $40 : 20‘8

A concise and easy Method of casting Compound Interest, at 6 per cent, on any sum in Federal Money.

Rule :—Multiply the given sum, if

For 2 years,	by 112,36	For 7 years,	by 150,3630
——3 years,	— 119,1016	—— 8 years,	— 159,3848
——4 years,	— 126,2476	—— 9 years,	— 168,9478
——5 years,	— 133,8225	——10 years,	— 179,0847
——6 years,	— 141,8519	——11 years,	— 189,8298

Note. Three of the first or highest decimals, in the above numbers, will be sufficiently accurate for most operations; the product, remembering to move the separatrix *two* figures from its natural place towards the left hand, will then show the amount of principal and compound interest for the given number of years. Subtract the principal from the amount and it will show the compound interest.

INVOLUTION.

When a number is multiplied into itself, it is said to be *involved*, and the process is called *Involution*.

The product which is obtained by multiplying a number into itself is called a Power. The power is often indicated by a figure placed at the right of the number, thus 2^4 which is called the *index* or *exponent* of that power. The number involved is called the Root, or first power. When the root is used as a factor twice, it is called the second power; when three times, the third power, &c. The different powers have other names beside their numbers, viz. The square, 2d power; cube, 3d power; biquadrate, 4th power; sursolid, 5th power; square-cubed, 6th power, &c.

Involution is performed by the following

Rule :—Multiply the given root, or number by itself, and that product by the same number, and so on to the required power. Thus:

1. What is the 6th power of 2? *Ans.* 64

$2\times2=4$, the 2d power; $4\times2=8$, the 3d power; $8\times2=16$, the 4th power; $16\times2=32$, the 5th power; and $32\times2=64$, the 6th power.

What is the 4th power of 4? *Ans.* 256 What is the cube of 6? *Ans.* 216 What is the square of 14? *Ans.* 196. What is the 2d power of 64? *Ans.* 4096 What is the biquadrate of 5? *Ans.* 625

To involve a number, multiply it into itself, as often as there are units in the exponent, save once.

Note. The exponent shows, not how many times we are to multiply, but *how many times the root is used as a factor.*

Involve 9^3. Ans. 729 Involve 6^5. Ans. 7776 Involve 10^4. Ans. 10000 Involve 211^3. Ans. 9393931

OBS. A vulgar fraction is involved by multiplying the numerator by itself, and the denominator by itself.

A mixed number must first be reduced to an improper fraction, or a decimal before involving it.

A decimal, is involved the same as a whole number—point off the same as in multiplication of decimals.

What is the 4th power of $\frac{3}{5}$? Ans. $\frac{81}{625}$ What is the square of $\frac{2}{3}$? Ans. $\frac{4}{9}$ What is the cube of $\frac{4}{7}$? Ans. $\frac{64}{343}$ Involve $\frac{4}{39}$ to the 4th power. Ans. $\frac{64}{59319}$

What is the square of $5\frac{1}{2}$? Ans. $30\frac{1}{4}$ What is the square of $4\frac{1}{4}$? Ans. $18\frac{1}{16}$ What is the square of $30\frac{1}{4}$? Ans. $915\frac{1}{16}$

What is the square of ,5? Ans. ,25 What is the square of 1,2? Ans. 1,44 What is the square of 37,5? Ans. 1406,25 What is the 6th power of 5,03? Ans. 16196,005304479729

NOTE. Involving a vulgar or decimal fraction diminishes it in the same proportion, as a whole number becomes increased.

EVOLUTION.

This is the extracting or finding the roots of any given powers; or it is exactly the reverse of Involution. There, *a root was given to find a power.* Here, *a power is given to find a root.*

The root of any number or power, is such a number, as being multiplied into itself a certain number of times, will produce that power. Thus, 2 is the square root, or 2d root of 4, because $2^2 = 2\times2 = 4$; and 3 is the cube root, or 3d root of 27, because $3^3 = 3\times3\times3 = 27$.

The power of any given number or root may be found exactly by multiplying the number continually into itself. But there are numbers, of which a proposed root can never be exactly found. Yet, by means of decimals, we may approximate or approach towards the root, to any degree of exactness. Those numbers whose roots only approximate towards the true roots, are called *surd numbers*; but those whose roots can be exactly found, are called *rational numbers.*

The roots are sometimes denoted by writing the character $\surd$, before the power, with the index of the root against it. Thus, the square root of 25 is expressed $\surd 25$, and the cube root of 64 is expressed $\surd^3 64$; and the 5th root of 16807, $\surd^5 \overline{16807}$. The index to the square root is always omitted; the character only, being placed before it; thus, $\surd 16$, the index 2, being omitted.

When the power is expressed by several numbers, with the sign +, or —, between, a line is drawn from the top of the sign over all the parts of it; thus the square root of 41—5, is $\sqrt{41—5}$ or thus, $\sqrt{}$ (41—5,) enclosing the numbers in a parenthesis.

But all roots are now frequently distinguished by fractional indices; thus, the square root of 8, is $8^{\frac{1}{2}}$, the cube root of 64, is $64^{\frac{1}{3}}$, and the square root of 41—5, is $\overline{41—5}^{\frac{1}{2}}$ or $(41—5)^{\frac{1}{2}}$.

It is very necessary for practical purposes, to be able to find the amount of *surface* there is in any given quantity.

The rule for finding the *amount of surface*, is to *multiply the length by the breadth*, and this will give the amount of square *inches*, *feet*, or *yards*.

It is important for the pupil to learn the distinction between a *square quantity*, and a certain extent that is in the form of a *square*. For example, *four square inches*, and *four inches square* are different quantities.

A four inch square, then, is a square whose *sides* are four inches long, but it makes a square which is on each side, only *two inches*. *Four square inches* are four squares that are each an inch on every side. But it contains *sixteen square inches*.

When we wish to find the *square contents* of any quantity, we seek to know how many square *inches*, or *feet*, or *yards*, there are in the quantity given, and this is always found by multiplying the length by the breadth.

When the length and breadth of any quantity are given, we find its *square contents*, or the amount of surface it will cover, by multiplying the *length* by the *breadth*.

If any quantity is placed in a *square form* the length of one side is the *square root* of the square contents of this figure. The length of the side of a square is the square root, made by the given quantity.

If we have *one side* of a square given, by the process of *Involution*, we find what are the *square contents* of the quantity given.

If, on the contrary, we have the *square contents* given, by the process of *Evolution*, we find what is the *length of one side* of the square, which can be made by the quantity given.

Thus if we have a *square* whose side is four inches, by *Involution* we find the surface, or *square contents* to be 16 *square inche*

But if we have 16 *square inches* given, by *Evolution* we fin what is *the length of one side* of the square made by these 16 inches

EXTRACTION OF THE SQUARE ROOT.

Extracting the square root is finding a number, which, multiplied into itself, will produce the given number; or, it is finding the length of *one side* of a certain quantity, when that quantity is placed in an exact square.

It will be found by trial, that the root always contains ju *half* as many, or one figure *more* than half as many figures a are in the given quantity. To ascertain, therefore, the numb of figures in the required root, we point off the given numb into periods of two figures each, beginning at the right, an there will always be as many figures in the root as there are periods.

RULE FOR EXTRACTING THE SQUARE ROOT.

1. Point off the given number, into periods of two figures each, beginning at the right.

2. Find the greatest square in the first left hand period, and subtract it from that period. Place the root of this square in the quotient. To the remainder bring down the next period for a dividend.

3. Double the root already found (understanding a cipher at the right) for a divisor. Divide the dividend by it, and place the quotient figure in the root, and also in the divisor.

4. Multiply the divisor, thus increased, by the last figure of the root, and subtract the product from the dividend. To the remainder bring down the next period, for a new dividend. Double the root already found, for a new divisor, and proceed as before.

Ex. Required the square root of 729.

OPERATION.

$\dot{7}2\dot{9}(27$ the root.
$\underline{4}$
$47)329$
329

EXPLANATION.

The given number being divided into periods, seek the greatest square number in the left hand period, (7) which is 4, of which the root (2) being placed to the right hand of the given number, after the manner of a quotient, and the square number (4) subtracted from the period, (7) to the remainder, (3) bring down the next period, (29) making for a dividend 329. Then the double of the root (4) being placed to the left hand for a divisor, say how often 4 in 32? (excepting 9 the right hand figure) the answer is 7, which place in the root for the second figure of it, and also to the right hand of the divisor; then multiplying the divisor thus increased by the figure (7) last obtained in the root, place the product underneath the dividend, and subtract it therefrom, and the work is finished.

The *proof* may be seen by involution, thus; 27×27=729; when there is a remainder add it to the result.

1. What is the square root of 117649? *Ans.* 343
2. What is the square root of 262144? *Ans.* 512
3. What is the square root of 531441? *Ans.* 729
4. What is the square root of 1679616? *Ans.* 1296
5. What is the square root of 5764801? *Ans.* 2401

NOTE. If there be a remainder after all the periods are brought down, the operation may be continued, at pleasure, by annexing periods of ciphers.

What is the square root of 42? *Ans.* 6,48
What is the square root of 30? *Ans.* 5,477
What is the square root of 625,895? *Ans.* 25,017

```
        .  .  .  .
       625,8950(25,017
       4
      ----
   45)225
      225
      --------
 5001) 8950
       5001
      ---------
50027)394900
      350189
      --------
       44711 rem.
```

Note. The first period in decimals must be placed over tenths, and so on, to the right, placing a period over every second figure. And if the decimals be odd, a cipher must be joined to the right hand of the last period to complete it; joining a cipher to the right of a decimal does not alter its value.

1. What is the square root of 262,44? *Ans.* 16,2
2. What is the square root of 24,7009? *Ans.* 4,97

To extract the square root of a Vulgar Fraction.

Rule:—Reduce the fraction to its lowest terms for this, and all other roots; then extract the root of the numerator for a new numerator, and the root of the denominator, for a new denominator.

Note. If the fraction be a surd, reduce it to a decimal, and extract its root; if a mixed number, reduce it to an improper fraction, and then proceed the same as with a simple fraction, or reduce the fraction to a decimal, and proceed as in whole numbers and decimals.

1. What is the square root of $\frac{25}{36}$? *Ans.* $\frac{5}{6}$
2. What is the square root of $\frac{27}{147}$? *Ans.* $\frac{3}{7}$
3. What is the square root of $\frac{112}{175}$? *Ans.* $\frac{4}{5}$

SURDS.

4. What is the square root of $\frac{11}{12}$? *Ans.* ,9574
5. What is the square root of $\frac{5}{12}$? *Ans.* ,64549
6. What is the square root of $\frac{7}{9}$? *Ans.* ,88191

MIXED NUMBERS.

7. What is the square root of 30$\frac{1}{4}$? *Ans.* 5$\frac{1}{2}$
8. What is the square root of 20$\frac{1}{4}$? *Ans.* 4$\frac{1}{2}$
9. What is the square root of 272$\frac{1}{4}$? *Ans.* 16,5

EXTRACTION OF THE CUBE ROOT.

A *Cube* is a solid body, having six equal sides, each of which is an exact square. Thus a solid, which is 1 foot long, 1 foot high, and 1 foot wide, is a *cubic foot;* and a solid whose length, breadth, and thickness are each 1 yard, is called a *cubic yard.*

The root of a cube is always the *length* of one of its sides; for as the length, breadth, and thickness of such a body are the same, the length of one side, raised to the third power, will show the contents of the whole.

Extracting the Cube Root of any quantity, therefore, is finding a number, which multiplied into itself, *twice*, will produce that quantity;—or it is finding the length of one side of a given quantity, when that quantity is placed in an exact cube.

To ascertain the number of figures in a cube root, we point off the given number, into periods of three figures each, beginning at the right, and there will be as many figures in the required root as there are periods.

To facilitate the extraction of the square and cube roots, it may be proper for the pupil to be familiar with the following tables.

Table I. The square of 1=1; of 2=4; of 3=9; of 4=16; of 5=25; of 6=36; of 7=49; of 8=64; of 9=81.

Table II. The cube of 1=1; of 2=8; of 3=27; of 4=64; of 5=125; of 6=216; of 7=343; of 8=512; of 9=729.

RULE FOR EXTRACTING THE CUBE ROOT.

1. Point off the given number into periods of three figures each, beginning at the right.

2. Find the greatest Cube in the left hand period; place the root in the quotient; then subtract the cube thus found from said period, and then bring down the next period, for a *dividend.*

3. *Square* the first quotient figure; then multiply its square by 300, for a divisor: then seek how many times the divisor will go in the dividend, place the result in the quotient [root]; then multiply the divisor by the 2d quotient figure, place the product under the *dividend* to form a part of the *subtrahend.*

4. *Square* said quotient figure; multiply its square by the 1st figure of the quotient, and its product by 30, which product, place under the former partial subtrahend; then *Cube* the same quotient figure, and place the *Cube* under the 2d partial *subtrahend,* which parts add together, calling their amount the *total subtrahend.*

5. Subtract said *subtrahend* from the dividend, and to the remainder bring down the next period, for another *dividend.*

6. To find a *divisor* for said dividend, *square* the *figures* in the quotient, and multiply their product by 300, which will be the divisor sought: then seek how many times it can be contained in said dividend, place the number in the quotient, by which multiply the divisor, *last found,* the product will be the first part of the 2d *subtrahend.*

7. *Square* the last quotient figure; multiply its square by the former figure of the quotient, and their product by 30, (placing the product as before) then *cube* the same figure, of the quotient, place its cube as before; then add all their products, as before, for another subtrahend, which subtract from the last *dividend,* and if ciphers be the result the work is done.

Obs. To find the *square*, there is only 1 multiplication, or two factors—in the CUBE two multiplications, or three or more factors. The square of any figure can never be more than two places, nor can the cube of any figure exceed three places. The places in the root, therefore, will always equal the periods in the power. Any power of a given number may be found exactly; but we cannot, conversely, find every root of a given number exactly.

Whenever any dividend is too small to contain the divisor, a cipher must be placed in the root, and another period brought down. When the subtrahend happens to be larger than the dividend, the quotient figure must be made one less, and we must find a new subtrahend.

Ex. What is the cube root of 34645976? *Ans.* 326

OPERATION.

34645976(326 *Ans.*
27
3×3=9×300=2700) 7645 1*st. dividend.*
2700×2=5400
2×2=4×3=12×30= 360
2×2=4×2= 8
5768 1*st. subtrahend.*
32×32=1024×300=307200)1877976 2*d dividend.*
307200×6=1843200
6×6=36×32=1152×30= 34560
6×6=36×6= 216
1877976 2*d. subtrahend.*

Proof. Involve the root found to the third power, thus; 326×326×326=34645976; or add the contents of the several parts together.

1. What is the cube root of 729? *Ans.* 9
2. Extract the cube root of 48228544? *Ans.* 364
3. What is the cube root of 99252847? *Ans.* 463
4. What is the cube root of 373248? *Ans.* 72
5. What is the cube root of 997002999? *Ans.* 999
6. What is the cube root of 729000000? *Ans.* 900

Note. When there are decimals in a given number, point off the whole numbers the same as if there were no decimals belonging to the given number, then place a period over tenths in decimals, and one over every third figure beyond it, counting to the right, and if the right hand period should not be complete, annex ciphers to complete the period; then extract the root the same as in whole numbers. The periods over whole numbers, show that the root must have so many figures in whole numbers; the rest will be decimals.

7. What is the cube root of 41421,736? *Ans.* 34,6
8. What is the cube root of 85766,121? *Ans.* 44,1
9. What is the cube root of 117,649? *Ans.* 4,9
10. What is the cube root of 84,604519? *Ans.* 4,39

Note. If there be a remainder after all the periods are brought down, the operation may be continued, at pleasure, by annexing periods of ciphers.

11. What is the cube root of 2? *Ans.* 1,2599
12. What is the cube root of 1? *Ans.* 1
13. What is the cube root of 3? *Ans.* 1,442

Note. To extract the cube root of a Vulgar Fraction, first extract the cube root of the numerator for a new numerator, and then extract the cube root of the denominator for a new denominator.—Or reduce the fraction to a decimal and extract the cube root of the decimal.

If the fraction be a surd, (i. e.) a number whose root can never be exactly found, reduce it to a decimal, and extract the root from it.

14. What is the cube root of $\frac{8}{27}$? *Ans.* $\frac{2}{3}$
15. What is the cube root of $\frac{125}{343}$? *Ans.* $\frac{5}{7}$
16. What is the cube root of $\frac{1}{125}$? *Ans.* $\frac{1}{5}$

SURDS.

17. What the cube root of $\frac{5}{8}$? *Ans.* ,8549879
18. What the cube root of $\frac{7}{40}$? *Ans.* ,5593445
19. What the cube root of $\frac{124}{125}$? *Ans.* ,9973262

Note. A mixed number may be reduced to an improper fraction or a decimal, and the root thereof extracted.

20. What is the cube root of $12\frac{19}{27}$? *Ans.* $2\frac{1}{3}$
21. What is the cube root of $42\frac{7}{8}$? *Ans.* $3\frac{1}{2}$
22. What is the cube root of $5\frac{104}{125}$? *Ans.* $1\frac{4}{5}$

SURDS.

23. What is the cube root of $8\frac{2}{13}$? *Ans.* 2,013+
24. What is the cube root of $7\frac{3}{5}$? *Ans.* 1,966+

Note. To extract the Biquadrate Root is to find out a number, which being involved four times into itself, will produce the given number.—First extract the square root of the given number, then extract the square root of that square root, and it will give the biquadrate root required.

1. What is the biquadrate root of 33362176? *Ans.* 76
2. What is the biquadrate root of 5719140625? *Ans.* 275

Practical exercises in the square and cube roots.

1. A.'s cellar is of the same length, breadth and depth, and 1728 cubic feet was thrown from it; what is the length of one side? *Ans.* 12 *ft.*

2. The contents of a cubical stick of timber, are 103823 solid inches; how many inches is it each way? *Ans.* 47

3. The floor of a certain room is made exactly square, each side of which contains 75 feet; I demand how many square feet are contained therein? *Ans.* 5625 *feet.*

4. What is the side of a cube or solid square containing 64 solid feet? *Ans.* 4 *feet.*

5. A certain pavement is made exactly square, each side of which contains 97 feet, I demand how many square feet are contained therein? *Ans.* 9409

6. An army consists of 331776 men, I desire to know how many in rank and file? *Ans.* 576

7. How many solid feet in a cubic block, measuring 5 feet on each side? *Ans.* 125 *feet.*

8. A man has 13824 feet of timber, in separate blocks, each containing one cubic foot, and having a desire to place them in a cubic pile, he wishes to know the length of each side of such a pile. *Ans.* 24 *feet.*

9. If 1369 fruit trees be planted in a square orchard, how many must be in a row each way? *Ans.* 37

10. What are the solid contents of a cubic block, of which each side measures 2 feet? *Ans.* 8 *feet.*

11A certain square pavement contains 48841 square stones, all of the same size, I demand how many are contained in one of the sides? *Ans.* 221

STERLING MONEY.

This is the money of account now in England. And it was the only mode of reckoning in the United States till after an act of congress in 1792, establishing a mint and regulating the standard of our coins, but since, it has gradually grown out of use.

The denominations of ***English money*** are ***pound, shilling, penny, halfpenny*** and *farthing.*

The characters used are

£	for Pounds.
s.	for Shillings.
d.	for Pence.
qrs.	for Farthings.

A Pound Sterling, is equal to $4 44⁴⁄₉ Fed. Money.
An English Shilling, = 22²⁄₉
4s. 6d. Sterling, = $1
A Groat, is 4 Pence, both here, and in England.

TABLE OF ENGLISH MONEY.

4 Farthing (qrs.)	make	1 Penny,	marked	*d.*
12 Pence	"	1 Shilling,	"	*s.*
20 Shillings	"	1 Pound	"	£

Note. Farthings are often written thus, 1 farthing ¼ of a penny, 2 farthings, ½ of a penny, and 3 farthings, ¾ of a penny.

The least Piece of *Money* used in *England* is a *Farthing*, And
5*s*. is a *Crown*.
10*s*. is an *Angel*.
6*s*. 8*d*. a *Noble*.
13*s*. 4*d*. a *Mark*.

Examples for Practice.

1. A man bought land to the amount of £69 13s. 5d.; farming implements to the amount of £11 10s.; a yoke of oxen for £15 6s.; a horse for £13 0s. 4d.; a cart for £4 17s. 8d. and a saddle for 19s. 4d. 2 qrs. What did the whole cost? *Ans.* £115 6s. 9½d.

2. I pay for cloth £14 19s. 6d. for flannel, £11 4s. 9d. for grain, £25 10s. for sugar, £4 0s. 6d. for coffee, £3 6s. 8d. and for melasses 19s. 6d. What cost the whole? *Ans.* £60 0s. 11d.

3. Subtract one hundred and forty-nine pounds, fifteen shillings, six pence, and three farthings—from eight hundred and fifty-seven pounds. *Ans.* £707 4s. 5¼d.

4. A man purchased cloth to the amount of £27 11s. In lieu he gave flour to the amount of £19 17s. 6d. and the rest in money. How much money did he give? *Ans.* £7 13s. 6d.

5. A farmer brought to market butter, which brought him 54s.; cheese, which brought 59s.; a load of wood, 49s. 9d. 2 qrs.; eggs, 39s. 8d.; and apples, 47s. 9d. In part payment he received 4 lb. of tea, at 4s. 9d. pr. lb.; 12 lb. sugar, at 8d. pr. lb.; 3 shovels at 6s. a-piece; 5 hoes, at 3s. a-piece; 8 yds, of cloth, at £1 3s. 6d. pr. yd., and the rest in money. How much money did he receive? *Ans.* 2s. 2¼d.

6. A merchant bought 5 yards of cloth for £2 6s. 1d. 3 qrs. per yard; what did the whole cost? *Ans.* £11 10s. 8d. 3f.

7. How many shillings, at 2 farthings a gill, will 5 T. 1. p. 1 hhd. 2 gals. 2 qts. 1 pt. 3 gills, cost? *Ans.* 1935s. 7d. 2 qrs.

8. At 9 pence per quart, what will 16 gals. 2 qts. of melasses come to in pence? *Ans.* 594

9. Divide £115 10s. by 90. *Ans.* £1 5s. 8d.

10. Divide £136 16s. 6d. by 108. *Ans.* £1 1s. 4d.

11. Bought a hhd. of wine for £67 4s. What was that a gal.? *Ans.* £1 1s. 4d.

12. Bought 48 yds. of cloth for £11 2s. What was that a yd.? *Ans.* 4s. 7½d.

REDUCTION OF ENGLISH MONEY.

Rule for reducing this money from a higher to a lower denomination.

Pounds multiplied by 20 give shillings.
Shillings × 12 give pence.
Pence × 4 give farthings.
Pence × 2 give halfpence.

Examples for Practice.

1. In £6; how many shillings? *Ans.* 120*s.*
2. In £1; how many shillings, and pence? *Ans.* 20*s.* 240*d.*
3. In 2 pounds; how many pence? *Ans.* 480*d.*
4. In £3; how many farthings? *Ans.* 2880 *qrs.*
5. In £16 14s. 6d.; how many pence? *Ans.* 4014*d.*
6. In £34; how many shillings, pence, and farthings? *Ans.* 680*s.* 8160*d.* 32640 *qrs.*
7. In £96; how many half pence? *Ans.* 46080
8. In £46 12s. and 6d.; how many farthings? *Ans.* 44760 *qrs.*
9. In £86 14s. 6d. 2 qrs.; how many farthings? *Ans.* 83258 *qrs.*
10. In £39; how many shillings, pence, and farthings? *Ans.* 780*s.* 9360*d.* 37440 *qrs.*

Rule for changing English money from a lower denomination to a higher.

Farthings divided by 4 give pence.
Pence ÷ 12 give shillings.
Shillings ÷ 20 give pounds.

1. In 46382 farthings; how many pence? *Ans.* 11595*d.* 2 *qrs.*
2. In 16486 pence; how many shillings? *Ans.* 1373*s.* 10*d.*
3. In 85 shillings; how many pounds? *Ans.* £4 5*s.*
4. In 2880 farthings; how many pounds? *Ans.* £3
5. In 46080 half pence; how many pounds? *Ans.* £96
6. In 36463 pence; how many pounds? *Ans.* £151 18*s.* 7*d.*
7. In 74981 half pence; how many pounds? *Ans.* £156 4*s.* 2½*d.*
8. In 3452 sixpences; how many pounds? *Ans.* £86 6*s.* 0*d.*

REDUCTION OF CURRENCIES.

Reduction of Currencies is finding the value of the coin or currency of one state or country in that of another. Although the same denominations and coin are generally used in the different countries and states, yet the standard value differs in almost every country.

Thus, a dollar is reckoned in

New York, Ohio, and North Carolina,	8*s.* called *New York* currency.
The New England States, Virginia, Kentucky, and Tennessee,	6*s.* called *New England* currency.
South Carolina, and Georgia,	4*s.* 8*d.* called *Georgia* currency.
New Jersey, Pennsylvania, Delaware, and Maryland,	7*s.* 6*d.* called *Pennsylvania* currency.
England,	4*s.* 6*d.* called *English* or *Sterling* currency.
Canada, and Nova Scotia,	5*s.* called *Canada* currency.

An easy method of Reducing the Currency of any Country to Dollars, and back to the given Currency.

Rule:—1. Reduce the given currency to its lowest or any convenient term, and reduce the dollar to the same term.

2. Divide by the term produced from the dollar, and the quotient will be in dollars.

3. Add to the remainder, if any, one cipher for dimes, one for cents, and a third for mills, and continue the division.

A. has £86 6s. 5¼d. New England money, and would convert it to dollars. 86×20+6=1726×12+5=20717×4+1= 82869 qrs. And, 6×12=72×4=288 qrs. Then; 82869+288 =$287 plus 213, and 213×100=21300+288=74 cents nearly.

Ans. $287 : 74

Obs. 1. The converse of this rule, will reduce dollars to £., that is, reduce the dollars to the term of pence, or farthings, as the case may require, and divide by the pence in a £.

Change $287 : 74 to £s. $287 : 74×6=1726 : 44×12= 20717 : 28; £1=20×12=240d. Then, 20717 : 28+240.=£86 plus 77 : 28×20+240=6s. plus 105 : 60×12+240=5d. plus 67 : 20×4+240=1 qr. *Ans.* £86 6*s.* 5¼*d.*

Obs. Some particular cases may admit of shorter methods, but this will meet every possible case; hence, it is deemed better than to burden the child's mind with a variety of rules, applicable only to particular cases.

New England Currency.

1. Reduce £240 to Federal Money. *Ans.* $800
2. Reduce £73 to Federal Money. *Ans.* $243 : 33'3
3. Reduce £45 15*s*. to Federal Money. *Ans.* $152 : 50
4. Reduce £345 10*s*. 11*d*. to Federal Money. *Ans.* $1151 : 68
5. Reduce £2 3*s*. 9*d*. to Federal Money. *Ans.* $7 : 29'1

New York Currency.

1. Reduce £461 to Federal Money. *Ans.* $1152 : 50
2. Reduce £419 10*s*. to Federal Money. *Ans.* $1048 : 75
3. Reduce £8 4*s*. 9*d*. to Federal Money. *Ans.* $20 : 59'3
4. Reduce £6 to Federal Money. *Ans.* $15

Various Currencies.

1. Change £471 Penn. Cur. to Federal Money. *Ans.* $1256
2. Change £480 19*s*. 9*d*. Penn. Cur. to Federal Money. *Ans.* $1282 : 63
3. Change £28 Georgia Cur. to Federal Money. *Ans.* $120
4. Change £11 4*s*. 8*d*. Georgia Cur. to Federal Money. *Ans.* $48 : 14'2
5. Change £36 Canada Cur. to Federal Money. *Ans.* $144
6. Change £528 12*s*. 6*d*. Canada Cur. to Federal Money. *Ans.* $2114 : 50
7. Reduce £5 9*s*. 0*d*. 1 *far*. Eng. Cur. to Federal Money. *Ans.* $24 : 22
8. Change £22 10*s*. Sterling to Federal Money. *Ans.* $100

Change of Currencies.

1. Change $135 : 50 to N. York currency. *Ans.* £54 4*s*.
2. Change $629 to N. York currency. *Ans.* £251 12*s*.
3. Change $196 to N. England currency. *Ans.* £58 16*s*.
4. Change $152 : 50 to N. England cur. *Ans.* £45 15*s*.
5. Change $752 to N. England cur. *Ans.* £225 12*s*.
6. Change $224 : 60 to Canada currency. *Ans.* £56 3*s*.
7. Change $118 : 25 to English Money. *Ans.* £26 12*s*. 1½*d*.
8. Change $8 : 30 to Penn. currency. *Ans.* £3 2*s*. 3*d*.
9. Change $1561 : 35 to Georgia currency. *Ans.* £364 6*s*. 4*d*.

APPLICATION.

1. A. of Boston owes B. of Philadelphia, £250 of B's. currency, 7s. 6d. to the dollar; how many dollars must he send him to pay the debt? *Ans.* $666 : 66 *nearly.*

2. The dollar in Georgia is worth 4s. 8d., and B. of New-York owes £1000 there; how many dollars must he send to pay the debt? *Ans.* $4285 : 71'5

3. B. of New York, owes C. of London £652 10*s.*, for what number of dollars will he draw on B. at 4s. 6d. each? *Ans.* $2900

4. How many dollars must A. send from Boston to pay a debt of £720 in New York, where the dollar is 8s.? *Ans.* $1800

5. B., of Boston, owes D., of London, £762 14s. 6d. how many dollars must he draw for to discharge the debt? *Ans.* $3389 : 88'8

6. A. of Boston was in France and borrowed 500 pistoles;* for which he drew on a house in New York; how many dollars paid the demand? *Ans.* $1750

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* £1 8s.

ARITHMETIC.

PART II.

Explanatory of the Science of Numbers.

ART. 1. Arithmetic is a science which explains the properties of numbers, and shows the method or art of computing by them.

2. At what time this science was introduced into the world we can by no means determine. That some part of it was coeval with the human race, is absolutely certain. We cannot conceive how any one indued with reason can be without some knowledge of numbers. We are told of natives in America who had no word in their language to express a greater number than *three.*

3. It is probable that the four principal rules of arithmetic have been always known to some nation or other. The Greeks were the first European nation among whom arithmetic arrived at any degree of perfection.

4. The object to be accomplished by this study, is to take given numbers, and working according to specific rules, obtain other relative numbers. This is effected only in two ways—adding something to them, or taking something from them. This operation includes the rules of addition and subtraction.

5. Whatever is capable of increase or diminution, is called *magnitude* or *quantity.* Mathematics in general, is the *science* of *quantity;* or the science which investigates the means of measuring quantities.

6. To determine the magnitude or measure of all kinds, it is necessary to fix upon some known magnitude of a like kind, and consider it as the *measure or unit*, and thereby determine the proportion which exists between the proposed magnitude and known measure. This proportion is always expressed by numbers; *unit*, or *one*, being arbitrarily assumed as a fixed measure or term of comparison.

7. All magnitudes may be expressed by numbers.—All that relates to magnitude is the object of mathematics; and the foundation of the science is laid on the science of numbers, and consists in an accurate examination of the methods of calculation made thereby. Arithmetic treats of numbers in particular, and is the science of numbers, properly so called.

8. The idea of number is the latest and most difficult to form. The savage is lost in his attempts at numeration, and significantly expresses his inability to proceed by holding up his expanded fingers, or pointing to the hairs of his head.

9. Nature has furnished the great and universal standard for computation in the fingers of the hand. All nations have accord-

ingly reckoned by *fives;* and some barbarous tribes have scarcely advanced any further. After the fingers of one hand had been counted once, it was a second and perhaps a distant step to proceed to those of the other. The primitive words, expressing numbers, did not probably exceed five. To denote *six*, *seven*, *eight* and *nine*, the North American Indians repeat the five with the successive addition of one, two, three, and four; could we safely trace the descent and affinity of the abbreviated terms denoting the numbers from five to ten, it seems highly probable, that we should discover a similar process to have taken place in the formation of the most refined languages.

10. The ten digits of both hands being reckoned up, it then became necessary to repeat the operation. Such is the foundation of our decimal scale of arithmetic. Language still betrays by its structure the original mode of proceeding. To express the numbers beyond ten, the Laplanders combine an ordinal with a cardinal digit. Thus, eleven, twelve, &c. they denominate *first* ten and one, *first* ten and two, &c. and in like manner they call twenty-one, twenty-two, &c. *second* ten and one, *second* ten and two, &c. Our term eleven is supposed to be derived from *ein*, or *one*, and *liben*, *to* remain, and to signify *one*, *leave* or *set aside ten*. *Twelve* is of the like derivation, and means *two*, *laying aside the ten*. The same idea is suggested by our termination *ty* in the words *twenty*, *thirty*, *&c.* This syllable, altogether distinct from *ten* is derived from *zichin*, to *draw*, and the meaning of *twenty* is, strictly speaking, *two drawings*, that is, the hands have been twice closed and the fingers counted over.

11. After ten was firmly established, as the standard of numeration, it seemed the most easy and consistent to proceed by the same repeated composition. Both hands being closed ten times would carry the reckoning up to a hundred. This word, originally *hund*, is of uncertain derivation; but the term *thousand*, which occurs at the next stage of the progress, or the hundred added ten times, is clearly traced out, being only a contraction of *duis hund*, or *twice hundred*, that is, the *repetition*, or *collection of hundreds*.

12. A number is written by placing one unit after another, beginning at the left, and adding a number of units to each collection, as far as we please. Each addition of units increases the value of the number ten times, which is the fundamental law of our written numeration.

13. When a number is written in figures, it is necessary to substitute for each of the figures the word which it represents, and then mention the collection of units, to which it belongs according to the place it occupies.

14. The names which are usually applied to every 6th place of figures from units, are as follows, viz: Millions, Billions, Trillions, Quatrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, &c. Each of these are millions so many times removed from the place of units, that there is the same proportion, between a million and a single unit—a million, is a single unit repeated a thousand thousand times—a billion is a thousand thousand millions, or a million of millions, and so on to infinity.

ADDITION.

ART. 15. When several quantities are given, and we wish to find their amount, the process is performed by collecting and arranging the parts of which they are composed, and bringing them into one total sum, which will represent each part in its own denomination. To the performance of which, it is necessary that each denomination first be duly arranged according to the rules of numeration.

16. We commence with units, and add one to another, till we find the amount of the first column, which we virtually *divide by ten;* we find the quotient, and place the remainder under the column, and carry the quotient to the next denomination. This process of division is actually performed, as much when adding integers, as when we perform an operation in the *compound Rules;* although the pupil may not recognise it.

17. The use of addition is apparent when we view the various objects that surround us, and the necessity we are constantly under, of collecting them into one entire mass or quantity, to determine the number of those of a like name. The application of this rule to the solution of questions, and to the performance of business is self-evident.

SUBTRACTION.

ART. 18. When two quantities are given, and we wish to find how much one exceeds the other. The operation is performed by taking the less from the greater; and arranging both numbers relative to numeration, in the same order as they would stand if they were to be added.

19. There are several important principles relative to this rule, which need particular attention, viz: *When the minuend is a single quantity, and the subtrahend an aggregate.* It is then necessary to find the amount of the quantity used for a subtrahend, and when found, take it from the minuend—

Thus: Subtract 40, 35 and 20 from 100.

OPERATION.

$40+35+20=95$. Then $100-95=5$.

And conversely, *when the subtrahend is a single quantity, and the minuend an aggregate.* First find the amount of the quantities used for the minuend; then take the single quantity from the amount, the remainder will be the answer.

Thus:—Subtract 100 from 85, 31, and 10. *Operation,* $85+31+10=126$. Then $126-100=26$.

20. Sometimes both the minuend and subtrahend are aggregates. In this case the figures which comprise the quantities must be added together, and the amount of each will form two single quantities, one of which will be the minuend, and the other the subtrahend; the smaller of which taken from the larger will show the difference

between them.. The following may be given as an example. Subtract 20, 10 and 5 from 40, 31 and 10.—The operation will stand thus:

20+10+5=35. 40+31+10=81. Then 81—35=46 the remainder.

21. Recapitulating what has been said in the previous ARTICLES, we may reduce the operation of subtraction where aggregate numbers occur to this: Consider which quantity is a single number and which is an aggregate. Then find the amount of the aggregate and subtract it from the single quantity. If both be aggregates, find their amounts and subtract one from the other.

MULTIPLICATION.

ART. 22. When there are two or more equal numbers to be added together, the expression of their sum may be abridged: for example, how many is 3 times 4, or what is the sum of 4 added to itself 3 times. The operation would stand thus: 3×4=12.

23. If a number is to be multiplied by another, we only put one under the other, thus:

$$\begin{array}{r} 213 \\ 7 \\ \hline 1491 \end{array}$$

The order in which the numbers are placed is indifferent, for 3 times 4 is the same as 4 times 3. The same is true of a larger sum.

24. The results which arise from the multiplication of two or more numbers are called *products;* and the numbers or individual figures, are called *factors.*

25. We have observed that a product is generated by the multiplication of two or more numbers, and that these numbers are called *factors.* Thus, the numbers 213 and 7 are the factors of the product 1491.

26. If, therefore, we consider all whole numbers as products of two or more numbers multiplied together, we shall soon find that some cannot result from such a multiplication, and consequently have not any factors; while others may be the products of two or more numbers multiplied together, and may consequently have two or more factors.

27. All numbers, such as 2, 3, 5, 7, 11, 13, &c. which cannot be represented by factors, are called *simple* or *prime numbers;* whereas others, as 4, 6, 8, 9, 10, 12, 14, &c. which may be represented by factors, are called *compound numbers.*

28. The use of multiplication is to compute the amount of any number of equal articles, either in respect to measure, weight, value, or any other consideration. The multiplicand, expresses how much is to be reckoned for each article; and the multiplier, expresses how many times that is to be reckoned. As the multiplier points out the number of articles to be added, it is always an *abstract number;* and has no reference to any value or measure whatever. The product must be of the same nature as the multiplicand. If the multiplicand is weight, the product will be weight. Repeating a quantity does not alter its nature.

DIVSION.

Art. 29. When a number is to be separated into two, three or more equal parts, it is done by means of *division*, which enables us to determine the magnitude of one of those parts. When we wish, for example, to separate the number 12 into three equal parts, we find by division that each of these parts is equal to 4.

30. If we divide a number by 2, or into two equal parts, one of these parts, or the quotient, taken twice makes exactly the number proposed. The multiplication of the quotient by the divisor, must always reproduce the dividend.

31. It is for this reason that division is called a rule, which teaches us to find a number or quotient, which being multiplied by the divisor, will exactly produce the dividend. The dividend, therefore, may be considered as a product of which one of the factors is the divisor, and the other the quotient. And in all examples of division that can be proposed, if we divide the dividend by the quotient, we shall again obtain the divisor; for as 24 divided by 4 gives 6, so 24 divided by 6 will give 4. The whole operation consists in representing the dividend by two factors, of which one shall be equal to the divisor, and the other to the quotient.

32. It often happens that we cannot represent the dividend as the product of two factors of which one is equal to the divisor, and then the division cannot be performed in the manner we have described.

33. If we multiply the divisor by the quotient, and to the product add the remainder, we must obtain the dividend; this is the method of proving division, and of discovering whether the calculation is right or wrong.

34. As some numbers are divisible by certain divisors, while others are not; it is necessary to distinguish the numbers that are divisible by divisors from those which are not.

And first, let the divisor be 2; the numbers divisible by it are 2 4 6 8 10 12, &c. which, it appears, increase always by two. These numbers, as far as they can be continued, are called *even numbers*. but there are other numbers, namely, 1, 3, 5, 7, 9, 11, &c., which are uniformly less or greater than the former by unity, and which cannot be divided by 2, without the remainder 1; these are called *odd numbers*.

35. It must here be particularly observed; that every number is divisible by 1; and also that every number is divisible by itself; so that every number has at least two factors, or divisors, the number itself and unity; but every number, which has no other divisor than these two, belongs to the class of numbers, which we have before called *simple* or *prime numbers*.

36. Lastly, it ought to be observed, that 0 or *nothing* may be considered as a number which has the property of being divisible by all possible numbers; because by whatever number we divide 0, the quotient is always 0; for it must be remarked that the multiplication of any number by *nothing* produces nothing, and therefore 0 times 2, or 0 2, is 0.

37. The dividend expresses the price of several articles or quantities; the divisor the quantity itself: and the quotient shows the

price of one single article or quantity, of which the dividend is the price:—Then, dividing the price of several quantities by the quantity, it will give the price of one single article or quantity: whether it be of money, weight, or measure.

FRACTIONS.

Art. 38. A fraction is a part of a unit, and is so called in contradistinction to an integer or whole number.

There are two kinds of fractions, viz: *Vulgar* and *Decimal*.

39. All fractions have two numbers; one called the numerator, and the other the denominator. The former is always expressed; the latter is either expressed or understood.

40. The numerator expresses the number of single parts contained in the fraction; the denominator, the whole number of parts which compose the integer. Thus, $\frac{4}{5}$, and ,8; that is, four fifths and eight tenths. Here the first is a vulgar, and the latter a decimal fraction; the numbers 4 and 8 are the numerators, and 5 and 10 their denominators.

41. Vulgar and decimal fractions differ principally with respect to their *notation*, their *denominators*, their *modes* of operation, and their utility.

42. In a vulgar fraction, the numerator and denominator are both noted, or written down, and the one set over the other: in decimals, the numerator only is expressed. In vulgar fractions, the denominator may be any number whatever; in decimal fractions, the denominator is ever a decimal number, or unity, with one or more ciphers annexed.

43. Vulgar fractions admit of *improper* fractions, or fractions greater than an integer, such as $\frac{8}{7}$; but decimals are always proper fractions, or less than an integer.

DECIMAL FRACTIONS.

Art. 44. Notation.—Decimal fractions are distinguished by a point, (thus.,) set at the left hand of the fractional number, which may be called the *sinister* point. This is of great and necessary use in decimals, as it supplies the place of the denominator, and designates the magnitude of the fractions. As it divides the fraction from the integral figures, it is termed, the *separatrix*.

45. Decimal fractions decrease infinitely towards the right hand; whole numbers increase infinitely towards the left; both starting from unity as a fixed central point.

46. Therefore it is evident, that the magnitude of a decimal fraction, compared with another, does not depend upon the number of figures, but upon the value of its first left hand figure; for instance, a fraction beginning with any figure less than 9, such as 8, 7, 5, 3, 0, 2, 4, &c., if extended to an infinite number of figures, will not equal ,9.

47. To read decimal fractions, numerate them as you would the same figures in a whole number. To find the denominator, consider the sinister point as standing for ,1; and all the figures upon

its right hand for so many ciphers; then, by the common method numerate and read them accordingly, which will give you the denominator of the fraction. Thus, the following fractions, ,1 ,15 ,236 ,0001 by this rule are to be read, one tenth, fifteen hundredths, two hundred and thirty-six thousandths, one ten thousandth; which, written in the manner of vulgar fractions, would stand, $\frac{1}{10}$, $\frac{15}{100}$, $\frac{236}{1000}$, $\frac{1}{10000}$.

48. If you remove the sinister point towards the right hand, it multiplies the fraction by 10 at every figure; thus ,235 being 2,35 becomes two integers and thirty-five hundredths: remove it to another figure, and it is 23,5 twenty-three and five tenths: remove it another figure, and it becomes a simple whole number 235.

49. But removing the point towards the left hand has directly the contrary effect: It divides and makes the quantity ten times less at every figure it is removed, thus: 235 becomes 23,5 2,35 ,235 ,0235 and so on.

50. Ciphers at the right hand neither increase nor diminish the value of a decimal; but upon the left hand they make it ten times the less for every cipher prefixed.

The reason of this, is because the numerator is increased in the same proportion with the denominator, and ,1 *one tenth*, becomes ,10 *ten hundredths*; but in the other case the denominator only is increased, which diminishes the fraction, and makes ,1 *one tenth*, no more than ,01 *one hundredth*. Therefore,

Any two or more decimal fractions, however differing in their magnitude, may be expressed by an equal number of figures, and have the same common denominator.

So these decimals ,8 ,05 ,456 ,0003 may be written ,8000 ,0500 ,4560 ,0003

51. A whole number may be decimally expressed by annexing one or more ciphers, separated by the sinister point thus, 39,0 24,000.

DECIMAL POINTS.

Art. 52. The points are,

The *Dexter* point, the *Surdal* point, and the *Antisurdal* point.

1. The *Dexter* point, placed thus, 5·, when applied to the dividend, denotes that the figures upon its left hand will be quotient integers, and those upon its right, fractions. It is also used in reducing a decimal fraction into its proper value, and then it shows that the figures upon the left hand only are made use of, and those upon its right thrown away, being smaller than the ultimate fraction, or lowest denomination of that table.

2. The *Surdal* point is a colon, (:) This is a sign of rejection: it is used in division, in case the dividend cannot be exactly measured by the divisor; and being applied to the last remainder, (thus 537:) denotes it to be a surd number, of no further account or use in the operation.

3. The *Antisurdal* point placed over a figure (thus $\dot{5}$) denotes that figure to be read one larger than its nominal value. It is placed over the right hand quotient figure, to force a stop in division.

where the remainder is such as would continue the division for ever, without bringing out the fraction with perfect exactness.

It is also used in reducing decimals, for the same purpose as the dexter point, where its right hand figure is as large as 5, or larger.

This is ever the case with the middle denominations in mixed quantities. For instance, a dime is a fraction of a dollar, and an integer of a cent and a mill. The highest denomination is never a fraction, and is called the *ultimate integer:* The lowest denomination is never an integer, and is called the *ultimate fraction.* The intermediate denominations, are in relation to each other, integers or fractions.

ART. 53. The addition of decimals is performed by placing the sinister points in a perpendicular line, and the first left hand figure of each fraction in the next column, and so on.

Add the several numbers together, &c.

Place the sinister point in the sum total, directly under those in the statement, and the left hand figures, will be integers; those upon the right, decimals.

54. The addition of integers and fractions is performed by placing the fractions as before, the integers upon the left hand of the point; then add all together as whole numbers, and place the sinister point in the sum total as before.

55. The subtraction of decimals is performed by setting down the numbers as in addition; then perform the subtraction as in whole numbers, and place the sinister point in the remainder, directly under that in the subtrahend.

56. The multiplication of decimals is performed exactly as in whole numbers, whether they are pure or mixed fractions; and then place the sinister point in the product as many places to the left hand as there are decimals in both factors; but if the product should not have so many, supply the defect by prefixing ciphers.

57. It is peculiar that a fraction multiplied by a fraction should diminish its value, and make the product less than either factor; and to multiply an integer by a fraction invariably makes the product less than the multiplicand.

The reason of this is, because a fraction in its nature and operation is converse to an integer, having single unity, for the middle term. Thus, to multiply by 1, neither adds nor diminishes, but to multiply by any number more than 1, increases; therefore to multiply by any part or parts, less than unity or 1, of course diminishes; so that to multiply one half by one half is the same thing as dividing it by 2.

58. The parts in decimal division are placed in the same order as in division of whole numbers. Annex a cipher to the last remainder, and continue the division until you have exhausted the dividend, and completed the fraction in the quotient.

59. When the dividend may be exactly measured by the divisor, it is called a terminate decimal. When the last remainder is such a number as will not give the quotient fraction with perfect exactness, although ciphers be annexed to any extent, such decimals, in contradistinction to the former, are called *infinite* or *circulating* decimals, and sometimes repeating decimals, because one or more

figures always repeat *ad infinitum;* as ,666; ,32573257 &c. in such a case it is unnecessary to protract the quotient to any greater number of figures, than what will contain the ultimate fraction sought; therefore apply the surdal point to the remainder; and if the number in the last remainder equal , or exceed the one half of that expressed in the divisor, set the antisurdal point over the last figure in the quotient.

This method reduces the circulating into a terminate decimal; and though it does not ascertain the fraction with perfect exactness, yet the variation is very inconsiderable.

60. The nature of decimal division is the converse of whole numbers, in that, dividing an integer by a fraction, or a fraction by a fraction, instead of diminishing, increases its value, or the quotient will ever be greater than the dividend.

61. The method of dividing decimals, is to divide as in whole numbers, enlarging the number of the dividend, if necessary, by annexing one or more ciphers; then, in the question, count off as many decimal places, as the dividend has more than the divisor; and if there are not so many places in the quotient, supply the defect with ciphers.

VULGAR FRACTIONS.

ART. 62. When a number is said not to be divisible by another number, this only means, that the quotient cannot be expressed by an integral number; this leads us to consider a particular species of numbers, called *fractions* or *broken* numbers. The following is an illustration. If we divide 7 by 3, we easily conceive the quotient which should result, and express it by $\frac{7}{3}$; placing the divisor under the dividend, and separating the two numbers by a line.

63. So, in general, when the number 7 is to be divided by the number 3, we represent the quotient by $\frac{7}{3}$ and call this form of expression a fraction. We cannot, therefore, give a better idea of a fraction than by saying that we express the quotient resulting from the division of the upper number by the lower, (thus $\frac{7}{3}$.)

64. In order to obtain a more perfect knowledge of the nature of fractions, we shall begin by considering the case in which the numerator is equal to the denominator, as in $\frac{2}{2}$. Now, since this expresses the quotient obtained by dividing 2 by 2, it is evident that this quotient is exactly unity, and that consequently this fraction $\frac{2}{2}$ is equal to 1, or an integer; the same is true of all fractions whose numerators and denominators are alike.

65. We have seen that a fraction, whose numerator is equal to the denominator, is equal to unity. All fractions, therefore, whose numerators are less than the denominators, have a value less than unity.

66. If the numerator, on the contrary, is greater than the denominator, the value of the fraction is greater than unity. Thus $\frac{3}{2}$ is greater than 1, for $\frac{3}{2}$ is equal to $\frac{2}{2}$, and $\frac{1}{2}$. Now $\frac{2}{2}$ is exactly 1, consequently $\frac{3}{2}$ is equal to $1 + \frac{1}{2}$, that is, to an integer and a half. It is sufficient in such cases to divide the upper number by the lower, and to add to the quotient a fraction having the remainder

for the numerator, and the divisor for the denominator. For example, $\frac{43}{12}$, we should have for the quotient 3, and 7 for the remainder; hence, we conclude that $\frac{43}{12}$ is the same as $3\frac{7}{12}$.

67. Thus we see how fractions, whose numerators are greater than the denominators, are resolved into two parts; one of which is an integer, and the other a fractional number. Such fractions are called *improper fractions*, to distinguish them from common fractions.

68. The nature of fractions is frequently considered in another way, which may throw additional light on the subject. For example, the fraction $\frac{3}{4}$, it is evident is three times as great as $\frac{1}{4}$. Now this fraction $\frac{1}{4}$ means that, if we divide 1 into 4 equal parts this will be the value of one of those parts; it is obvious then that by taking 3 of those parts, we shall have the value of the fraction $\frac{3}{4}$.

69. As it is easy to understand what $\frac{3}{4}$ is, when we know the signification of $\frac{1}{4}$, we may consider the fractions, whose numerator is unity, as the foundation of all others. And it is observable that these fractions go on continually diminishing; for the more you divide an integer, or the greater number of parts into which you distribute it, the less does each of those parts become. Thus $\frac{1}{100}$ is less than $\frac{1}{10}$.

70. As we have seen, that the more we increase the denominator of such fractions, their value become less; it may be asked—whether it is possible to make the denominator so great, that the fraction shall be reduced to nothing? I answer no; for into whatever number of parts unity (the length of a foot for instance) is divided; let those parts be ever so small, they will still preserve a certain magnitude, and therefore can never be absolutely reduced to nothing. This circumstance has introduced the expression, that the denominator must be *infinite*, or infinitely great, in order that the fraction may be reduced to 0, or to nothing; and the word *infinite* in reality signifies here, that we should never arrive at the end of the series of the above mentioned fractions.

71. It may be necessary also in this place to correct the mistake of those who assert, that a number infinitely great is not susceptible of increase. This opinion is inconsistent with just principles, for $\frac{1}{0}$ signifies a number infinitely great, and $\frac{2}{0}$ being incontestibly the double of $\frac{1}{0}$, it is evident that a number, though infinitely great may still become two or more times greater.

72. To reduce a given fraction to its least terms, it is required to find a number by which both the numerator and denominator may be divided. Such a number is called a *common divisor*, and so long as we can find a common divisor to the numerator and the denominator, it is certain that the fraction may be reduced to a lower form; but, on the contrary, when we see that except unity no other common divisor can be found, this shows that the fraction is already in the simplest form that it admits of.

73. This property of fractions preserving an invariable value, whether we divide or multiply the numerator and denominator by the same number, is of the greatest importance, and is the principal foundation of the doctrine of fractions. For example, we can scarcely add together two fractions, or subtract them from each

other, before we have, by means of this property, reduced them to other forms, that is to say, to expressions whose denominations are equal.

Addition and Subtraction of Fractions.

ART. 74. When fractions have equal denominators, there is no difficulty in adding and subtracting them; for $\frac{2}{7}$, + $\frac{3}{7}$ is equal to $\frac{5}{7}$, and $\frac{4}{7} - \frac{2}{7}$ is equal to $\frac{2}{7}$. In this case, either for addition or subtraction, we alter only the numerators, and place the common denominator under the line, thus $\frac{1}{3} + \frac{2}{3}$ is equal to $\frac{3}{3}$, or 1, that is to say, an integer; and $\frac{3}{4} - \frac{2}{4} - \frac{1}{4}$ is equal to $\frac{0}{4}$, that is to say, nothing, or 0.

75. But when fractions have not equal denominators, we can always change them into other fractions that have the same denominators. For example, add together $\frac{1}{2}$ and $\frac{1}{3}$, we must consider that $\frac{1}{2}$ is the same as $\frac{3}{6}$, and that $\frac{1}{3}$ is equivalent to $\frac{2}{6}$, we have therefore $\frac{3}{6}+\frac{2}{6}$, the sum of which is $\frac{5}{6}$.

76. We may have a greater number of fractions to be reduced to a common denominator, for example $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$; in this case the whole depends on finding a number which may be divisible by all the denominators of these fractions. In this instance, 60 is the number which has that property, and which consequently becomes the common denominator. We shall therefore have $\frac{30}{60}$ instead of $\frac{1}{2}$; $\frac{40}{60}$ instead of $\frac{2}{3}$; $\frac{45}{60}$ instead of $\frac{3}{4}$; $\frac{48}{60}$ instead of $\frac{4}{5}$; and $\frac{50}{60}$ instead of $\frac{5}{6}$. Now if it be required to add together all these fractions $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, and $\frac{50}{60}$, we have only to add all the numerators, and under the sum place the common denominator 60; that is to say, we shall have $\frac{213}{60}$, or three integers, and $\frac{33}{60}$, or $3\frac{11}{20}$.

77. When it is required to subtract a fraction from an integer, it is sufficient to change one of the units of that integer into a fraction having the same denominator as the fraction to be subtracted; the rest of the operation is performed without any difficulty. If it be required, for example, to subtract $\frac{2}{3}$ from 1, we write $\frac{3}{3}$ instead of 1, and say, that $\frac{2}{3}$ taken from $\frac{3}{3}$ leave $\frac{1}{3}$. So $\frac{5}{12}$ subtracted from 1, leave $\frac{7}{12}$.

If it were required to subtract $\frac{3}{4}$ from 2, we should write 1 and $\frac{4}{4}$ instead of 2, and we should immediately see that after the subtraction there must remain $1\frac{1}{4}$.

78. It sometimes happens, that having added two or more fractions together, we obtain more than an integer; that is to say, a numerator greater than the denominator; for example $\frac{2}{3}+\frac{3}{4}$, or $\frac{8}{12}+\frac{9}{12}$, makes $\frac{17}{12}$, or $1\frac{5}{12}$. We have only to perform the actual division of the numerator by the denominator, to see how many integers there are for the quotient, and to set down the remainder. Nearly the same must be done to add together numbers compounded of integers and fractions; (called mixed numbers;) we first add the fractions, and if their sum produces one or more integers, these are added to the other integers. For example, add together $3\frac{1}{2}$ and $2\frac{2}{3}$; we first take the sum of $\frac{1}{2}$ and $\frac{2}{3}$, or of $\frac{3}{6}$ and $\frac{4}{6}$. It is $\frac{7}{6}$ or $1\frac{1}{6}$; then the sum total is $6\frac{1}{6}$.

Multiplication and Division of Fractions.

ART. 79. The rule for the multiplication of a fraction by an integer is to multiply the numerator by the given number, and not to change the denominator, thus:

2 times, or twice $\frac{1}{2}$ makes $\frac{2}{2}$, or one integer.

But instead of this rule, we may use that of dividing the denominator by the given integer; and this is preferable when it can be used, because it shortens the operation. If it be required for example, to multiply $\frac{8}{9}$ by 3; we divide the denominator by the integer, and find immediately $\frac{8}{3}$ or $2\frac{2}{3}$ for the given product.

80. We have shown how a fraction is to be multiplied by an integer; let us now consider how a fraction is to be divided by an integer. It is evident if we have to divide the fraction $\frac{2}{3}$ by 2, that the result must be $\frac{1}{3}$; and that the quotient of $\frac{6}{7}$ divided by 3 is $\frac{2}{7}$. The rule therefore is, to divide the numerator by the integer without changing the denominator. Thus:

$\frac{12}{25}$ divided by 2 gives $\frac{6}{25}$.

This rule may be easily practiced, provided the numerator be divisible by the number proposed; but very often it is not: for example, to divide $\frac{3}{4}$ by 2, we should change the fraction into $\frac{6}{8}$, and then dividing the numerator by 2, we should immediately have $\frac{3}{8}$ for the quotient sought.

81. When a fraction is to be divided by an integer, we have only to multiply the denominator by that number, and leave the numerator as it is. Thus $\frac{5}{9}$ divided by 4 gives $\frac{5}{36}$.

This operation becomes easier when the numerator itself is divisible by the integer. For example, $\frac{9}{16}$ divided by 3 would give, according to the last rule, $\frac{9}{48}$; but by the first rule, which is applicable here. we obtain $\frac{3}{16}$, an expression equivalent to $\frac{9}{48}$, but more simple.

Hence the following rule for multiplying fractions:—multiply separately the numerators, and the denominators together. Thus: $\frac{1}{2}$ by $\frac{2}{3}$ gives the product $\frac{2}{6}$, or $\frac{1}{3}$.

82. It remains to show how one fraction may be divided by another. We remark first, that if the two fractions have the same number for a denominator, the division takes place only with respect to the numerators; it is evident, that $\frac{3}{12}$ is contained as many times in $\frac{9}{12}$ as 3 in 9, that is to say, thrice; and in the same manner, in order to divide $\frac{8}{12}$ by $\frac{9}{12}$, we have only to divide 8 by 9, which gives $\frac{8}{9}$.

83. But when the fractions have not equal denominators, we must have recourse to the method already mentioned for reducing them to a common denominator.

Hence the following rule: Multiply the numerator of the dividend by the denominator of the divisor, and the denominator of the dividend by the numerator of the divisor; the first product will be the numerator of the quotient, and the second will be its denominator.

84. Applying this rule to the division of $\frac{5}{8}$ by $\frac{2}{3}$, we shall have the quotient $\frac{15}{16}$; the division of $\frac{3}{4}$ by $\frac{1}{2}$ will give $\frac{6}{4}$, or $\frac{3}{2}$ or 1 and $\frac{1}{2}$.

85. This rule is often represented in a manner more easily remem-

bered as follows: Invert the fraction which is the divisor, so that the denominator may be in the place of the numerator, and the latter be written under the line: then multiply the fraction, which is the dividend by this inverted fraction, and the product will be the quotient sought. Thus $\frac{3}{4}$ divided by $\frac{2}{3}$ is the same as $\frac{3}{4}$ multiplied $\frac{3}{2}$.

86. Every number when divided by itself produces unity, and it is evident that a fraction divided by itself must also give 1 for the quotient. The same follows from our rule, for, in order to divide $\frac{3}{4}$ by $\frac{3}{4}$, we must multiply $\frac{3}{4}$ by $\frac{4}{3}$, and we obtain $\frac{12}{12}$ or 1; and if it be required to divide $\frac{a}{b}$ by $\frac{a}{b}$, we multiply $\frac{a}{b}$ by $\frac{b}{a}$; now the product $\frac{ab}{ab}$ is equal to 1.

87. We have still to explain an expression which is frequently used. It may be asked, for example, what is the half of $\frac{3}{4}$; this means that we must multiply $\frac{3}{4}$ by $\frac{1}{2}$. So likewise, if the value of $\frac{2}{3}$ of $\frac{5}{8}$ were required, we should multiply $\frac{5}{8}$ by $\frac{2}{3}$, which produces $\frac{10}{24}$; and $\frac{3}{4}$ of $\frac{9}{16}$ is the same as $\frac{9}{16}$ multiplied by $\frac{3}{4}$, which produces $\frac{27}{64}$.

A DICTIONARY OF

ARITHMETICAL TERMS.

A.

A, the first letter of the alphabet, denotes one or unity. A. is an abbreviation of *Anno*, or *Ante*, as A. D. Anno Domini, in the year of our Lord.

A. or Ans. when annexed to an example, is a contraction for Answer, the result of an operation.

A. B. &c. are used hypothetically for different individuals, as A. bought of B. &c.

Abate, to lessen, to diminish.

Absolute, real value; without being joined or connected with any thing else.

Abstract, separate, distinct from any thing else. Abstract numbers are those used without application to things, as 6, 7, 8, 10: but when applied to any thing, as 6 feet, 7 men, they become *concrete* numbers.

Account, the bill of a tradesman; the statement of a merchant's dealings.

Addition, the first of the four fundamental rules in Arithmetic, whereby several small sums are added or collected into one that is larger. The only method of proving addition, which can properly be called a proof is by subtraction.

Ad infinitum, indefinitely, or to infinity.

Ad valorem, according to the value.

Advance, net gain; money paid before the equivalent is received.

Adulteration, the debasing of coin by mixture of impure metals.

Aggregate, formed by a collection of particular numbers into one sum; it differs from compound.

Aliquant parts, such numbers as will not divide or measure a whole number exactly, as 7, which is the aliquant part of 16.

Aliquot parts, such part of a numas will divide or measure a whole number exactly, as 2, the aliquot part of 4; 3 of 9.

All, the whole number, every one. The whole quantity; every part.

Alligation, a rule teaching how to compound several ingredients for a design proposed. It is either medial or alternate. Questions of this kind are better solved by algebra.

Alloy, a proportion of any baser metal mixed with one that is finer.

Alternate, to change one thing for another reciprocally.

A. M. Anno Mundi, in the year of the world.

Amount, the sum total of several quantities; the interest and principal added together.

Analysis, is the method of finding out truth, and synthesis is the method of explaining that truth to others.

Annex, to unite to at the end; thus, annex two ciphers to $27, and it becomes 2700 cents.

Annuity, a periodical payment; annuities generally are yearly, half-yearly, or quarterly. Annuities are of two kinds, *certain* and *contingent*.

Answer, to correspond to, to suit with; to be equivalent to; the result of an operation.

Antecedent, the *first term* of a *ratio* is called the *Antecedent*, the *second* the *Consequent;* the first and last terms of a *proportion* are called the *Extreme terms*, the second and third the *Mean terms*.

Arithmetic, the art of numbering or computing by certain rules, of which the four first and simplest are addition, subtraction, multiplication and division.

Arithmetic is properly the science which treats of numbers. It is called a science, because it investigates the properties upon which its rules depend; and an art, because it shows how to apply these rules to practice. Hence the *theory* of arithmetic explains the properties of numbers; *practical* arithmetic applies these properties to the performance of operations. Logarithmic arithmetic, is that which is performed by tables of logarithms. Vulgar arithmetic, is that which is conversant about integers and vulgar fractions. Decimal arithmetic, is the doctrine of decimal fractions. Political arithmetic, is the application of arithmetical calculations to political uses; as in estimating the revenues, resources, and population of a country.

Assemblage, the uniting or joining of things together, or the things so united or joined.

Augment, to increase in value.

Axiom a self evident proposition, or one requiring no proof, as that 'the whole is greater than the part.'

B.

B. C., an abbreviation for Before Christ.

Balance of Trade, a term in commerce denoting the equality between the value of the commodities bought of foreigners, and the value of the native productions transported into other countries. Balance in a merchant's account is when the debtor and creditor's account are made even. Bankrupt, a trader who fails or breaks, so as to be unable to carry on his business or pay his debts.

Barter, the exchanging of one commodity for another.

Bill of Exchange, a note containing an order for the payment of a sum of money, to a person called the drawer, who when he has signed it with his name, and written the word accepted, is called the acceptor. The person in whose favor it is drawn, or to whom it is ordered to be paid, is called the drawee, or payee, who when he has indorsed it, is called the indorser. He who is in possession of the bill, is the holder.

Bill of Lading, or Invoice, a deed signed by the master of a ship, by which he acknowledges the receipt of the merchant's goods, and obliges himself to deliver them at the place to which they are consigned.

Biquadrate, the fourth power arising from the multiplication of a number by itself four times.

Bond, an obligation or covenant in writing to pay any sum, or perform any contract.

Broker, one who concludes bargains, or contracts for merchants, as exchange brokers, ship brokers, &c.

Brokerage, what is paid to a broker for his trouble.

Bullion, gold or silver in the mass, before it is wrought into coin.

C.

A. C., an abbreviation for After Christ.

Capital, is the stock or fund of a trading company; or the sum of money they jointly contribute to be employed in trade.

Carat, the weight of 24 grains; or one scruple, 24 carats makes one ounce. This is the weight by which the fineness of gold is distinguished.

Carry, adding one to each and every number that exceeds 9: or it is dividing compound numbers by different denominations. The reserving of the tens, hundreds, &c. and adding them with the other tens, hundreds, &c. is carrying.

Chaldron, a dry measure, consisting of 36 bushels.

Cipher, a character, (0) which shows the want of a number; (100) which supply a vacancy; (104) which when annexed to figures multiplies them by ten; (14,140) which if cut off from figures to which it is annexed, diminishes or divides that number by ten; (150,15|0.) It is not mentioned in adding: it is of no value when in the subtrahend, but in the minuend it is treated as other numbers are: in multiplication, when at the right hand of either of the factors, it is omitted in the operation, and annexed to the product: in the quotient it shows that the divisor is not contained in the dividual. In decimals, it serves to bring figures to the situation which shows their relative value, and diminishes its value tenfold; two a hundred fold, &c. for they remove the fraction further from unity, which always stands immediately before the point.

Cloff, is an allowance of two pounds on every 3 cwt.

Coin, a piece of metal stamped with certain marks, and made current at a certain value.

Commensurable, such numbers as may be measured by one and the same common divisor.

Commodity any merchandise or ware which a person trades in.

Common denominator, when the denominators of two or more fractions are alike. The process by which they are made alike, is called *reducing* them to a *common* denominator. When two or more numbers can be divided by the same number without a remainder, that number is called their *common divisor.*

Company, in commerce, a trading association having a joint stock.

Component, parts that make up the whole; as 12 × 2 are the component parts of 24.

Compound Interest, is that interest which arises from principal and interest put together.

Composite, a number which may be divided by some number less than itself, but greater than unity, as 6, 8, 9, 10, &c.

Consequent, the latter of the two terms of proportion.

Consignment, the sending or delivering over of goods to another person.

Contraction, the shortening of operations.

Converse, directly the opposite in principle, as subtraction is the converse of addition.

Cube, a figure of six equal sides, a number multiplied into itself and then again into the product, as $3\times3=9\times3=27$, the cube.

Cube root, the side of a cube, thus, 3 is the cube root of 27.

Currency, the local value of money.

D.

Deed, a written indenture, purporting to convey lands or other property, on certain specified conditions. The writing, signing and sealing, completes the deed, but it takes no effect until delivered.

Decimal, ten fold; increasing in a ten fold proportion.

Decimals, fractions whose denominator is unity with any number of ciphers annexed.

When a decimal is expressed either by the continual repetition of the same figure, or of the number expressed by two or more figures, it is called a *circulating decimal;* if it be continued to an unlimited number of figures, is called an *interminate decimal.* The figures are called *repetends,* and if one repeats, it is called a *single repetend;* a *compound repetend* hath the same figures circulating alternately; if other figures arise is called a *mixed repetend.*

Dem. contraction of demonstration, a proof, or chain of arguments serving to prove the truth.

Denominations, different parts which make up a compound number, as pounds, ounces, drams.

Denominator, that part of a fraction which stands below the line, as [illegible] in the fraction [illegible].

Difference what remains when one number is subtracted from another; the remainder in subtraction.

Digit from the latin word *digitus,* which signifies a figure; either of the nine significant figures—1, 2, 3, 4, 5, 6, 7, 8, 9. Taken collectively they are called 'the nine digits.'

Discount, an allowance made for paying a debt before it is due.

Dividend, the number proposed to be separated into equal parts.

Dividend of Stock, is a share or proportion of the interest or profit of stocks invested.

Dividual, a part of the dividend in division.

Division, one of the four first rules of arithmetic, by which we find how often one number is contained in another. Division is proved by multiplying the divisor by the quotient.

Divisible, that which may be divided without a remainder.

Divisor, the number by which the dividend is separated into parts.

Draft, a bill drawn by one person on another, for a sum of money.

Drawback, an allowance made to merchants on the exportation of goods which paid duty inward.

Duodecimal, increasing in a twelve fold proportion.

Duodecimals, a rule used in computing the contents of superficies and solids.

Duty, a certain sum paid to government for articles imported.

E.

Entire, whole, undivided unbroken, complete in all its parts.

Equation of Payments, a rule for finding a time when if a sum be paid which is equal to the sum of several others due at different times, no loss will be sustained by either party.

Evolution, the extraction of the root of any power.

Exchange, the method of finding what quantity of the money of one place is equal to a given sum of another, according to a given course of exchange.

Exchange Brokers, men who give merchants information how the exchange goes.

Excise Duties, inland taxes on commodities of general consumption.

Exports, goods sent out from one's own country.

Extraction, the finding out the true root of any number.

Extremes, the first and last terms of a proportion.

F.

Factor, an agent commissioned by merchants to buy or sell goods on their account.

Factors, numbers multiplied together which produce a product.

Fellowship, a rule by which the loss or gain of each partner in a joint stock, is discovered.

Figure, one of the nine digits, as 1, 2, 3, &c. When figures stand for whole things, they are called Integers, or whole numbers; but when they stand for parts of things, they are called fractions, or something less than a unit.

Formula, a general theorem or statement for resolving questions in proportion, thus 6 : 18 :: 20

Fraction, a part of an integer—there are vulgar fractions, as $\frac{1}{2}$, $\frac{3}{4}$, which are called proper fractions; improper, as $\frac{8}{2}$; compound, as $\frac{2}{3}$ of $\frac{3}{4}$, &c.; mixed, as $4\frac{1}{4}$; and decimals, written ,5 ,05 2,50

Fund, the capital or stock invested in trade belonging to a public company.

G.

Gross Weight, the weight of goods together with the cask which contains them.

I.

Incommensurable, a term applied to one or more numbers which have no common measure by which they can be divided.

Index, the number that shows to what power the number is to be raised, as in 10^3, the figure ^{3} is the exponent or index.

Indicate division, to place the dividend over the divisor, thus $\frac{36}{3}$.

Inequality, the difference between two numbers.

Indorsing, writing on the back of a bill of exchange, check, or note.

Integer, a whole number, as 1, to distinguish it from a fraction, as $\frac{1}{2}$.

Integral Parts, parts of which the whole is made up.

Interest, money given for the use or loan of money.

Interest, a rule by which interest is computed, which is either simple or compound. It is generally calculated at so much on a hundred; as $100 is worth $6. When the principal and interest are incorporated; it is called compound interest.

Invoice, a bill or account of goods sent by a merchant to his correspondent in a foreign country.

Involution, the raising of any number to a given power, by multiplying it into itself the required number of times; thus, the cube of 4 is got by multiplying 4, the root, into itself twice, as 4×4×4=64.

Irreducible, any thing that cannot be separated into parts.

J.

Joint Stock, a stock or fund, formed by the union of several shares from different persons.

L.

Life Annuities, annual payments depending on the life of another.

M.

Magic Square, figures so disposed into parallel and equal ranks, that the sum of each row, diagonally and latterally, shall be equal.

4	9	2
3	5	7
8	1	6.

Mathematics, is that science which teaches or contemplates whatever is capable of being numbered or measured, in so far as it is computable or measurable; it accordingly is subdivided into arithmetic, which has numbers for its object, and geometry, which treats of magnitudes.

Mean arithmetical, half the sum of the extremes.

Mean geometrical, the square root of the product of the two extremes.

Measure, a certain number or quantity, which being repeated a certain number of times is equal to another that is larger, to which it has relation, as 7 is the measure of 56.

Mensuration, the art of measuring lines, superficies, and solids.

Minuend, the number which *has a number subtracted from it.*

Minus, means less,—plus, more.

Multiplicand, the number to be added to itself or repeated.

Multiplier, the number that repeats the multiplicand.

Multiple, a number which includes another a certain number of times, as 6, the multiple of 2.

Multiplication, one of the four simple rules of arithmetic, which consists in *repeating a number as often as there are units in another number*. The proper proof of multiplication is by division.

Multiplicand, the number to be added to itself or repeated.

Multiplier, the number that repeats the multiplicand.

N.

Negative quantities, quantities to be subtracted.

Negative sign, the sign of subtraction, marked thus, —, thus; 8—2=6.

Net weight, the weight of a commodity without the bag, &c.

Notation, the method of expressing numbers or quantities by characters appropriated for that purpose.

Note, a promissory note is a written evidence of debt, with an engagement to pay.

Number, an assemblage of several units, or of several things of the same kind. Whole numbers are otherwise called integers, as 1, 2, 3. Broken numbers are fractions, as ½. Cardinal numbers express the number of things, as 1, 2, 3. Ordinal numbers denote the order of things, as 1st, 2d, 3d. Even numbers are those which may be divided into two equal parts without a fraction, as 4, 8, 10. Uneven or odd numbers, are such as leave a remainder after being divided by 2, as 3, 5, 7. Prime numbers are those which cannot be represented by factors; or in other words, which cannot be divided without a remainder, as 2, 3, 5, 7, 13. Compound numbers are such as may be represented by factors, as 4, 6, 8, 9, 10, 12. A concrete number expresses or denotes a particular subject, as *three* men; but when we use no subject, as *three*, or *four*, we use the term in the abstract. A *composite* number, a number which is a product of two or more numbers. A *perfect number* is that which is equal to the sum of all its aliquot parts. A square number is the product of a number multiplied by itself, as 9, the product of 3×3. A cubic number is the product of a square number multiplied by its root, as 9 multiplied by its root 3, makes 27.

Numeration, the art expressing in figures any number proposed in words, or expressing in words any number expressed in figures.

Numerator, in a fraction, the number above the line; it denotes the number of the given parts taken, as 1 in the fraction ¼.

Numerical, relating to numbers, that which is performed by the help of numbers.

O.

Order, a species of draft implying a command, as from A. to B. to pay over, deliver money, or some specific article to a third person, as C.:—there are several kinds of them.

P.

Par, equality; worth the nominal value. A bank note that passes current for the amount mentioned thereon, is at par; if not worth so much, it is below par. Specie or stock sometimes commands a premium on its original or regular price; it is then above par.

Per, by the; as per cent, by the hundred: per annum, by the year.

Per centum or per cent, rate.

Plus, i. e. more, a term to denote addition, marked by the sign +, as $4+5=9$, that is, 4 plus 5 equal to 9.

Policy, a deed of insurance.

Position, a rule where supposed numbers are used in solving questions.

Power, the product of a number multiplied by itself any number of times.

Premium, a reward for the use of money, &c. at so much per cent.

Prime, one-twelfth of a foot.

Principal, the sum on which interest is computed.

Proceeds, the sum arising from the sale of goods, &c.

Product, the number produced by multiplying two or more numbers together, as $7\times 6=42$, the product required.

Profit, net gain.

Progression, a series of numbers keeping a certain ratio among themselves: when they decrease or increase with equal differences, it is called Arithmetical Progression, as 1, 3, 5, 7; when they increase by a common multiplier, or decrease by a common divisor, it is called Geometrical Progression, as 1, 2, 4, 8, 16.

Proof, an operation whereby the truth and justness of a calculation are examined and ascertained.

Proportion, the equality or similitude of ratios: thus the four numbers, 4, 8, 15, 30, are proportionals, or in proportion, because the ratio of 4 to 8 is equal or similar to that of 15 to 30, it being in both cases as 1 to 2. Between Proportion and Ratio, there is this difference; proportion consists always of four terms, and ratio of only two.

Proposition, a thing proposed to be demonstrated, proved, or made out, either a problem or theorem.

Q.

Quantity, any thing capable of estimation or mensuration, which, being compared with another thing of the same nature, may be said to be greater or less, equal or unequal to it.

Quotient, what results from dividing any number by another, as $12\div 4=3$ quotient.

R.

Radix, the number which is selected to constitute units of the higher orders, in the system of numeration.

Ratio, it means *in what relation does one number stand to another.* When quantities have the *same ratio*, they are said to be *proportional* to each other.

Reversion, an annuity that does not come into possession till a given time has elapsed.

Receipt, a discharge from debt, either in full or in part.

Reduction, a rule by which money, weights, and measures are changed from one denomination to another.

Remainder, see difference.

Resolution, a method by which the truth or falsehood of a proposition is discovered.

Root, a number which being multiplied into itself a certain number of times will produce the given number, as the cube root of 8 is 2, and the square root 4.

Rule, a certain maxim to be observed in any art or science, or principle to go by.

Rule of Three, or Proportion, teaches by means of three numbers to find a fourth.

S.

Sexagesimal, is that which proceeds by 60's, as the division of circles, &c. into 60 degrees, the degrees each into 60 minutes, and every minute into 60 seconds.

Sign, any mark used in operations, as + for addition, — for subtraction, × for multiplication, ÷ for division, = for equality.

Square Root, see Root.

Stock, shares in the funds of a bank, or trading establishment. The *nominal value* of a share is the amount paid, when the stock was first created. The *real value* is the sum for which a share *will sell*.

Stock Broker, a dealer in stock.

Stock exchange, the place where stock is bought and sold.

Submultiple, a number contained in another a certain number of times exactly, as 2, 3, 4, or 6 in 12.

Subtraction, the second primary rule in arithmetic, by which a less sum is taken from a greater.

Subtrahend, the number to be subtracted.

Superficies, a magnitude bounded by lines.

Surd, a number that is incommensurable, as the square root of 2.

Suttle, what remains after one or two allowances are deducted.

T.

Tare, is an allowance to the buyer for the weight of the box, barrel, bag, &c. which contains the goods bought.

Tret is an allowance of 4 lb. in every 104 lb. for waste, dust, &c.

Theory, a doctrine which confines itself to the speculative parts of a subject, without regard to its practice or application.

U. V. Z.

Unit, or unity, a figure expressing the least whole number, 1.

Unity is that by which every thing in nature is called one.

Usury, taking more interest for money than is allowed by law.

V. for *vide*, consult, see.

Zero, the character 0.

THE KEY,

Containing Answers referred to in the Preceding Pages.

Page.	*Sect.*	Ex.	Answer.	Ex.	Answer.	Ex.	Answer.
32	§3	1	225253884	2	280168977	3	270336348
38	§7	1	475265	2	611865	3	294312
"	"	4	287202	5	233945	6	607523
48	§17	0	77420006	0	1311522	0	822101
50	§19	1	3768079	2	339790188	3	590180075
"	"	4	106799280	5	9001		
"	"	6	368489577655733811				
"	"	7	250188778	8	12896	9	2002
"	"	10	65			11	8999
"	"	12	909091	13	8999899	14	94760000
51	"	15	66643	16	6013568	17	3493688
"	"	18	32102	19	1190972871	20	68789
"	"	21	1367789	22	10008882	23	2309513
"	"	24	782709963				
"	"	25	2210717091 2				
"	"	26	14454578112778				

Page.	*Sect.*	Ex.	Quotients.	Ex.	Quotients.
66	§30	2	10293—13152	3	21303—10523
"	"	4	20340—64140	5	17865 —6188
"	"	6	188058—157775	7	118454—105145
"	"	8	15506—11917		

RECAPITULATION,—Page, 85, 86, 87.

Ex.	Answer.	Ex.	Answer.	Ex.	Answer.
1	2431	9	857	17	65 *acres.*
2	6	10	4315	18	(*see No.* 18* *below.*)
3	6. 777	11	203*dols.*	19	4664*lbs.*
4	1865	12	1554	20	187785*lbs.*
5	31*dols.*	13	48	21	114*lbs.*
6	28463563	14	10000000	22	300*dols.*
7	9 *boots* ; 3 *shoes.*	15	109	23	————
8	821	16	292	24	5713*dols.*

25 amount of years, 5223
26 *Ans.* A. 46 *dols.* ; B. 92 ; C. 368 [11 shares.
27 *Ans.* 95 *dols.* [first take 15 from 300 dols.
28 *Ans.* A. 30 ; B. 33 ; C. 37 [find 10 and subt. it from 100 dols.
29 *Ans.* 2d 150 *dols.* ; 1st 200 ; 3d 250 [4 shares.
30 *Ans.* A. 317 ; B. 951 ; C. 1268 ; D. 2219 [15 shares.
31 *Ans.* har. 40 *dols.* ; hor. 80 ; ch 240 [9 shares.
18* *Ans.* sug. 34 *lbs.* ; coffee 220 *lbs.* ; tea 102 *lbs.* ; pep. 4 *lbs.* ; mace 1 *oz.* ; rice 31 *lbs.* ; oil 12 *gals.*

FEDERAL MONEY.

Page.	Ex.	Answer.	Page.	Ex.	Answer.
89	7	$999 : 99'9	90	9	$1101 : 06
89	8	$39 : 59'5	90	10	$9999 : 99
90	11	$40 : 71			

PROPORTION. PARTICULAR CASES,—Page 182.

Ex. 1, 25 : 15 :: 10 Ex. 2, 20 : 1 :: 500
Ex. 3, 33 : 18 :: 15 Ex. 4, 23 : 10 :: 13

A series of forms of notes, receipts, &c.—exhibiting a species of *knowledge which should be familiar to every man in community.*

1. Promissory Notes.

1. *Form of a Note on demand.*

$100 : 00 For value received, I promise to pay A. B., or order, one hundred dollars, on demand, with interest. P.
Hartford, Oct. 15, 1833.

2. *Form of a Note on time.*

One day after date, for value received, I promise to pay A. B. or bearer, at my store in Hartford, one hundred dollars, with interest.
Hartford, Oct. 15, 1833. P.

3. *Form of a Note on settlement of Account.*

$100 : 00 This day reckoned and settled book accounts with A. B., and found due to him a balance of one hundred dollars, which I hereby promise to pay to him or his order, in sixty days from date, with interest.
Hartford, Oct. 15, 1833. P.

4. *Promissory Note on time.*

$100 : 00 Ninety days after date, for value received, I promise to pay to the order of A. B., at the bank of Hartford, one hundred dollars.
Hartford, Oct. 15, 1833. P.

5. *A joint and several Note by three persons.*

$100 : 00 For value received, we jointly and severally promise to pay to the order of A. B., at the bank of Hartford, four months after date, one hundred dollars. H. P.
Hartford, Oct. 15, 1833. C. C.
D. D.

6. *Note of hand for a specific article.*

Six months after date, I promise to pay A. B., at his store in Hartford, one hundred bushels of merchantable wheat, at ninety-five cents a bushel. Hartford, Oct. 15, 1833. P.
100 bushels of wheat.

2. Receipts.

1. *Receipt for money on Account.*

Received, Hartford, Oct. 15, 1833, of A. B., the sum of one hundred dollars, to apply on account. P.
$100 : 00

2. *A receipt in full.*

Received, Hartford, Oct. 15, 1833, of A. B., the sum of one dollar, in full of all demands to this date. P.
$1 : 00

3. *A partial payment on a note or bond.*

$50 : 00 Received, Hartford, Oct. 15, 1833, of A. B., fifty dollars in part payment of the within obligation. P.

4. *Receipt for interest on a bond.*

$50 : 00 Received, Hartford, Oct. 15, 1833, of A. B., fifty dollars in full for one years' interest, due the first inst. on his bond, upon which the same is this day indorsed. P.

5. *A receipt for the payment of a lost obligation.*

$100 : 00 Received, Hartford, Oct. 15, 1833, of A. B., one hundred dollars in full for his note of the same amount, dated Hartford, at two months, payable to the order of C. D., and by him indorsed, which note appears now to be mislaid or lost. And I hereby obligate and bind myself to save harmless the said A. B., and C. D., or either of them, from all costs and damages which they or either of them, may sustain in regard to said note. P.

3. ORDERS.

1. *An order to pay money on sight.*

$100 : 00 Hartford, Oct. 15, 1833.

Sir :—On sight hereof; pay to A. B., or his order, the sum of one hundred dollars, being for value received, and place the same to the account of Your obedient servant,

C. D., Esq. P.

2. *Order and Discharge of Debt.*

$100 : 00 Hartford, Oct. 15, 1833.

Sir :—Please pay to the order of A. B., the sum of one hundred dollars, and that amount indorsed hereon, shall be your discharge in full of all demands, from

C. D., Esq. Your obedient servant,

P.

3. *The form of a Check.*

$100 : 00

Cashier of the Bank of Hartford, Oct. 15, 1833.

Pay A. B., or bearer,

one hundred dollars. P.

TABLES

OF WEIGHTS AND MEASURES.

TROY WEIGHT.

24	Grains (*gr.*)	make	1 Penny-weight,	*pwt.*
20	Penny-weights	"	1 Ounce,	*oz.*
12	Ounces	"	1 Pound.	*lb.*

By Troy weight are weighed gold, silver, jewels, and all liquors. The denominators are pounds, ounces, penny-weights, and grains.

In a pound Troy, are 5760 grains, 240 penny-weights, 12 ounces.

What is the use of Troy weight? The denominations? What do you carry by, in an example marked *lb. oz. pwt. gr.?* Place 24 over *gr.* 20 over *pwt.* 12 over *oz.* and nothing over *lb.*

APOTHECARIES' WEIGHT.

20	Grains (*gr.*)	make	1 Scruple,	*sc.*
3	Scruples	"	1 Dram,	*dr.*
8	Drams	"	1 Ounce,	*oz.*
12	Ounces	"	1 Pound,	℔.

Apothecaries' weight is used by apothecaries and physicians, in compounding medicine. The denominations are pounds, ounces, drams, scruples, and grains.

In a pound are 5760 grains, 288 scruples, 96 drams, 12 ounces.

What is the use of Apothecaries' weight? The denominations? What do you carry by in an example marked *lb. oz. dr. sc. gr.?* Place 20 over *gr.* 3 over *sc.* 8 over *dr.* 12 over *oz.*, and nothing over *lb.*

CLOTH MEASURE.

2¼	Inches (*in.*)	make	1 Nail,	*na.*
4	Nails	"	¼ Of a yd. or 1 qr.	*qr.*
4	Quarters	"	1 Yard,	*yd.*
3	Quarters	"	1 Ell Flemish,	*E. Fl.*
5	Quarters	"	1 Ell English,	*E. E.*
6	Quarters	"	1 Ell French,	*E. Fr.*

Cloth measure is used in selling cloths and other goods, sold by the yard, or ell. The denominations are ells, yards, quarters, and nails.

In a yard are 16 na. 1 ell Flemish, 12 na. 1 ell English, 20 na. 1 ell French, 24.

What is the use of Cloth measure? The denominations? What do you carry by in an example marked *yds. qrs. na.?* Place 4 over *na.* 4 over *qrs.* and nothing over *yds.*

LONG MEASURE.

3	Barley corns (*bc.*)	make	1 Inch,	*in.*
12	Inches	"	1 Foot	*ft.*
3	Feet	"	1 Yard,	*yd.*
5½	Yards, or 16½ feet	"	1 Rod, pole or perch,	*p.*
40	Poles	"	1 Furlong,	*fur.*
8	Fur. or 320 poles,	"	1 Mile	*mi.*
3	Miles,	"	1 League,	*L.*
60	Geo. or 69½ Stat. M.	"	1 Degree,	*deg.*
360	Degs. or equal parts,		1 Great circle of the earth.	

Long measure is used in measuring distances or other things, where length is considered without breadth.—The denominations are degrees, leagues, miles, furlongs, rods, yards, feet, inches, and barleycorns.

In a mile are, 190080 barleycorns, 63360 inches, 5280 feet, 1760 yards, 320 poles, 8 furlongs, 80 chains, 1056 paces.

What is the use of Long measure? The denominations? What do you carry by in an example marked *L. mi. fur. po.*? Place 40 over *po.* 8 over *fur.* 3 over *mi.* and nothing over *L.*

LAND MEASURE.

144	Inches (*in.*)	make	1 Foot,	*ft.*
9	Feet	"	1 Yard,	*yd.*
272¼	Feet, (*or* 30¼ *yds.*)	"	1 Rod or perch,	*rd.*
40	Rods, (*or* 14400 *ft.*)	"	1 Rood,	*r.*
4	Roods (*or* 57600) *ft.*	"	1 Acre,	*A.*
640	Acres	"	1 Mile,	*mi.*

Square measure is used in measuring land, and any other thing, where *length* and *breadth* are considered.—The denominations are miles, acres, roods, rods, perches, yards, feet, and inches.

In a square mile, are 4014489600 inches, 27878400 feet, 3097600 yards, 102400 poles, 2560 roods, 640 acres.

What is the use of Land Measure? The denominations? What do you carry by in an example marked *A. rd. r.?* Place 40 over *rd.* 4 over *r.* &c.

SOLID MEASURE.

1728	Inches (*in.*)	make	1 Solid foot,	*ft.*
27	Solid feet	"	1 Solid yard,	*yd.*
40 50	Ft. of round timber or Ft. of hewn timber,	make	1 Tun,	*T.*
128	Solid ft. or 8 ft. long, 4 high and 4 wide,	"	1 cord of wood,	*c.*

Solid measure is used in measuring things that have length, breadth and thickness; such as timber, wood, stone, bales of goods, &c. The denominations are cords, tuns, yards, feet and inches.

What is the use of Solid measure? The denominations? What do you carry by in an example marked *T. ft. in.?* Place 1728 over *in,* 40 over *ft.* &c.

LIQUID MEASURE.

4	Gills (*gi.*)	make	1	Pint,	*pt.*
2	Pints	"	1	Quart,	*qt.*
4	Quarts	"	1	Gallon	*gal.*
31½	Gallons	"	1	Barrel,	*bar.*
2	Barrels (*or* 63 *gals.*)	"	1	Hogshead,	*hhd.*
2	Hogsheads	"	1	Pipe,	*pi.*
2	Pipes, (*or* 252 *gals.*)		1	Tun,	*T.*

Liquid measure is used in measuring all spirituous liquors, ale and beer excepted, also, vinegar and oil. The denominations are tuns, pipes, hogsheads, barrels, gallons, quarts, pints, and gills.

In a tun are 2016 pints, 1008 quarts, 252 gallons, 8 barrels, 4 hogsheads.

What is the use of Liquid Measure? The denominations? What do you carry by in an example marked *hhd. gals. qts. pts.?* Place 2 over *pts.* 4 over *qts.* 63 over *gals. &c.*

DRY MEASURE.

2	Pints (*pts.*)	make	1	Quart,	*qt.*
4	Quarts	"	1	Gallon,	*gal.*
2	Gallons, (8 *qts.*)	"	1	Peck,	*pk.*
4	Pecks	"	1	Bushel,	*bu.*
36	Bushels	"	1	Chaldron,	*ch.*

Dry measure is used in measuring all commodities such as grain, fruit, roots, salt, coal, &c. The denominations are chaldrons, bushels, pecks, gallons, quarts, and pints.

In a chaldron are, 2304 pints, 1152 quarts, 288 gallons, 144 pecks.

What is the use of Dry measure? The denominations? What do you carry by in an example marked *bu. pks. qts. pts.?* Place 2 over *pts.* 8 over *qts.* 4 over *pk. &c.*

TIME.

60	Seconds (*sec.*)	make	1	Minute,	*min.*
60	Minutes	"	1	Hour,	*h.*
24	Hours	"	1	Day,	*da.*
7	Days	"	1	Week,	*w.*
4	Weeks	"	1	Month,	*mo.*
12	Months	"	1	Year,	*yr.*

The denominations of time are years, months, weeks, days, hours, minutes, and seconds.

In a year are, 31557600 seconds, 525960 minutes, 8766 hours, 365 days, 52 weeks. A Century is 100 Years.

What are the denominations of Time? What do you carry by in an example marked *yr. mo. w. da.?* Place 7 over *da.*, 4 over *w.*, 12 over *mo. &c.*

MOTION.

60 Seconds	(″)	make	1 Minute,	*min.*
60 Minutes		"	1 Degree,	*deg.*
30 Degrees		"	1 Sign of the Zodiac,	*S.*
12 Signs, or (360 deg.)		"	1 Great circle of the Zodiac.	

Motion is used in reckoning latitude and longitude; also in computing the revolution of the earth and other planets round the sun. The denominations are circles, signs, degrees, minutes, and seconds.

What is the use of motion? The denominations? What do you carry by in an example marked *S. deg. min.?* Place 60 over *m.* 30 over *deg. &c.*

TABLES OF PARTICULARS.

12 units make	1 Dozen.
12 dozen	1 Gross.
144 dozen	1 Great Gross.
20 units	1 Score.
5 score	1 Hundred.
24 sheets	1 Quire.
20 quires	1 Ream.
2 reams	1 Bundle.
10 reams	1 Bale.

A barrel of flour is lbs.	196
Pork is	200
Beef is	200
Gunpowder is	112
Raisins is	112
Figs from 98 to	108

30 days make 1 month.

365 Days } make 1 year.
52 Weeks }

Folio	2 leaves, or	4 pages make	1	sheet.
Quarto, 4to.	4	8	1	do.
Octavo, 8vo.	8	16	1	do.
Duodecimo, 12 mo.	12	24	1	do.
Octodecimo, 18 mo.	18	36	1	do.

Note. The smaller books are called 18's, 24's, 32's, 36's, &c. according to the number of leaves in a sheet.

Paper is called by the following names, viz.—Drawing, Writing, Printing, Sheathing, Wrapping, &c.

The sizes of Paper are designated by the following names, viz.—Pot, Fools-cap, Letter, Post-folio, Crown, Demy, Medium, Royal, Super-royal, Imperial, Elephant, &c.

Months.	Names.	Days.	Months.	Names.	Days.
1st Month,	January,	31	7th Month,	July,	31
2d -	February,	28	8th -	August,	31
3d -	March,	31	9th -	September,	30
4th -	April,	30	10th -	October,	31
5th -	May,	31	11th -	November,	30
6th -	June,	30	12th -	December,	31

TABLES OF ALIQUOT PARTS.

cts.		
50 =	1/2	Of a dollar.
25 =	1/4	
20 =	1/5	
12½ =	1/8	
6¼ =	1/16	
5 =	1/20	
4 =	1/25	
2½ =	1/40	
2 =	1/50	
1 =	1/100	

ms.		
5 =	1/2	Of a cent.
2 =	1/5	
2½ =	1/4	
1 =	1/10	

s.	d.		
10	0 =	1/2	Of a pound.
5	0 =	1/4	
4	0 =	1/5	
3	4 =	1/6	
2	6 =	1/8	
2	0 =	1/10	
1	8 =	1/12	
1	4 =	1/15	
1	3 =	1/16	
1	0 =	1/20	
0	6 =	1/40	

d.		
6 =	1/2	Of a shilling.
4 =	1/3	
3 =	1/4	
2 =	1/6	
1½ =	1/8	
1 =	1/12	
¾ =	1/16	
½ =	1/24	

PENCE TABLE.

d.		s.	d.	d.		s.	d.
20	=	1	8	80	=	6	8
30		2	6	90		7	6
40		3	4	100		8	4
50		4	2	110		9	2
60		5	0	120		10	0
70		5	10				

SHILLINGS TABLE.

s.		d.	s.		d.
1	=	12	7	=	84
2		24	8		96
3		36	9		108
4		48	10		120
5		60	11		132
6		72	12		144

A TABLE OF FEDERAL MONEY.

	N. E. cur.			*N. Y. cur.*				*N. E. cur.*			*N. Y. cur.*		
	$	cts.	m.	$	cts.	m.		$	cts.	m.	$	cts.	m.
*f*1	0	00	3	0	00	2	*s*4	0	66	7	0	50	2
2	0	00	7	0	00	5	5	0	83	3	0	62	5
3	0	01		0	00	8	6	1			0	75	
*d*1	0	01	4	0	01		7	1	16	7	0	87	5
2	0	02	8	0	02		8	1	33	3	1		
3	0	04	2	0	03	1	9	1	50		1	12	5
4	0	05	6	0	04	2	10	1	66	7	1	25	
5	0	07		0	05	2	11	1	83	3	1	37	5
6	0	08	3	0	06	2	12	2			1	50	
7	0	09	7	0	07	3	13	2	16	7	1	62	5
8	0	11	1	0	08	3	14	2	33	3	1	75	
9	0	12	5	0	09	4	15	2	50		1	87	5
10	0	13	9	0	10	4	16	2	66	7	2		
11	0	15	3	0	11	4	17	2	83	3	2	12	5
*s*1	0	16	7	0	12	5	18	3			2	25	
2	0	33	3	0	25		19	3	16	7	2	37	5
3	0	50		0	37	5	20	3	33	3	2	50	

TABLE OF FEDERAL MONEY.

N. E. Cur.			N. Y. Cur.	N. E. Cur.			N. Y. Cur.	N. E. Cur.			N. Y. Cur.
s.	d.	$cts.m.	$cts.m.	s.	d.	$cts.m.	$cts.m.	s.	d.	$cts.m.	$cts.m.
1	1	0 18 5	0 13 5	5	1	0 84 7	0 63 5	9	1	1 51 4	1 13 5
1	2	19 5	14 5	5	2	86 1	64 5	9	2	1 52 8	1 14 5
1	3	20 9	15 6	5	3	87 5	65 4	9	3	1 54 2	1 15 6
1	4	22 3	16 7	5	4	88 9	66 7	9	4	1 55 6	1 16 7
1	5	23 7	17 7	5	5	90 3	67 7	9	5	1 57	1 17 7
1	6	25	18 7	5	6	91 6	68 7	9	6	1 58 3	1 18 7
1	7	26 4	19 8	5	7	93	69 8	9	7	1 59 7	1 19 8
1	8	27 8	20 8	5	8	94 4	70 8	9	8	1 61 1	1 20 8
1	9	29 2	21 9	5	9	95 8	71 9	9	9	1 62 5	1 21 9
1	10	30 6	22 9	5	10	97 2	72 9	9	10	1 63 9	1 22 9
1	11	32	23 9	5	11	98 6	73 9	9	11	1 65 3	1 23 9
2	1	34 7	26	6	1	1 01 4	76	10	1	1 68 1	1 26
2	2	36 1	27	6	2	1 02 8	77	10	2	1 69 5	1 27
2	3	37 5	28 1	6	3	1 04 2	78 1	10	3	1 70 9	1 28 1
2	4	38 9	29 2	6	4	1 05 6	79 2	10	4	1 72 3	1 29 2
2	5	40 3	30 2	6	5	1 07	80 2	10	5	1 73 7	1 30 2
2	6	41 6	31 2	6	6	1 08 3	81 2	10	6	1 75	1 31 2
2	7	43	32 3	6	7	1 09 7	82 3	10	7	1 76 4	1 32 3
2	8	44 4	33 3	6	8	1 11 1	83 3	10	8	1 77 8	1 33 3
2	9	45 8	34 4	6	9	1 12 5	84 4	10	9	1 79 2	1 34 4
2	10	47 2	35 4	6	10	1 13 9	85 4	10	10	1 80 6	1 35 4
2	11	48 6	36 4	6	11	1 15 3	86 4	10	11	1 82	1 36 4
3	1	51 4	38 6	7	1	1 18 1	88 5	11	1	1 84 7	1 38 5
3	2	52 8	39 6	7	2	1 19 5	89 5	11	2	1 86 1	1 39 5
3	3	54 2	40 7	7	3	1 20 9	90 6	11	3	1 87 5	1 40 6
3	4	55 6	41 8	7	4	1 22 3	91 7	11	4	1 88 9	1 41 7
3	5	57	42 8	7	5	1 23 7	92 7	11	5	1 90 3	1 42 7
3	6	58 3	43 8	7	6	1 25	93 7	11	6	1 91 6	1 43 7
3	7	59 7	44 9	7	7	1 26 4	94 8	11	7	1 93	1 44 8
3	8	61 1	45 9	7	8	1 27 8	95 8	11	8	1 94 4	1 45 8
3	9	62 5	47	7	9	1 29 2	96 9	11	9	1 95 8	1 46 9
3	10	62 9	48	7	10	1 30 6	97 9	11	10	1 97 2	1 47 9
3	11	65 3	49	7	11	1 32	1 98 9	11	11	1 98 6	1 48 9
4	1	68 1	51 2	8	1	1 34 7	1 01	12	1	2 01 4	1 51
4	2	69 5	52 2	8	2	1 36 1	1 02	12	2	2 02 8	1 52
4	3	70 9	53 3	8	3	1 37 5	1 03 1	12	3	2 04 2	1 53 1
4	4	72 3	54 4	8	4	1 38 9	1 04 2	12	4	2 05 6	1 54 5
4	5	73 7	55 4	8	5	1 40 3	1 05 2	12	5	2 07	1 55 2
4	6	75	56 4	8	6	1 41 6	1 06 2	12	6	2 08 3	1 56 2
4	7	76 4	57 5	8	7	1 42	1 07 3	12	7	2 09 7	1 57 3
4	8	77 8	58 5	8	8	1 44 4	1 08 3	12	8	2 11 1	1 58 3
4	9	79 2	59 6	8	9	1 45 8	1 09 4	12	9	2 12 5	1 59 4
4	10	80 6	60 6	8	10	1 47 2	1 10 4	12	10	2 13 9	1 60 4
4	11	82	61 6	8	11	1 48 6	1 11 4	12	11	2 15 3	1 61 4

SHORT METHOD OF KEEPING ACCOUNTS.

Dr. *Solomon Thornton, in Account with Walter Underwood,* **Cr.**

1833.			$.	cts.
Feb.	3.	To 8 bushels of wheat, at $1 : 66	13	28
”	9.	To 29lb. beef, at 6½ cts.	1	88½
”	17.	To 3 bushels of rye, at $1 : 50	4	50
”	20.	To 5 bushels potatoes, at 25 cts.	1	25
”	25.	To 10 pair of shoes, at $1 : 75	17	50
March	8.	To 75lb. cheese, at 10 cts.	7	50
”	12.	To 3 thousand of clear boards, at 13 dols. a thousand,	39	00
”	18.	To 7 cords oak wood, at 3 dols.	21	00
”	25.	To 4 days' work done by myself and 1 yoke of oxen, at $1 : 40	5	60
May	7.	To 28 yds. tow-cloth, at 22 cts.	6	16
”	15.	To 19lb. butter, at 20 cts.	3	80
June	5.	To 2 days' work at hoeing. done by		
”	20.	myself, at 83 cts. a day.	1	66
”	29.	To 7lb. pork, 18½ cts.	1	29½
Sept.	25.	To 9 barrels of cider, at $1 : 33	11	97
			136	40

1833.			$.	cts.
March	8.	By 4 bush. salt, at 75 cts.	3	00
”	12.	By 1¾ cwt. flour, at 6 dols. per cwt.	10	50
”	19.	By 3 gal. M. wine, at $1 : 50	4	50
”	28.	By 18 yds. linen, at 40 cts.	7	20
April	2.	By 10lb. sugar, at 15 cts.	1	50
”	9.	By 2 pieces nankin, at $1 : 25	2	50
”	15.	By a side of sole-leather, weighing 28lb. at 20 cts.	5	60
”	23.	By 12 yds. broadcloth, at $4 : 50	51	00
”	29.	By 3 qts. Sherry wine, at 35 cts.	1	05
May	5.	By 4 doz. buttons, at 67 cts. a doz.	2	68
”	23.	By ½ cwt. of rice, at $3 : 34	1	67
July	5.	By a brass kettle, weighing 25lb. at 50 cts. per lb.	12	50
			106	70
Oct.	15.	By cash to balance,	29	70
			136	40

Oct. 15, 1833. Settled all book accounts heretofore contracted between us. WALTER UNDERWOOD,
SOLOMON THORNTON.

THE ARRANGEMENT.

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